

Superconducting circuits for quantum computing

Eisuke Abe

RIKEN Center for Quantum Computing

2026.8.31

RIKEN-NCHU Summer School

@National Chung Hsing University, Taichung, Taiwan

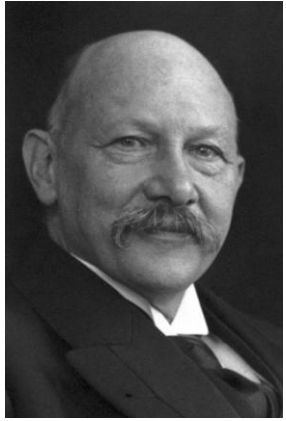
Contents

- Brief history: from discovery of superconductivity to dawn of quantum era
- Superconducting qubits
 - Qubit, coherence time, and protection from noise
 - Josephson junction
 - Plasma oscillation qubit: phase qubit and transmon (derived from charge qubit)
 - Loop-based qubit: flux qubit, fluxonium, and Blochonium (dual of transmon)
 - Protected qubit: $0-\pi$ qubit
- Quantum operations with transmons
 - Circuit QED
 - Dispersive readout
 - Single- and two-qubit gates
- Building quantum computers (→ Symposium talk)

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Superconductivity

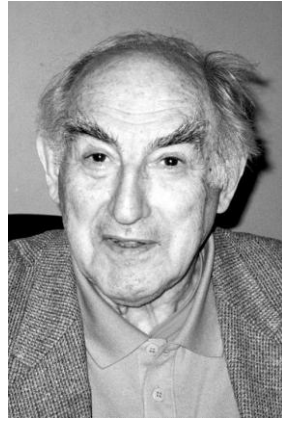


H. Kamerlingh Onnes



1913

1911: Discovery

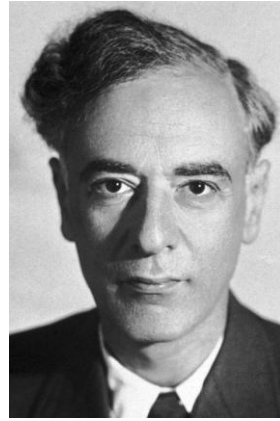


V. L. Ginzburg



2003

1950: GL theory (phenomenological)



L. D. Landau



1962

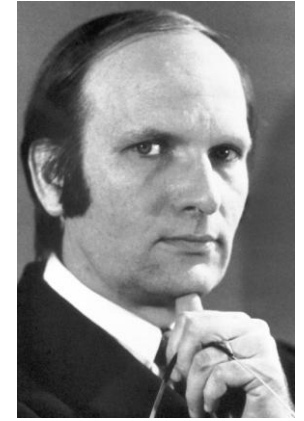
Superconductor



J. Bardeen



L. N. Cooper

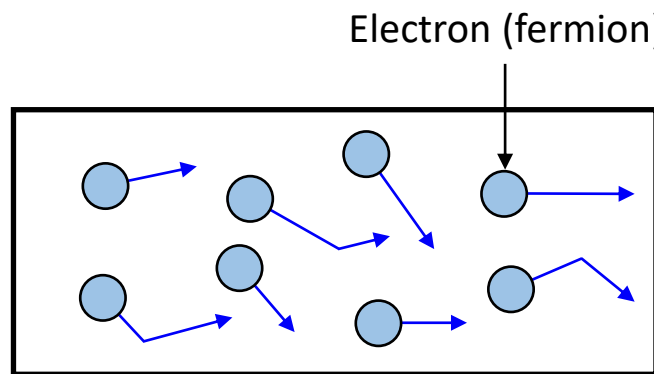


J. R. Schrieffer

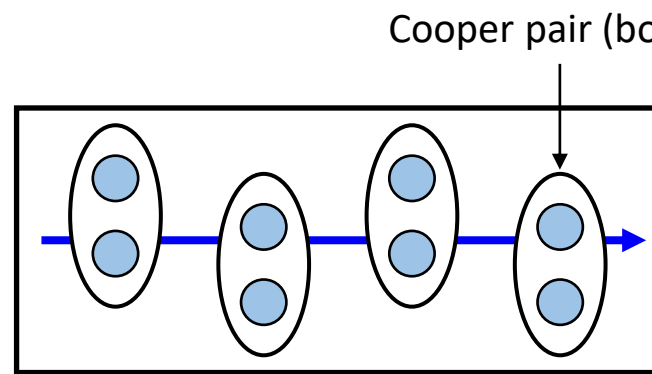


1972

1957: BCS theory (microscopic)

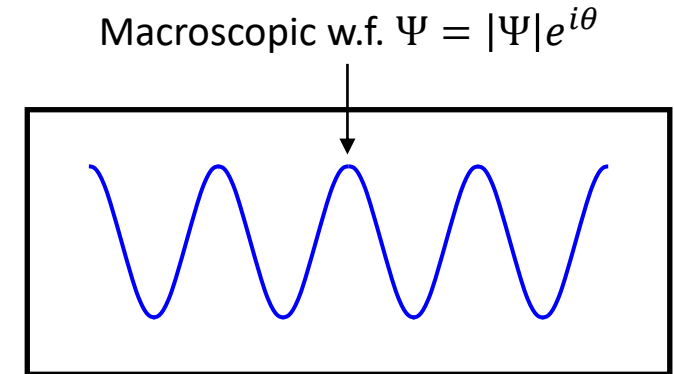


Metal

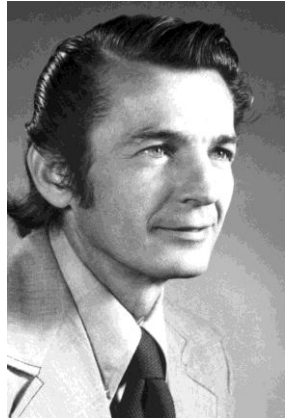


Superconductor

=



Superconducting gap and tunneling



I. Giaever



1973

1960: Single-electron tunneling

Phys. Rev. Lett. 5, 147 (1960) I. Giaever

VOLUME 5, NUMBER 4

PHYSICAL REVIEW LETTERS

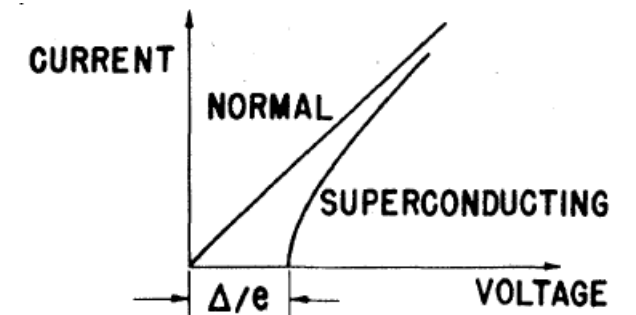
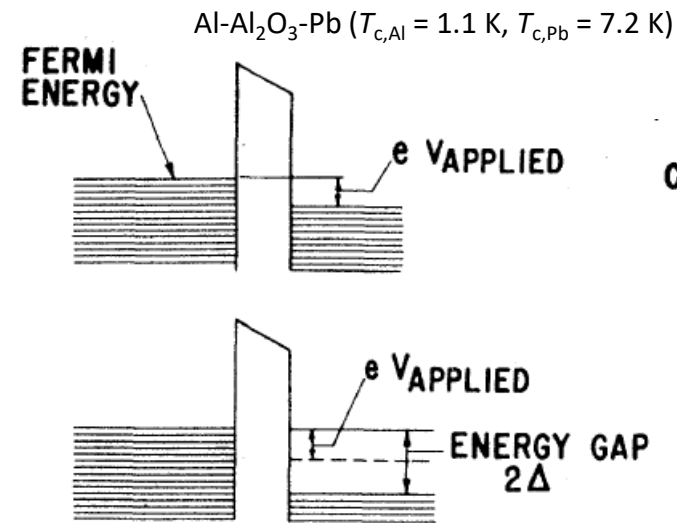
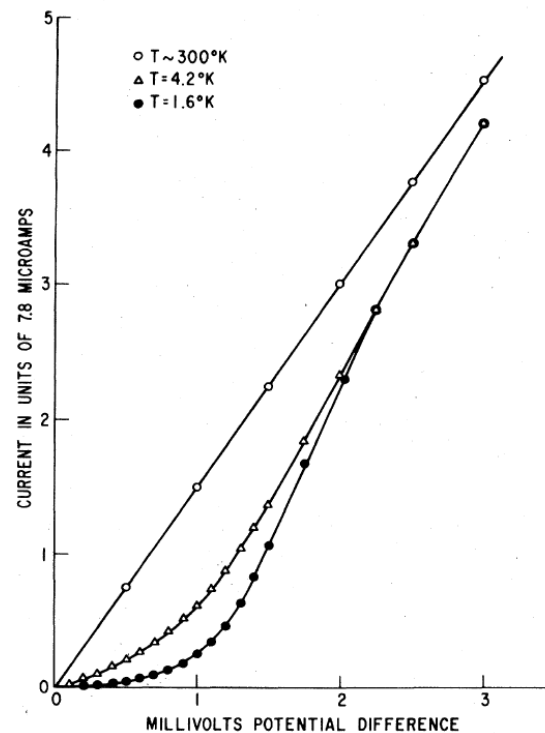
AUGUST 15, 1960

ENERGY GAP IN SUPERCONDUCTORS MEASURED BY ELECTRON TUNNELING

Ivar Giaever

General Electric Research Laboratory, Schenectady, New York

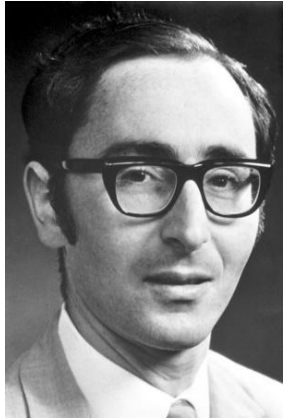
(Received July 5, 1960)



Rev. Mod. Phys. 46, 245 (1974) I. Giaever

Josephson effect

Phys. Lett. **1**, 251 (1962) B. D. Josephson



B. D. Josephson



1973

Volume 1, number 7

PHYSICS LETTERS

1 July 1962

POSSIBLE NEW EFFECTS IN SUPERCONDUCTIVE TUNNELLING *

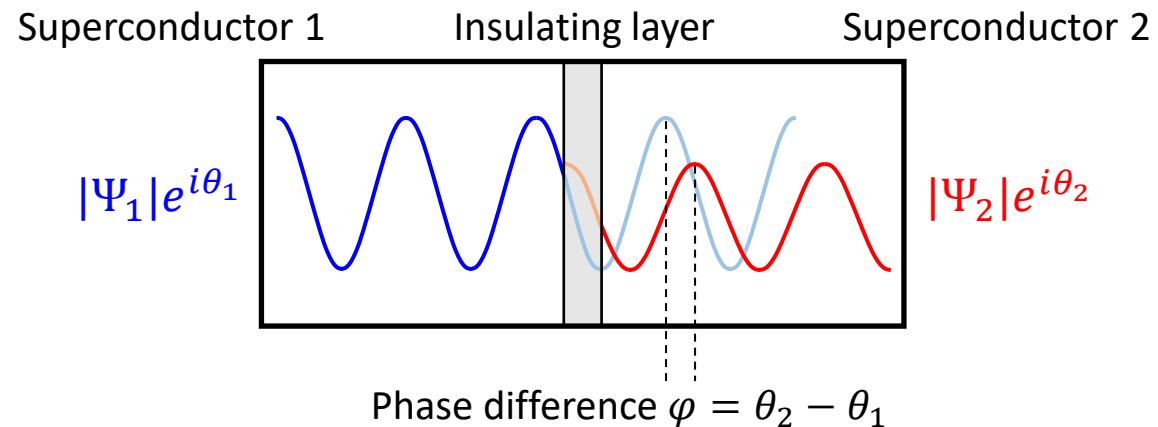
B. D. JOSEPHSON

Cavendish Laboratory, Cambridge, England

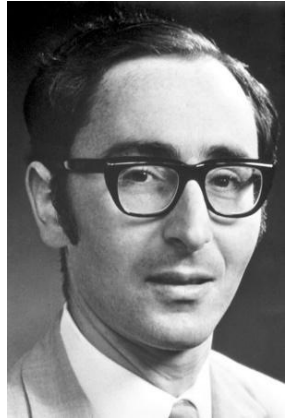
Received 8 June 1962

1962: Josephson effect

$$\begin{cases} V = \frac{\hbar}{2e} \frac{d\varphi}{dt} \\ I = I_c \sin \varphi \end{cases}$$



Josephson effect



B. D. Josephson



1973

1962: Josephson effect

$$\begin{cases} V = \frac{\hbar}{2e} \frac{d\varphi}{dt} \\ I = I_c \sin \varphi \end{cases}$$

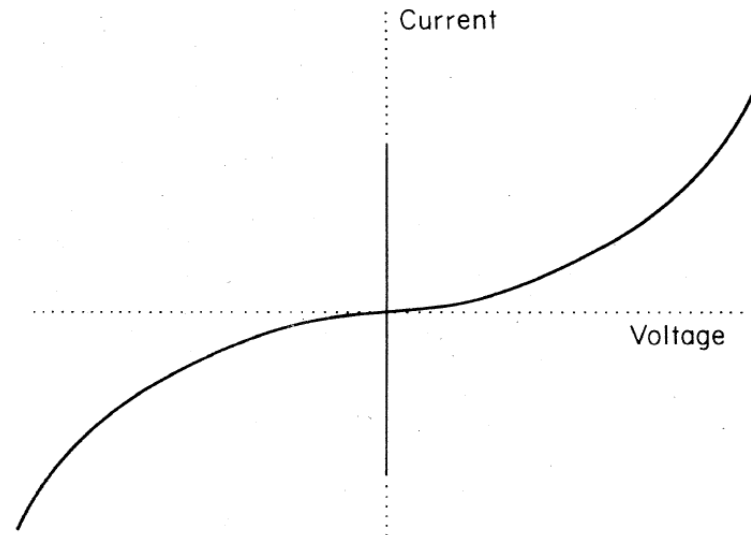
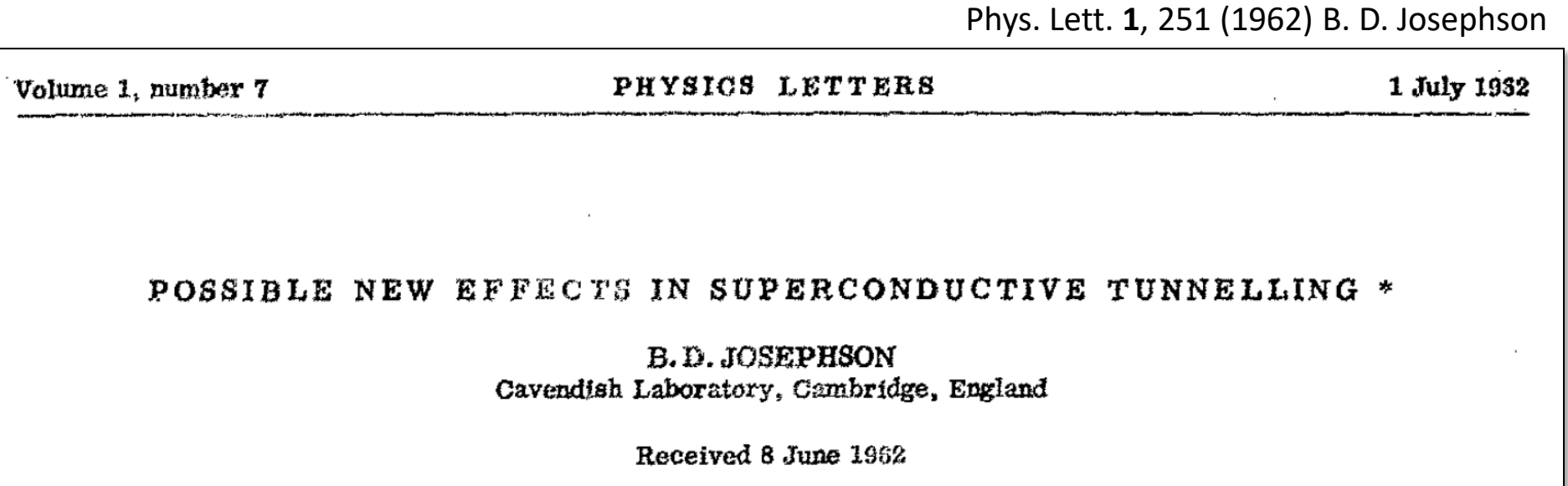


FIG. 2. Predicted two-part current-voltage characteristic of a superconducting tunnel junction.

Rev. Mod. Phys. **46**, 251 (1974) B. D. Josephson

Observation

Phys. Rev. Lett. **10**, 230 (1963) P. W. Anderson & J. M. Rowell



P. W. Anderson



1977

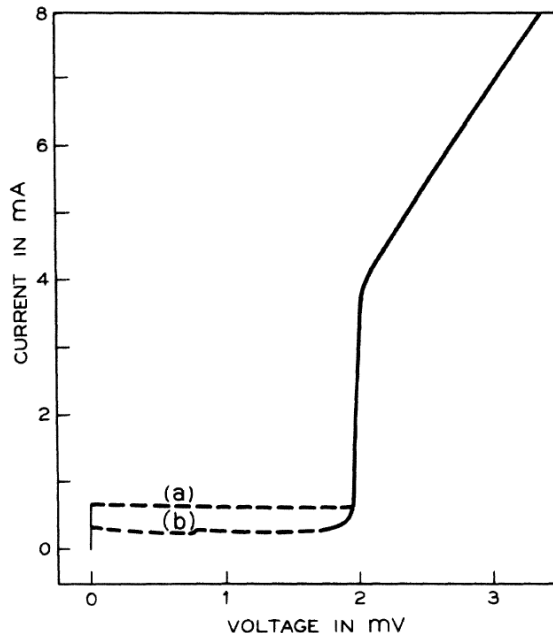


FIG. 1. Current-voltage characteristic for a tin-oxide-lead tunnel structure at $\sim 1.5^\circ\text{K}$, (a) for a field of 6×10^{-3} gauss and (b) for a field 0.4 gauss.

VOLUME 10, NUMBER 6

PHYSICAL REVIEW LETTERS

15 MARCH 1963

PROBABLE OBSERVATION OF THE JOSEPHSON SUPERCONDUCTING TUNNELING EFFECT

P. W. Anderson and J. M. Rowell
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 11 January 1963)

VOLUME 130, NUMBER 1

1 APRIL 1963

Plasmons, Gauge Invariance, and Mass

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 8 November 1962)

Phys. Rev. **130**, 439 (1963) P. W. Anderson

→ Broken symmetry & Anderson–Higgs mechanism

- Phys. Rev. Lett. **13**, 321 (1964) F. Englert & R. Brout
- Phys. Rev. Lett. **13**, 508 (1964) P. W. Higgs
- Phys. Rev. Lett. **13**, 585 (1964) G. S. Guralnik, C. R. Hagen, & T. W. B. Kibble

Reminiscence

Physics Today **23 (11)**, 23 (1970) P. W. Anderson



P. W. Anderson

About eight years ago a 22-year-old student showed his professor some calculations on quantum tunneling. Now that professor tells us

How Josephson discovered his effect

Philip W. Anderson

The discovery of what we have ever since called the “Josephson effect” took place while I was visiting Cambridge, England, for a sabbatical year in 1961–62. Recently, when I travelled to Kyoto to accept the London Prize on Brian Josephson’s behalf, I put together some reminiscences of that period, which may be more interesting than a mere recapitulation of the bare scientific facts. Lest I appear too central to this account, I should make it clear that at least two other people could have told a similar story from their own points of view. They are Brian Pippard, Josephson’s thesis advisor while the work was being done, and David Shoenberg, director at that time of the Mond Laboratory in which Josephson was a research student in experimental physics and where I served as nominal head of the solid-state theory group.

The discovery

Josephson had taken my course on solid-state and many-body theory. This was a disconcerting experience for a lecturer, I can assure you, because

everything had to be right or he would come up and explain it to me after class. Probably because of the course and some of the things I said, he showed me his calculations within a day or two after first making them. At first he followed directly some recent calculations, by Morrel Cohen, Leo Falicov and James Phillips,¹ of the current by the “tunneling Hamiltonian” method, applied to two superconductors separated by an insulating layer. But by the time I saw the calculations he had already worked out the rather sophisticated formalism he later published in which he kept track of particle charges.

By this time I knew Josephson well enough that I would have accepted anything else he said on faith. However, he himself seemed dubious, so I spent an evening checking one of the terms that make up the current. We were all—Josephson, Pippard and myself, as well as various other people who also habitually sat at the Mond tea table and participated in the discussions of the next few weeks—very much puzzled by the meaning of the fact that the current depends on the phase. (This is the famous $J = J_1 \sin \varphi$ relation—see box on page 24.) I think it was residual uneasiness on this score that caused the two Brians (Pippard



Brian D. Josephson in 1969

and Josephson) to decide to send the paper to *Physics Letters*,² which was just then starting publications, rather than to *Physical Review Letters*.

Earlier in my course I had made some remarks about broken symmetry, and later we discussed how broken symmetry made this peculiar behavior of the current possible. Josephson has always given me much more credit for these remarks than my understanding at the time deserved. Apart from these ideas about broken symmetry and some very minor points acknowledged explicitly in that *Physics Letters* paper, I want to emphasize that the whole achievement, from the conception to the explicit calculation to the publication, was completely Josephson’s.

Philip W. Anderson is at Bell Telephone Laboratories, Murray Hill, N.J.

Reminiscence

Physics Today **23 (11)**, 23 (1970) P. W. Anderson



P. W. Anderson

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Reminiscence



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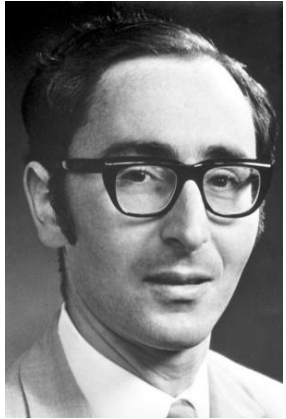


B. D. Josephson

Rev. Mod. Phys. **46**, 251 (1974) B. D. Josephson

I was fascinated by the idea of broken symmetry, and wondered whether there could be any way of observing it experimentally. The existence of the original symmetry implies that the absolute phase angle ϕ would be unobservable, but the possibility of observing phase differences between the F functions in two separate superconductors was not ruled out.

Reminiscence



B. D. Josephson

Rev. Mod. Phys. **46**, 251 (1974) B. D. Josephson

The embarrassing feature of the theory at this point was that the effects predicted were too large! (...) Examination of the literature showed that possibly dc supercurrents of this magnitude had been observed, for example in the first published observation of tunnelling between two evaporated-film superconductors by Nicol, Shapiro and Smith. Giaever had made a similar observation (...)

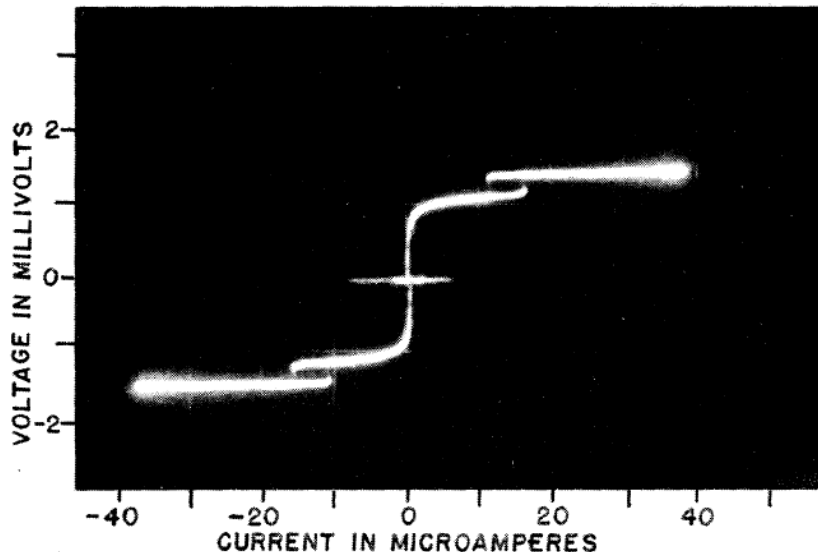


FIG. 1. Voltage vs tunneling current for an Al-Al₂O₃-Pb sandwich at 1°K.

Missed opportunity? Just two months after Giaever's paper

DIRECT MEASUREMENT OF THE SUPERCONDUCTING ENERGY GAP

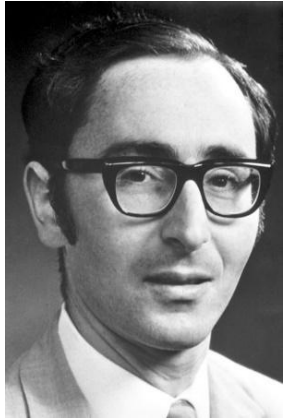
James Nicol, Sidney Shapiro, and Paul H. Smith

Advanced Research Division, Arthur D. Little, Incorporated, Cambridge, Massachusetts

(Received October 27, 1960)

Phys. Rev. Lett. **5**, 461 (1960) J. Nicol *et al.*

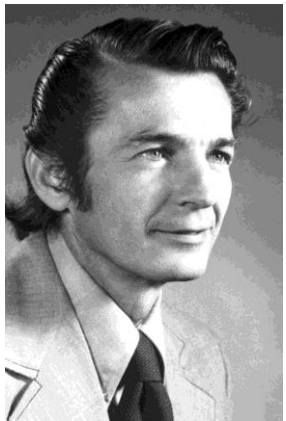
Reminiscence



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Rev. Mod. Phys. **46**, 251 (1974) B. D. Josephson

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I. Giaever

Rev. Mod. Phys. **46**, 245 (1974) I. Giaever

We had observed this behavior [dc Josephson effect] many times; matter of fact, it is difficult not to see this current when junctions are made of tin-tin oxide-tin or lead-lead oxide-lead. (...) all the samples we made showing the Josephson effect were discarded as having shorts. (...) Later I have been asked many times if I feel bad for missing the effect? The answer is clearly no, because to make an experimental discovery it is not enough to observe something, one must also realize the significance of the observation, and in this instance I was not even close.

SQUID

Superconducting QUantum Interference Device

Phys. Rev. Lett. **12**, 159 (1964) R. C. Jaklevic *et al.*

VOLUME 12

17 FEBRUARY 1964

NUMBER 7

QUANTUM INTERFERENCE EFFECTS IN JOSEPHSON TUNNELING

R. C. Jaklevic, John Lambe, A. H. Silver, and J. E. Mercereau

Ford Scientific Laboratories, Dearborn, Michigan

(Received 16 January 1964)

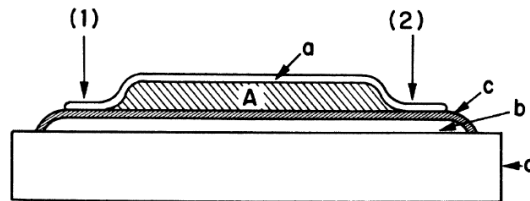


FIG. 1. Cross section of a Josephson junction pair vacuum-deposited on a quartz substrate (*d*). A thin oxide layer (*c*) separates thin ($\sim 1000\text{\AA}$) tin films (*a*) and (*b*). The junctions (1) and (2) are connected in parallel by superconducting thin film links forming an enclosed area (*A*) between junctions. Current flow is measured between films (*a*) and (*b*).

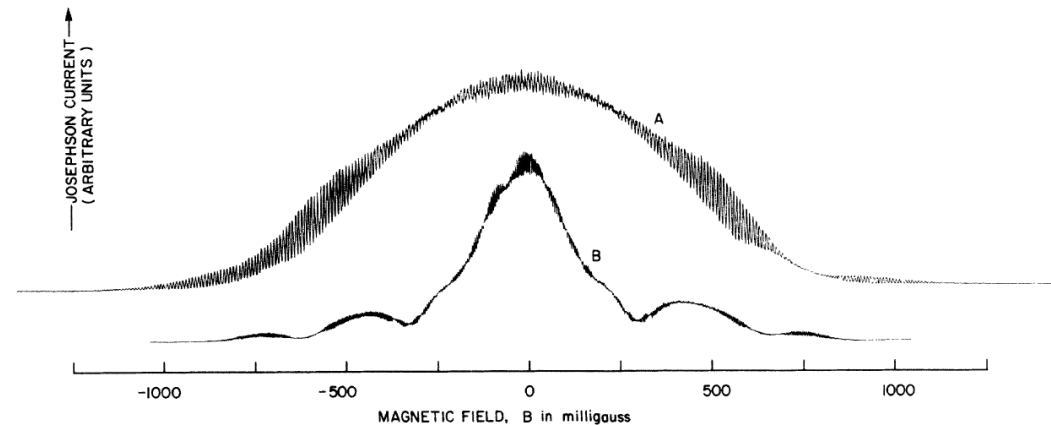


FIG. 2. Josephson current vs magnetic field for two junctions in parallel showing interference effects. Magnetic field applied normal to the area between junctions. Curve (*A*) shows interference maxima spaced at $\Delta B = 8.7 \times 10^{-3}$ G, curve (*B*) spacing $\Delta B = 4.8 \times 10^{-3}$ G. Maximum Josephson current indicated here is approximately 10^{-3} A.

SLUG

Superconducting Low-inductance Undulatory Galvanometer

Philos. Mag. **13**, 115 (1966) J. Clarke

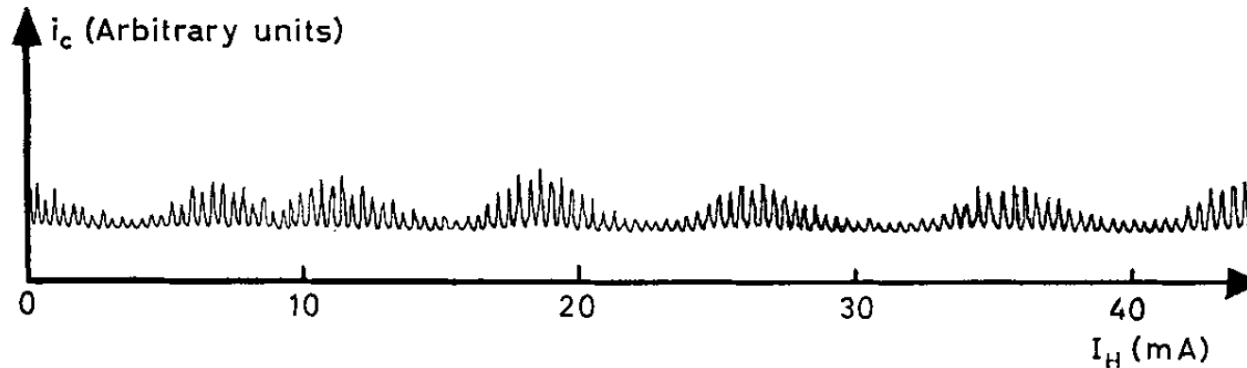
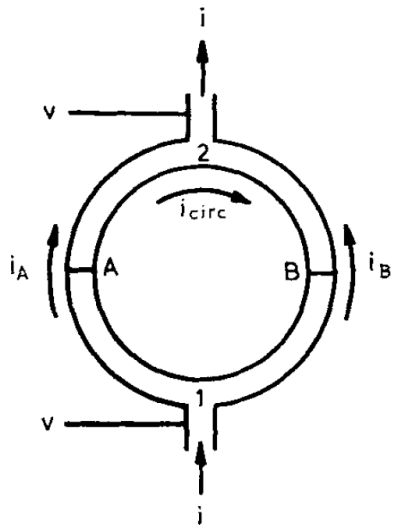
A Superconducting Galvanometer Employing Josephson Tunnelling

By J. CLARKE

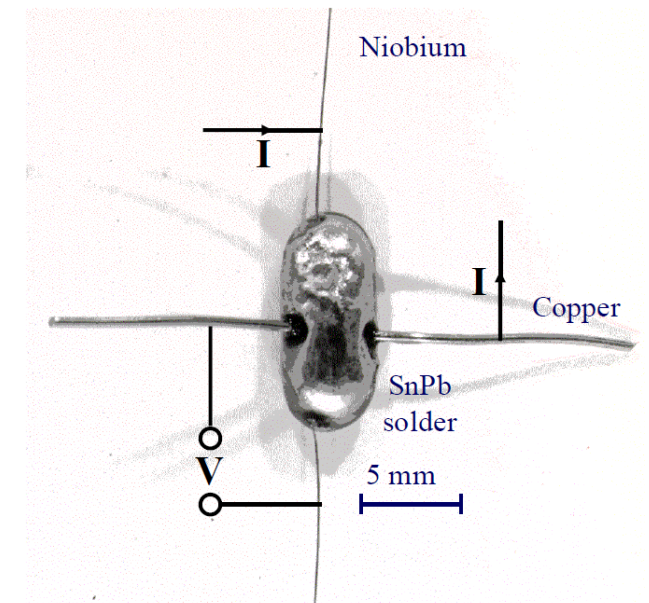
Cavendish Laboratory, University of Cambridge

[Received 13 August 1965]

Pippard's student, two years younger than Josephson, heard his lecture, and received the 2025 Nobel Prize in Physics



“The exact nature of the junctions and the mechanism by which the critical current is modulated are not fully understood. The device behaves as if a tunnelling junction were formed between the wire and the solder only at each end of the bead.”



Macroscopic quantum tunneling



A. J. Leggett



2003

Phys. Rev. Lett. **46**, 211 (1981) A.O Caldeira & A. J. Leggett

Influence of Dissipation on Quantum Tunneling in Macroscopic Systems

A. O. Caldeira and A. J. Leggett

School of Mathematical and Physical Sciences, University of Sussex, Brighton BN1 9QH, Sussex, United Kingdom

(Received 28 July 1980)

Ann. Phys. **149**, 374 (1983) A.O Caldeira & A. J. Leggett

Quantum Tunnelling in a Dissipative System

A. O. CALDEIRA

*Instituto de Fisica "Gleb Wataghin" Universidade Estadual de Campinas,
Cidade Universitaria, Barao Geraldo, 13-100 Campinas, Sao Paulo, Brazil*

AND

A. J. LEGGETT*

*School of Mathematical and Physical Sciences,
University of Sussex, Falmer, Brighton BN1 9QH, England*

Received July 22, 1982; revised December 20, 1982

Suppl. Prog. Theor. Phys. **69**, 80 (1980) A. J. Leggett

Macroscopic Quantum Systems and the Quantum Theory of Measurement

A. J. LEGGETT

*School of Mathematical and Physical Sciences
University of Sussex, Brighton BN1 9QH*

Experiments at Berkeley



J. Clarke



M. H. Devoret



J. M. Martinis



2025

© Nobel Foundation

“for the discovery of macroscopic quantum mechanical tunnelling and energy quantisation in an electric circuit”

Ph. D. Thesis, J. M. Martinis, December 1985

“Macroscopic Quantum Tunneling and Energy-Level Quantization in the Zero Voltage State of the **Current-Biased Josephson Junction**”

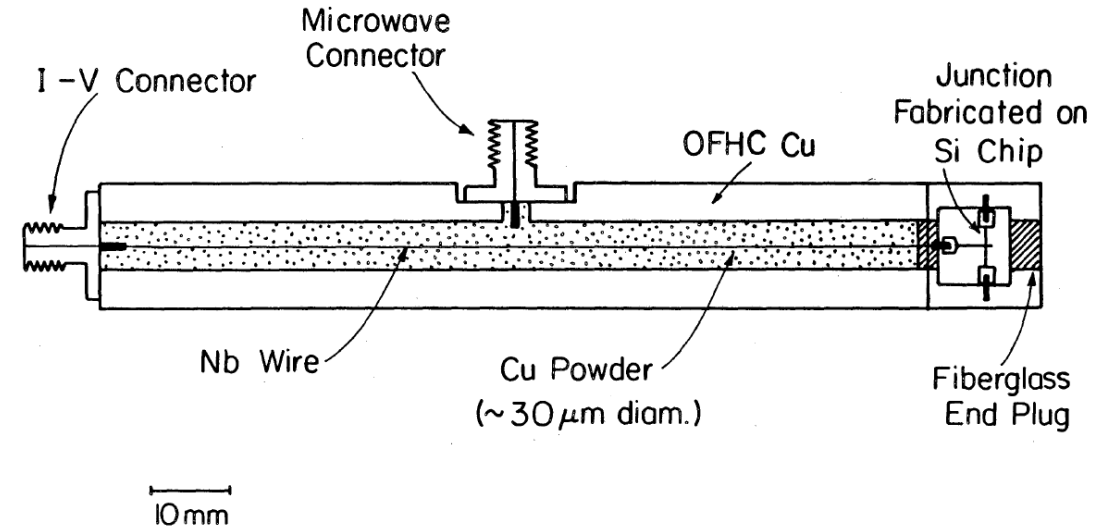


Fig. 6. Scale drawing of the junction mount.

XBL 8510-6744

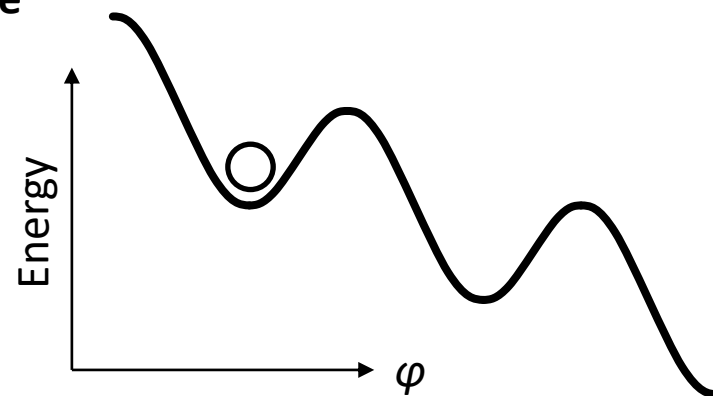
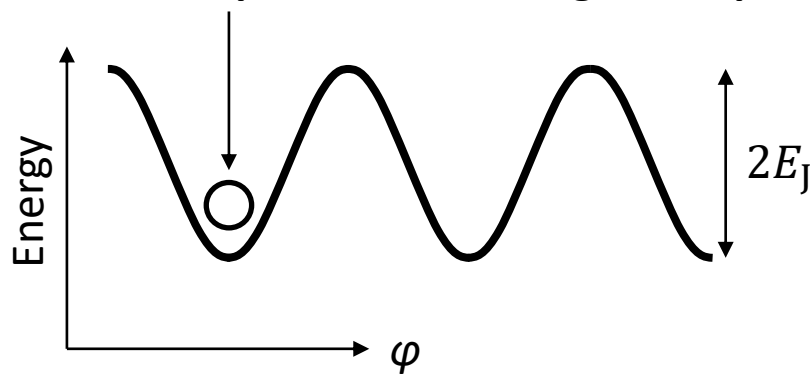
Current-biased Josephson junction

$$\begin{cases} V = \frac{\hbar}{2e} \frac{d\varphi}{dt} \\ I = I_c \sin \varphi \end{cases} \longrightarrow U = \int IV dt = \int \underbrace{\left(\frac{\hbar I_c}{2e} \right)}_{E_J} \sin \varphi \frac{d\varphi}{dt} dt = -E_J \cos \varphi$$

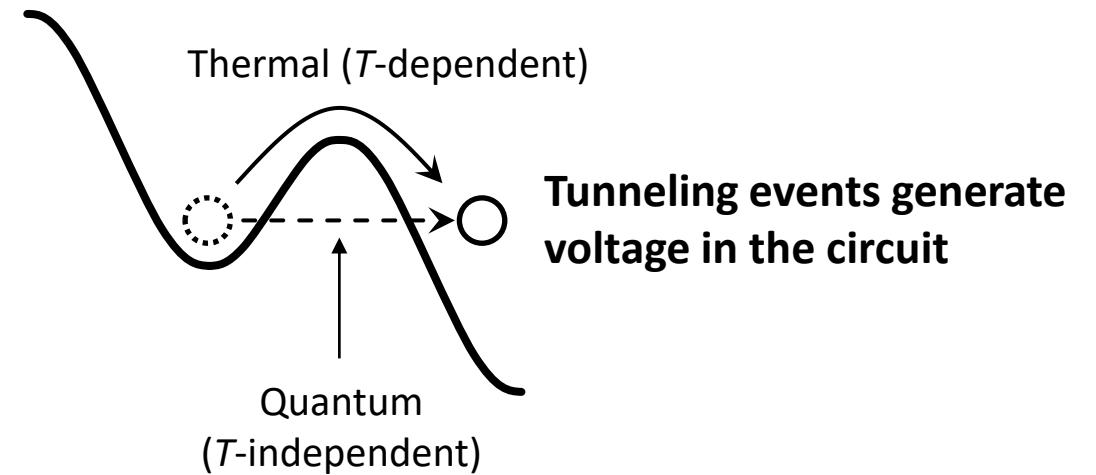
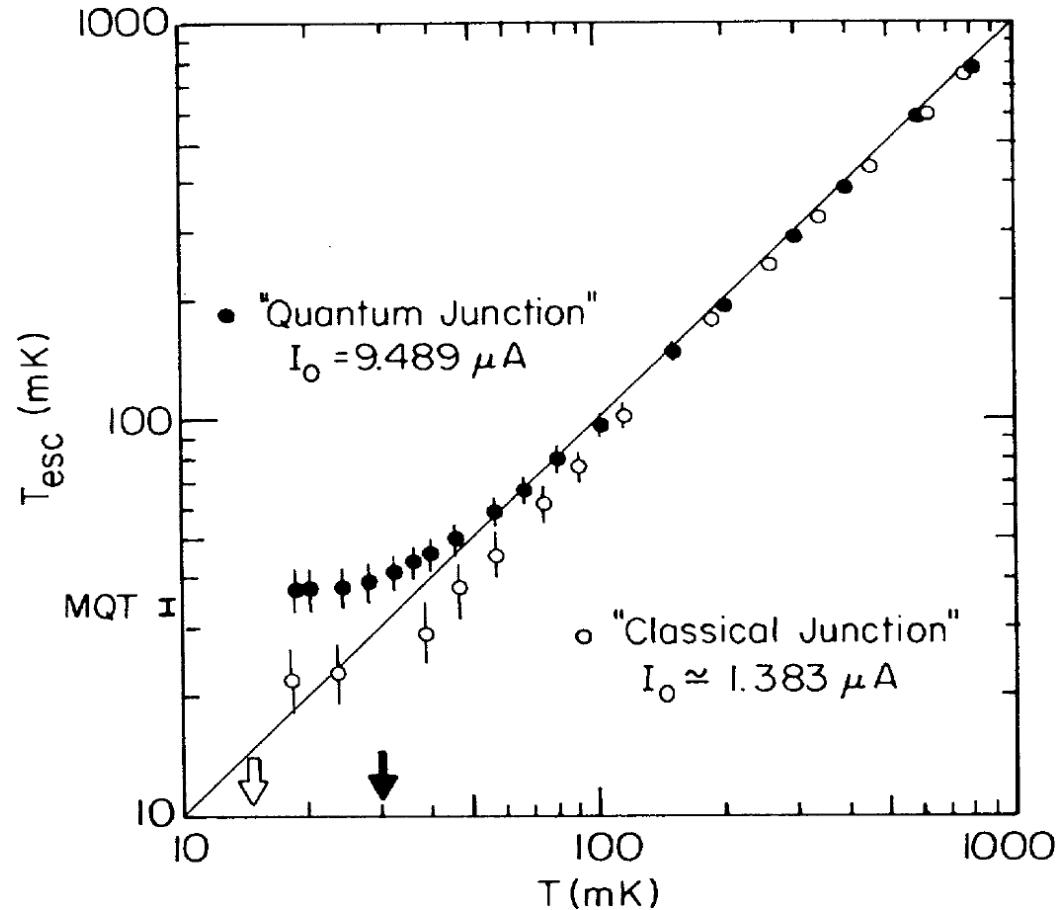
$$U = -E_J \cos \varphi$$

$$U = -E_J \cos \varphi - \frac{\hbar I_b}{2e} \varphi \quad \text{Titled washboard potential}$$

Virtual particle consisting of Cooper-pair condensate



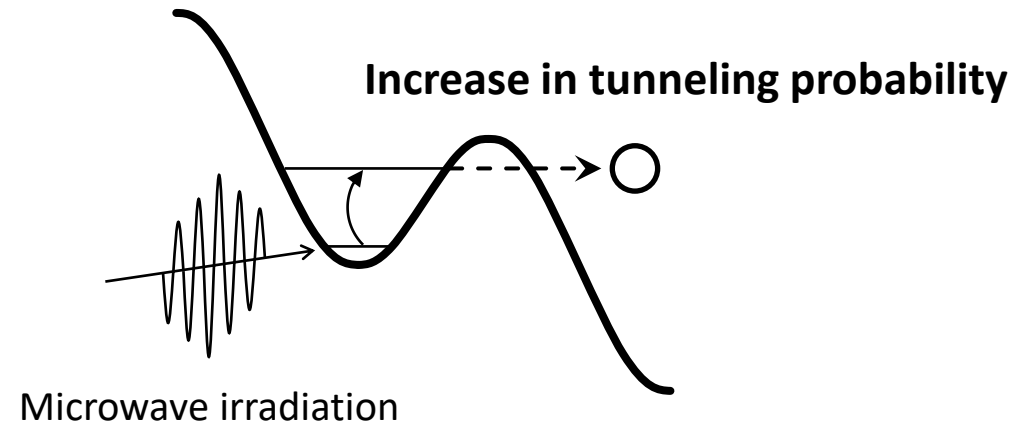
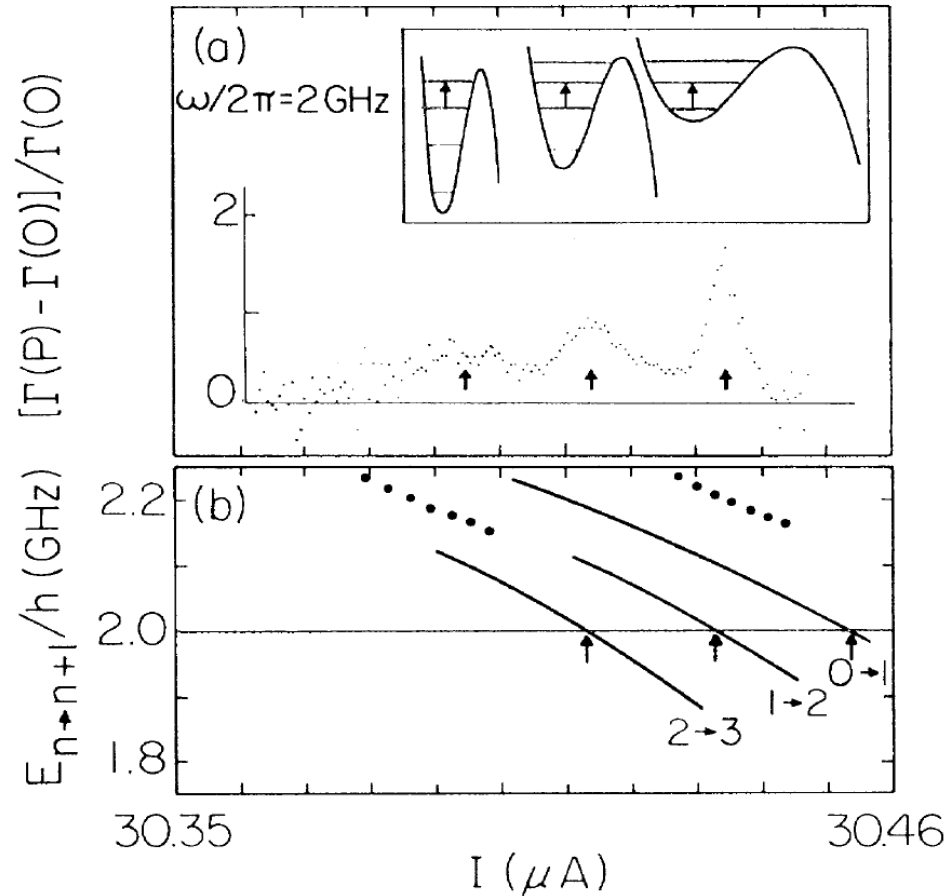
Macroscopic quantum tunneling



Phys. Rev. Lett. **55**, 1908 (1985) M. H. Devoret, J. M. Martinis, & J. Clarke

"Measurements of Macroscopic Quantum Tunneling out of the Zero-Voltage State of a Current-Biased Josephson Junction"

Energy quantization



Almost but not quite phase qubit

Phys. Rev. Lett. **55**, 1543 (1985) J. M. Martinis, M. H. Devoret, & J. Clarke

“Energy-Level Quantization in the Zero-Voltage State of a Current-Biased Josephson Junction”

Macroscopic quantum coherence

Phys. Rev. Lett. **54**, 857 (1985) A. J. Leggett & A. Garg

Quantum Mechanics versus Macroscopic Realism: Is the Flux There when Nobody Looks?

A. J. Leggett

Department of Physics,^(a) University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, and Department of Physics,
Harvard University, Cambridge, Massachusetts 02138

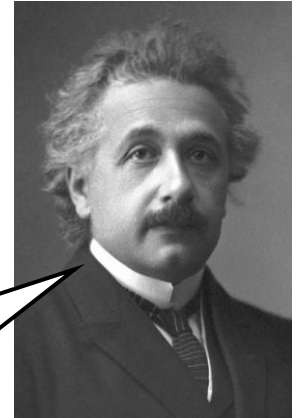
and

Anupam Garg

University of Illinois at Urbana-Champaign, Urbana, Illinois 61801

(Received 19 November 1984)

It is shown that, in the context of an idealized “macroscopic quantum coherence” experiment, the predictions of quantum mechanics are incompatible with the conjunction of two general assumptions which are designated “macroscopic realism” and “noninvasive measurability at the macroscopic level.” The conditions under which quantum mechanics can be tested against these assumptions in a realistic experiment are discussed.

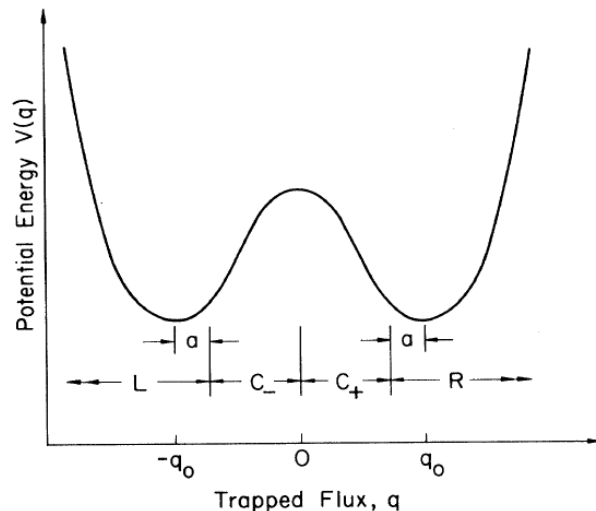


Is the Moon there
when nobody looks?

A. Einstein



1921



Ph. D. Thesis, J. M. Martinis, Dec. 1985

ACKNOWLEDGEMENTS

My collaborators in this work, M. Devoret, D. Esteve, and John Clarke, must be thanked since this experiment was possible only through the “coherent superposition” of all of our efforts.

⋮

Lastly, I would like to thank Dr. A. Leggett. This thesis is a result of his interest and insight to the theory of measurement.

Macroscopic quantum coherence

Charge qubit

letters to nature

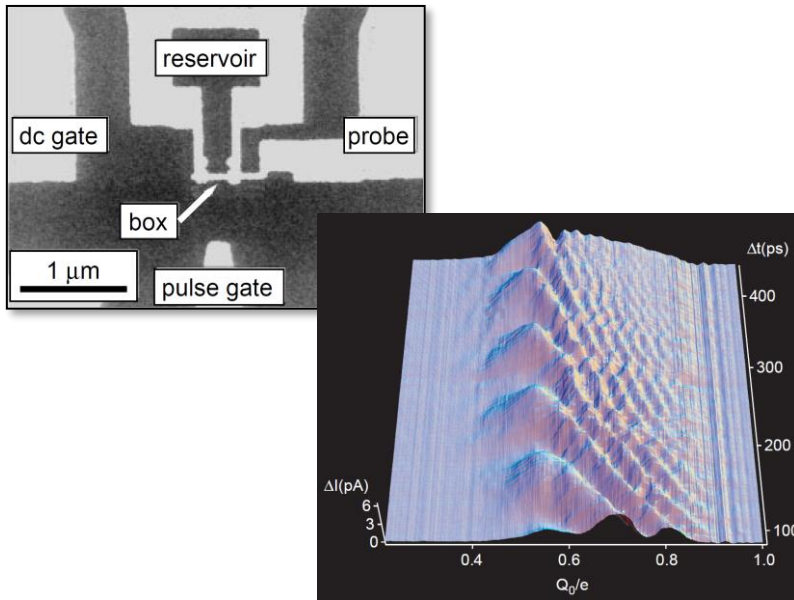
Coherent control of macroscopic quantum states in a single-Cooper-pair box

Y. Nakamura*, Yu. A. Pashkin† & J. S. Tsai*

* NEC Fundamental Research Laboratories, Tsukuba, Ibaraki 305-8051, Japan

† CREST, Japan Science and Technology Corporation (JST), Kawaguchi, Saitama 332-0012, Japan

Nature **398**, 786 (1999) Y. Nakamura *et al.*

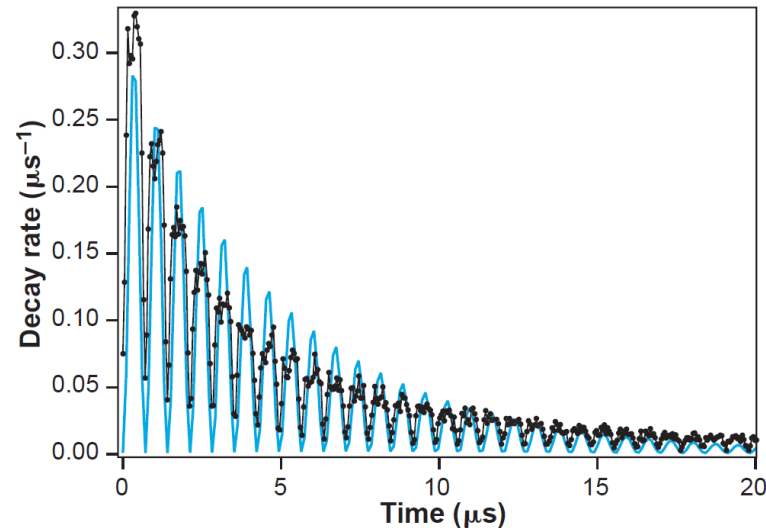


Phase qubit

Coherent Temporal Oscillations of Macroscopic Quantum States in a Josephson Junction

Yang Yu,¹ Siyuan Han,^{1*} Xi Chu,^{2,†} Shih-I Chu,² Zhen Wang³

Science **296**, 889 (2002) Y. Yu *et al.*



See also Phys. Rev. Lett. **89**, 117901 (2002)
J. M. Martinis *et al.*

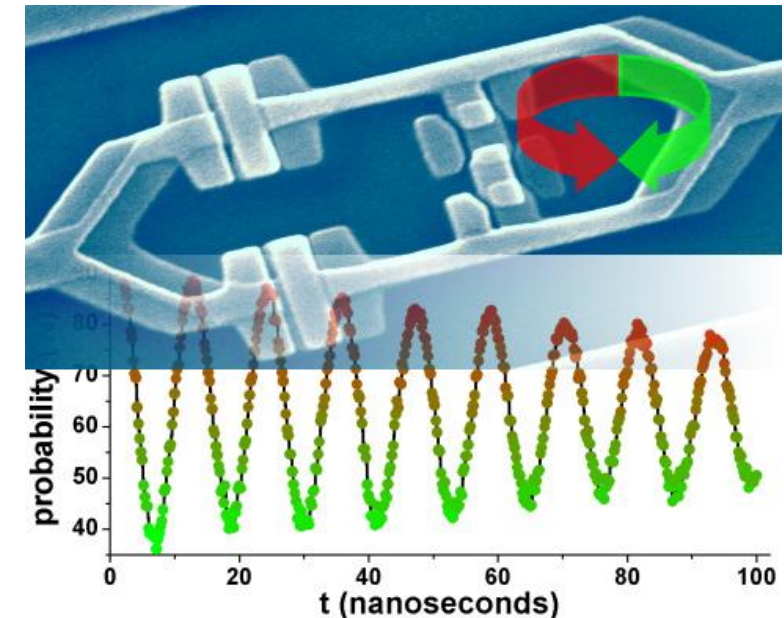
Flux qubit

REPORTS

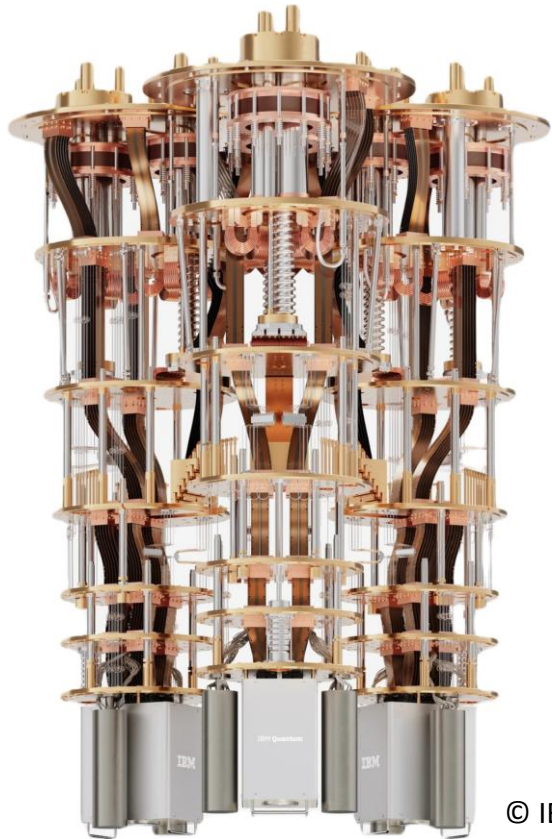
Coherent Quantum Dynamics of a Superconducting Flux Qubit

I. Chiorescu,^{1*} Y. Nakamura,^{1,2} C. J. P. M. Harmans,¹ J. E. Mooij¹

Science **299**, 1869 (2003) I. Chiorescu *et al.*



Quantum era



© IBM



Heron R2 (156Q) © IBM

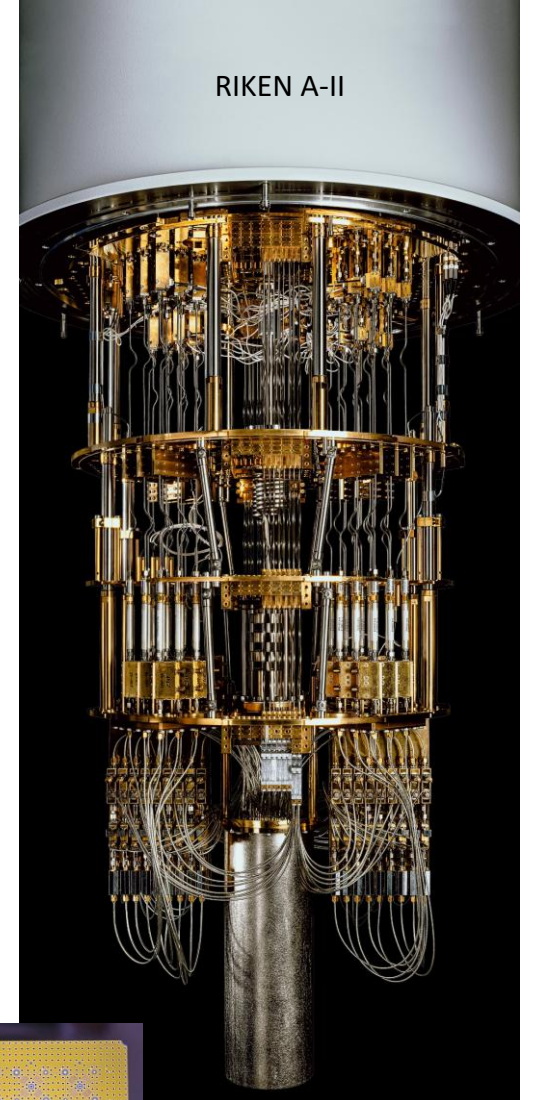
<https://quantumai.google/>
<https://research.ibm.com/quantum-computing>



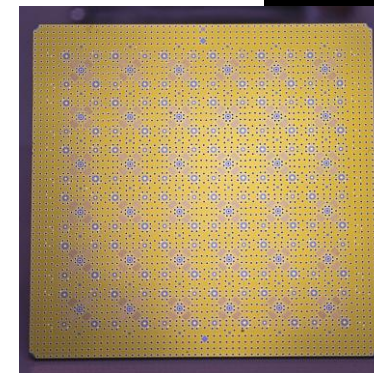
Willow (105Q) © Google



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RIKEN A-II



RIKEN 144Q chip

Contents

- Brief history: from discovery of superconductivity to dawn of quantum era
- **Superconducting qubits**
 - Qubit, coherence time, and protection from noise
 - Josephson junction
 - Plasma oscillation qubit: phase qubit and transmon (derived from charge qubit)
 - Loop-based qubit: flux qubit, fluxonium, and Blochonium (dual of transmon)
 - Protected qubit: $0-\pi$ qubit
- Quantum operations with transmons
 - Circuit QED
 - Dispersive readout
 - Single- and two-qubit gates
- Building quantum computers (→ Symposium talk)

DiVincenzo's criteria

1. A scalable physical system with well characterized qubits
2. The ability to initialize the state of the qubits to a simple fiducial state, such as $|000\dots\rangle$
3. Long relevant decoherence times, much longer than the gate operation time
4. A universal set of quantum gates
5. A qubit-specific measurement capability

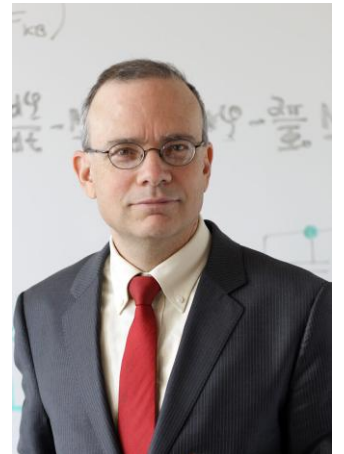
Fortschr. Phys. **48**, 771 (2000) D. P. DiVincenzo

“Universal”

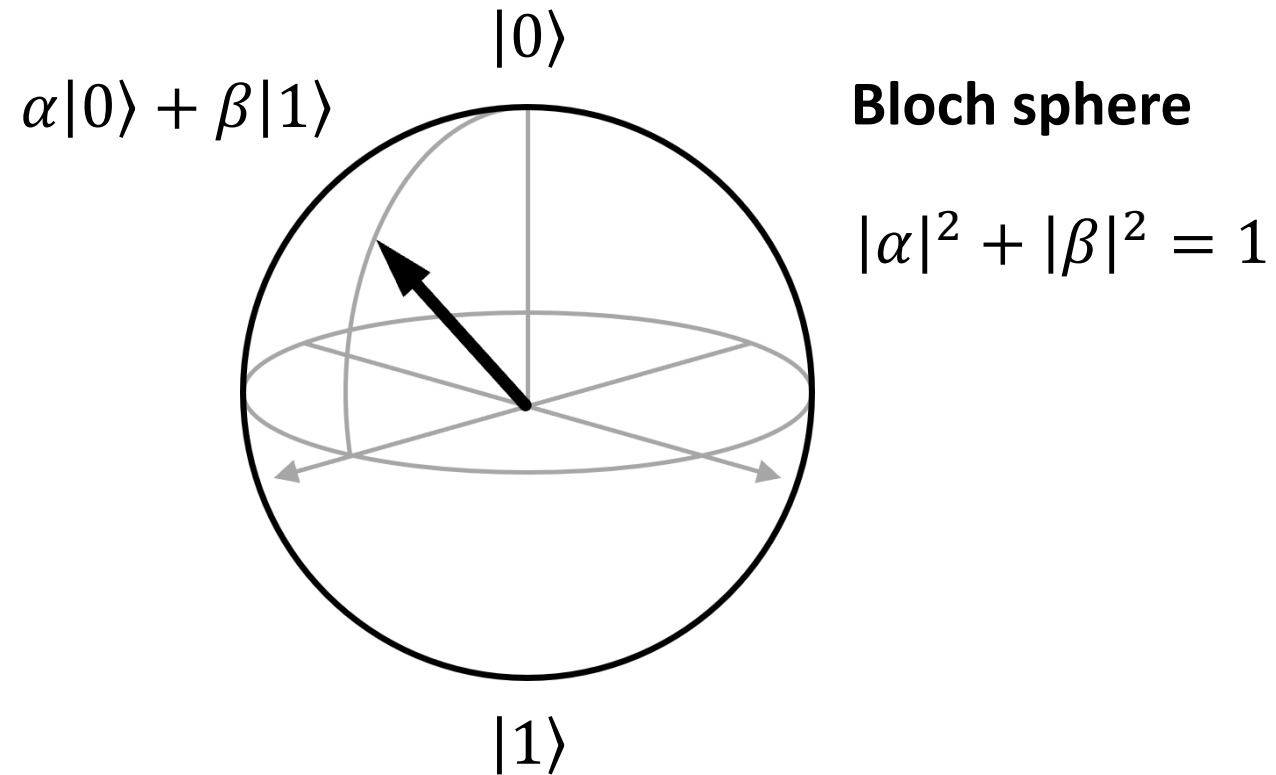
- 1Q gates + CNOT (can construct arbitrary n -qubit gates)
- T, H, S + CNOT (can approximate arbitrary n -qubit gates with arbitrary accuracy)

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad S = T^2 = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad \text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

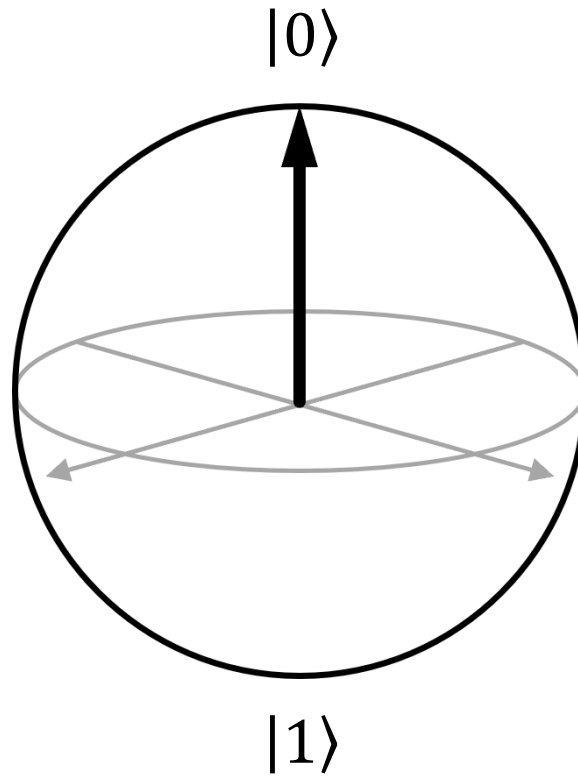
Clifford gates



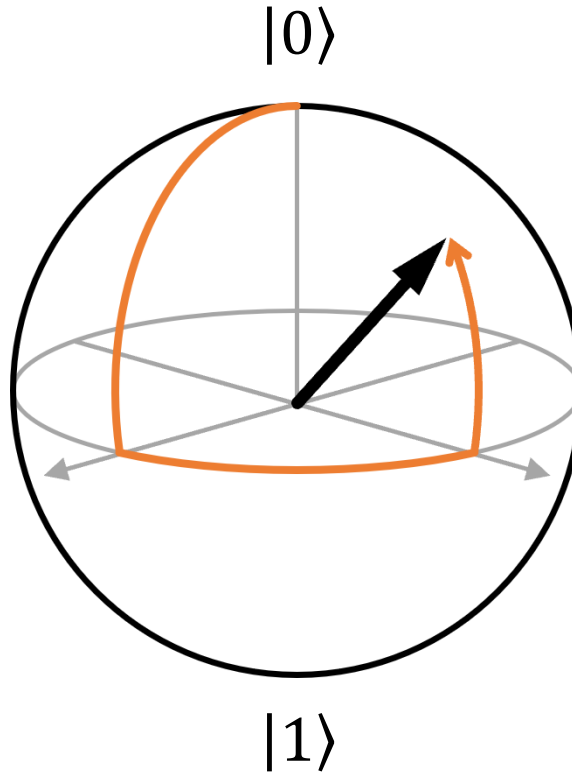
DiVincenzo's criteria: qubit



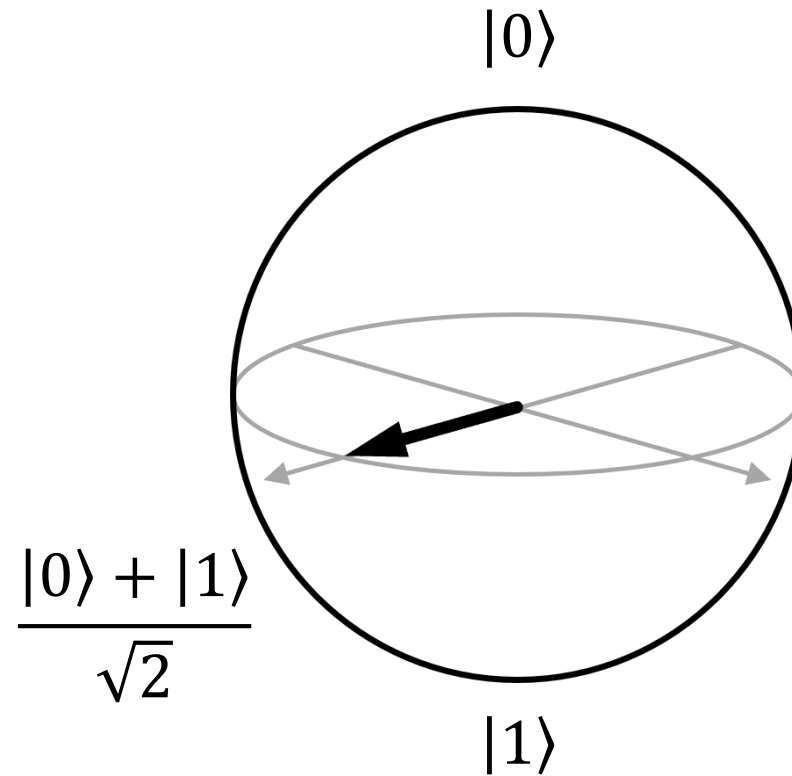
DiVincenzo's criteria: initialization



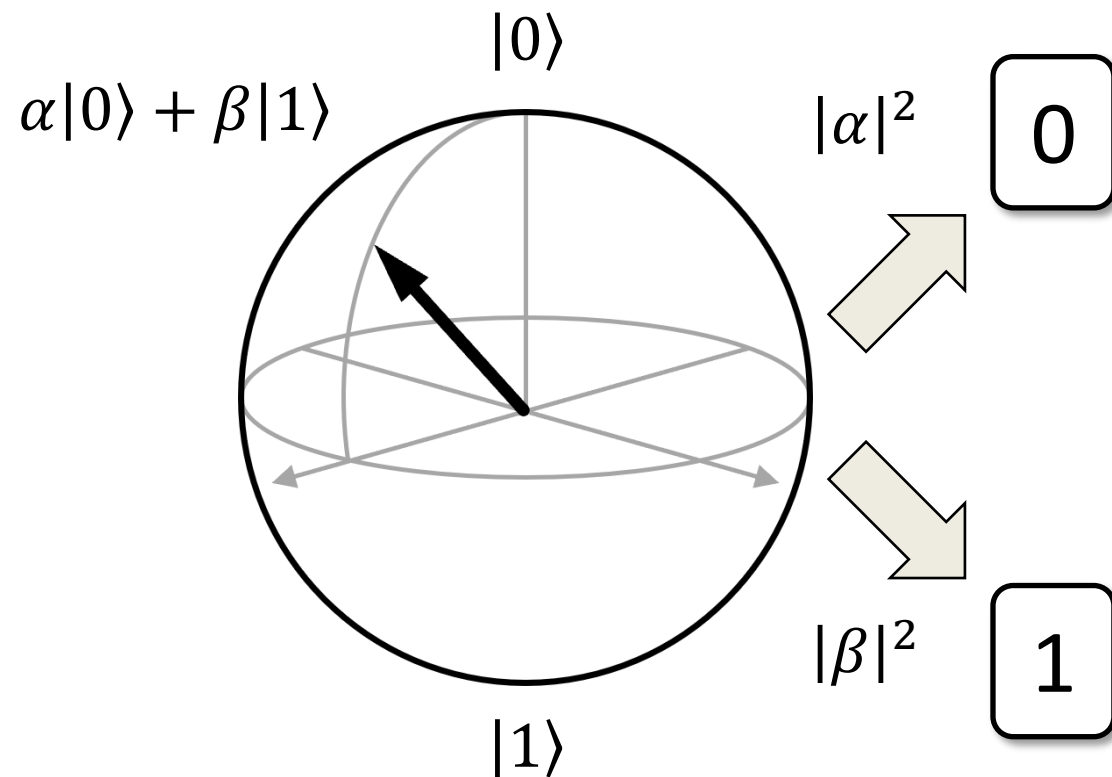
DiVincenzo's criteria: single-qubit gate



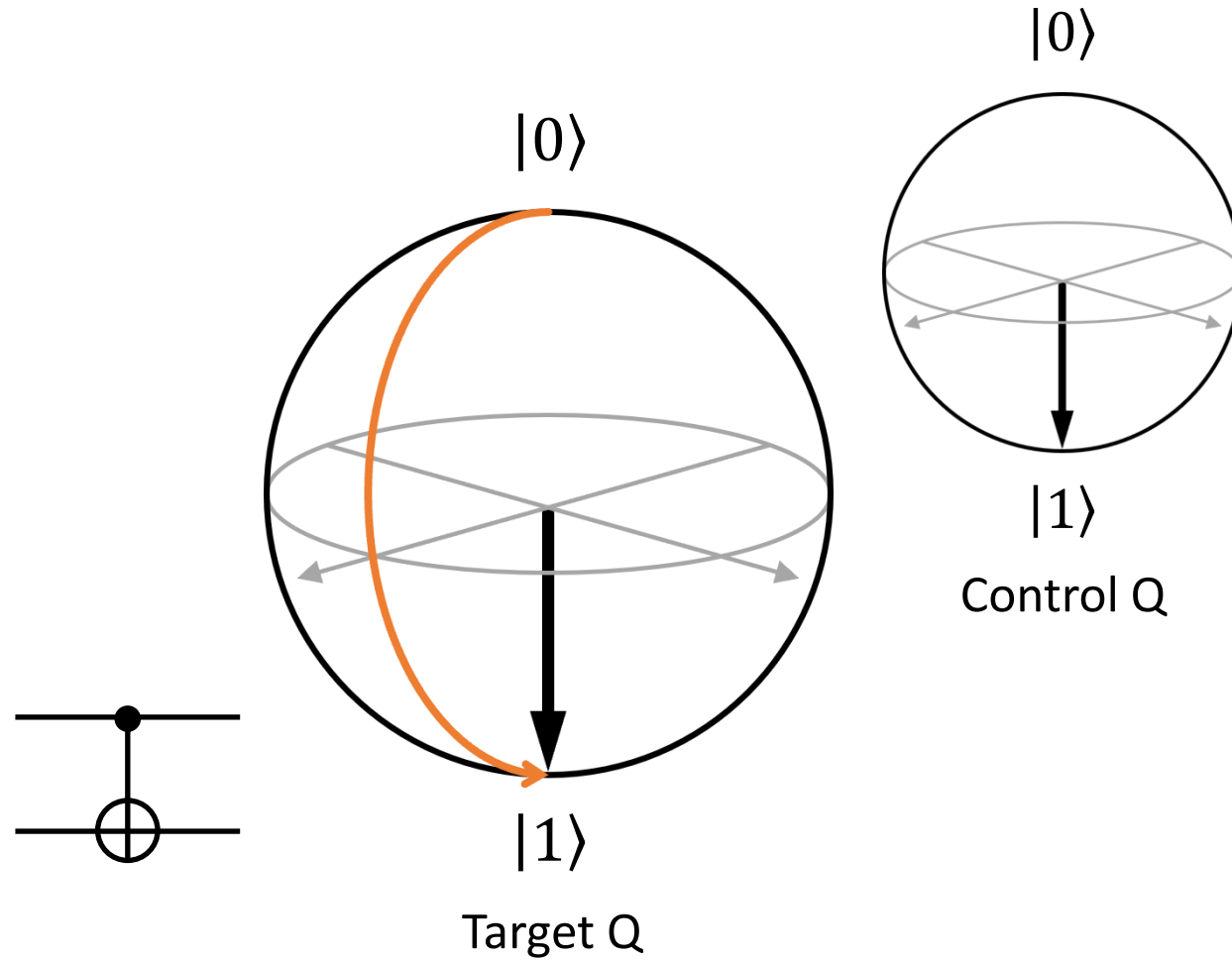
DiVincenzo's criteria: long coherence



DiVincenzo's criteria: readout



DiVincenzo's criteria: two-qubit gate



DiVincenzo's criteria

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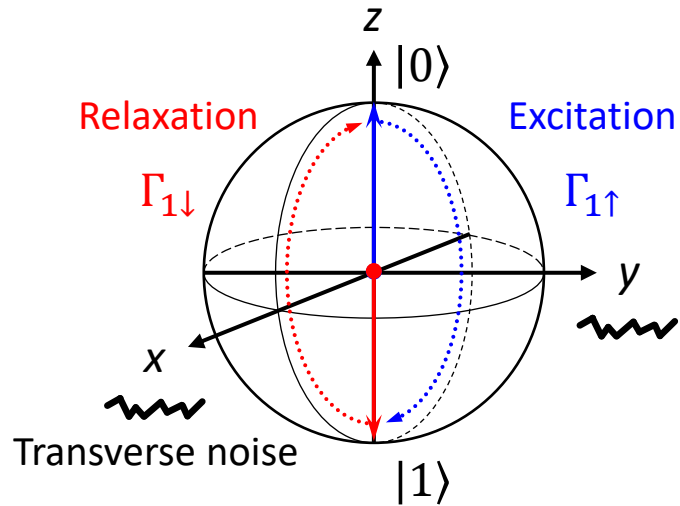
DiVincenzo's criteria

1. A scalable physical system with well characterized qubits
2. The ability to initialize the state of the qubits to a simple fiducial state, such as $|000\dots\rangle$
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T_1 and T_2

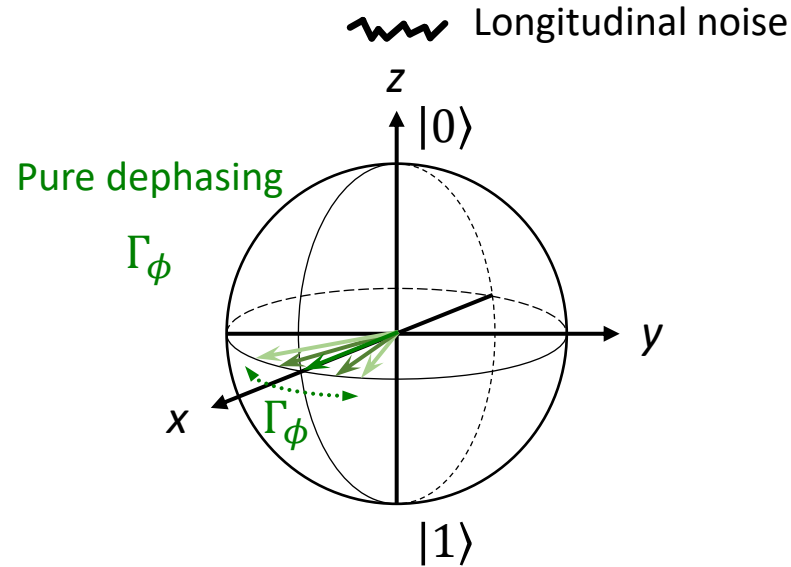
Longitudinal relaxation



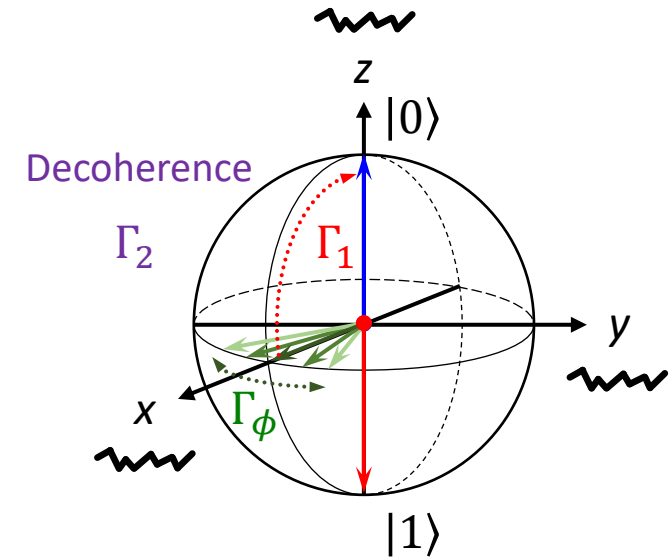
$$\Gamma_1 \equiv \frac{1}{T_1} = \Gamma_{1\downarrow} + \Gamma_{1\uparrow} \approx \Gamma_{1\downarrow}$$

$$\frac{\Gamma_{1\uparrow}}{\Gamma_{1\downarrow}} = \exp\left(-\frac{\hbar\omega_q}{k_B T}\right) \approx 10^{-21}$$

Transverse relaxation



$$\Gamma_\phi \equiv \frac{1}{T_\phi}$$



$$\Gamma_2 \equiv \frac{1}{T_2} = \frac{\Gamma_1}{2} + \Gamma_\phi$$

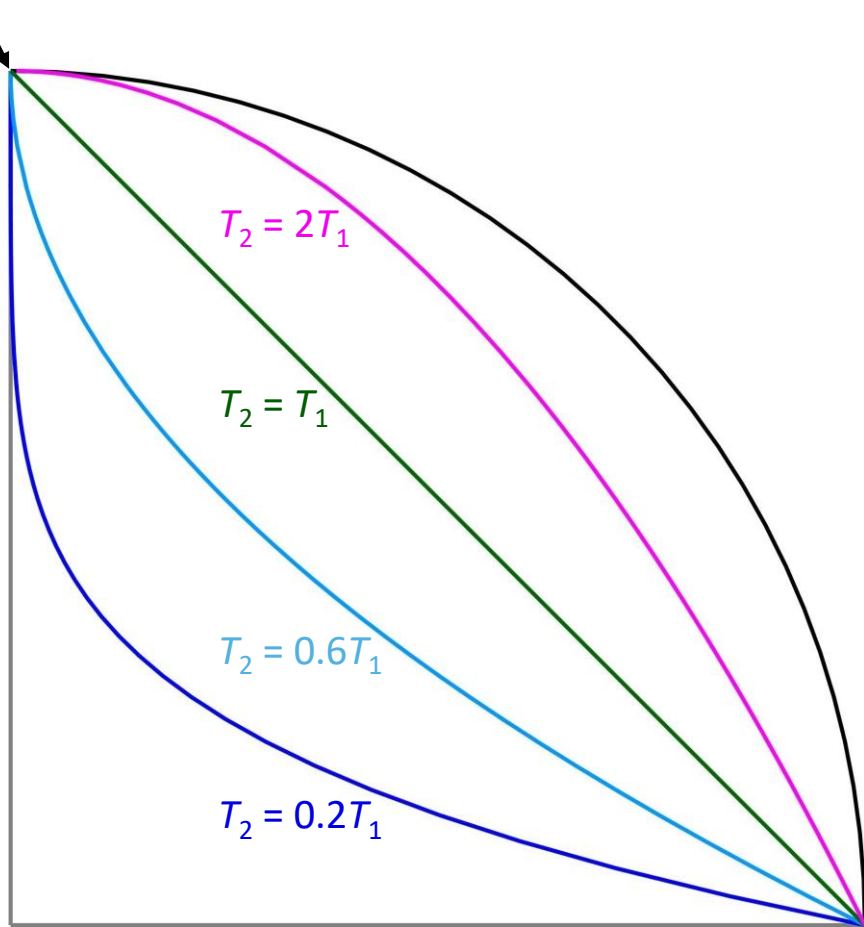
$$\longrightarrow 2T_1 \geq T_2$$

$2T_1$ limit of T_2

$$\rho(\infty) = |0\rangle\langle 0|$$

Qubit's density matrix during the relaxation

$$\rho(t) = \frac{1}{2} \begin{pmatrix} 2 - e^{-t/T_1} & e^{-t/T_2} \\ e^{-t/T_2} & e^{-t/T_1} \end{pmatrix}$$

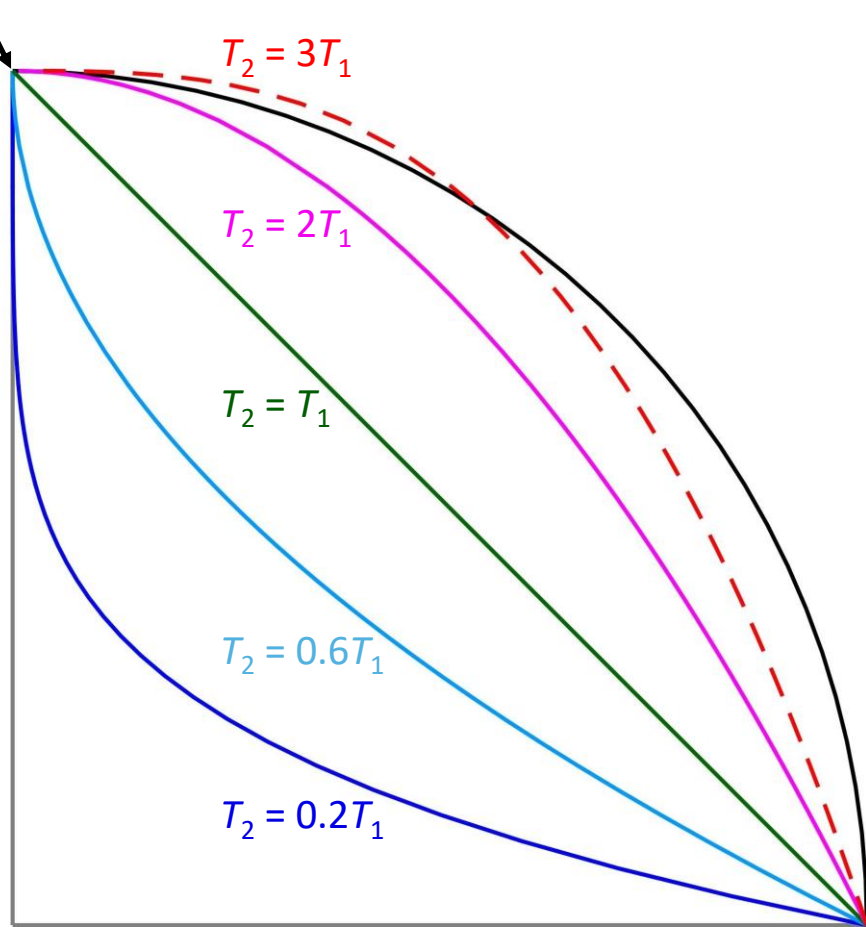


xz-plane of Bloch sphere

$$\rho(0) = \frac{1}{2} (|0\rangle + |1\rangle)(\langle 0| + \langle 1|)$$

$2T_1$ limit of T_2

$$\rho(\infty) = |0\rangle\langle 0|$$



xz-plane of Bloch sphere

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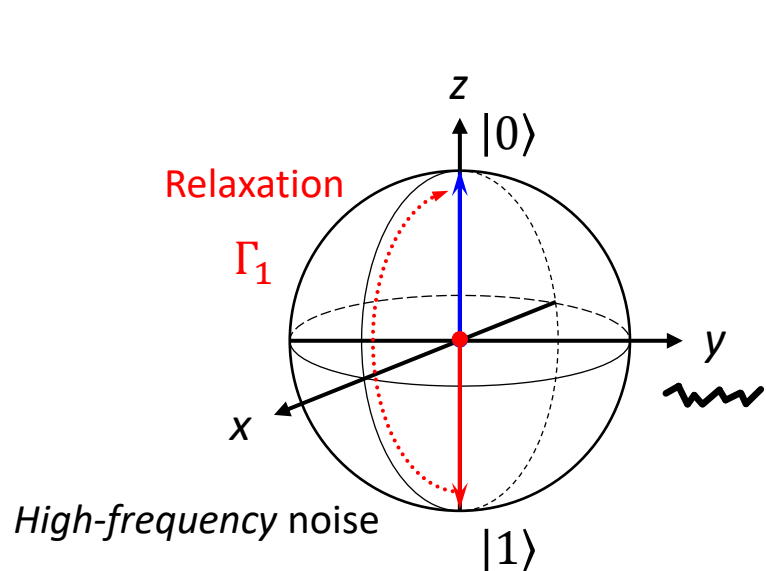
$$\text{Tr}(\rho^2) \leq 1$$

$$\rightarrow (e^{-t/T_1})^{2/k-1} + e^{-t/T_1} \leq 2 \quad (T_2 = kT_1)$$

$$\rightarrow \frac{2}{k} - 1 \geq 0 \quad (0 \leq e^{-t/T_1} \leq 1)$$

$$\rightarrow k \leq 2$$

T_1 and T_ϕ

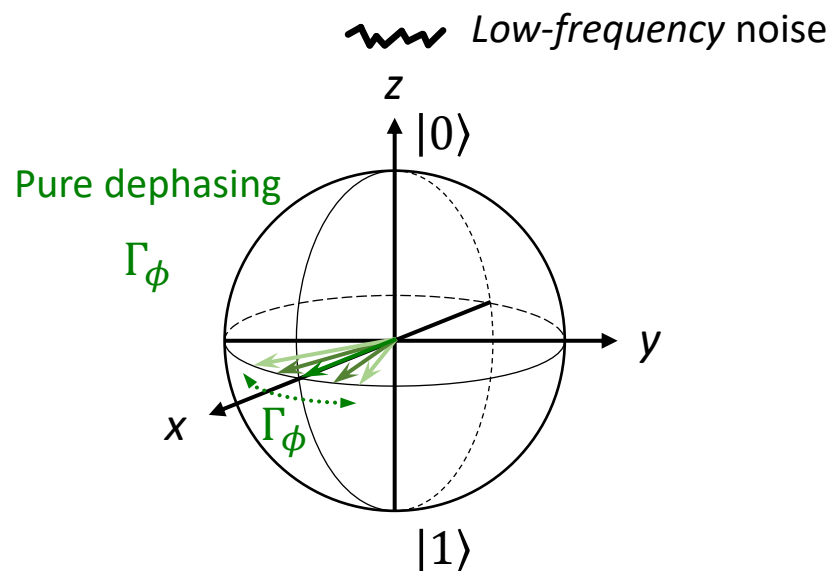


$$\frac{1}{T_1} \propto |\langle 0|O|1\rangle|^2 S_\lambda(\omega_{01})$$

Qubit operator
(charge, phase...)

Control parameter
(charge, flux...)

Fermi's golden rule

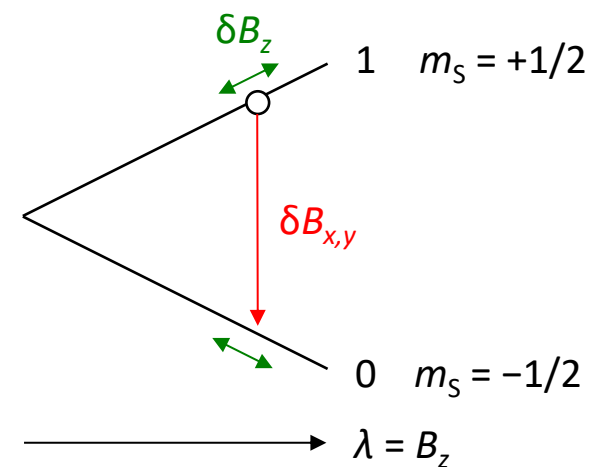


$$\frac{1}{T_\phi} \propto \left| \frac{\partial \omega_{01}}{\partial \lambda} \right|^2 S_\lambda(\omega \rightarrow 0)$$

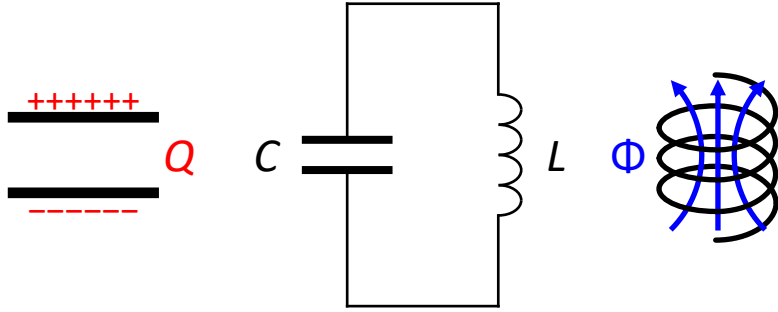
Noise spectral density

Zeeman effect: $E_{01} = h\gamma_e B_z$

$\gamma_e = 28 \text{ MHz/mT}$



Quantum LC circuit



Hamiltonian

$$H_0 = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

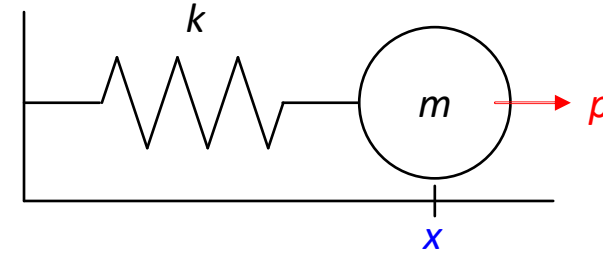
Commutation relation

$$[Q, \Phi] = i\hbar$$

Energy levels

$$E_n = \hbar\omega_0 \left(\frac{1}{2} + n \right) \quad \text{with} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Spring pendulum

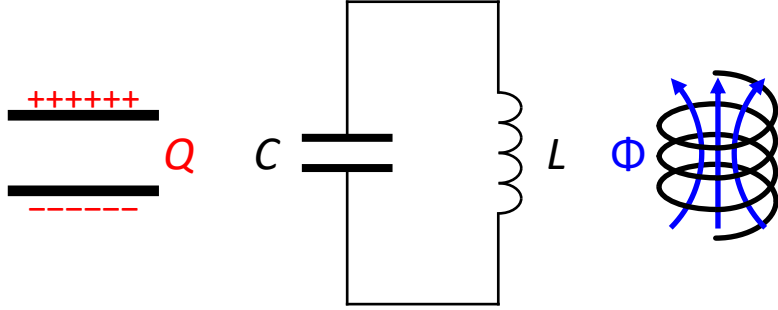


$$H_0 = \frac{p^2}{2m} + \frac{1}{2} kx^2$$

$$[x, p] = i\hbar$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

Quantum LC circuit



Hamiltonian

$$H_0 = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

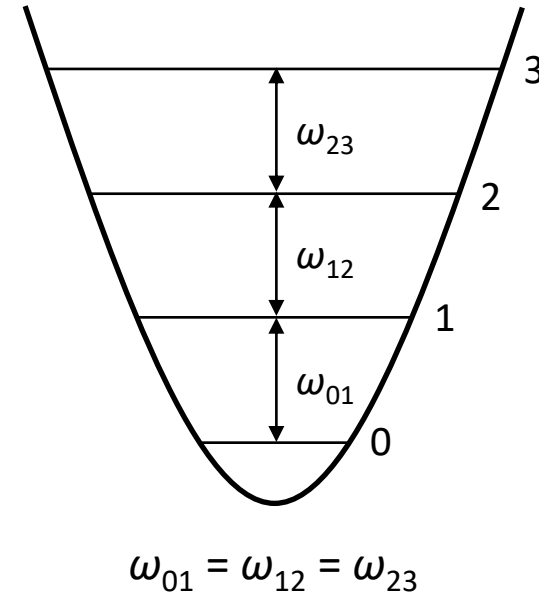
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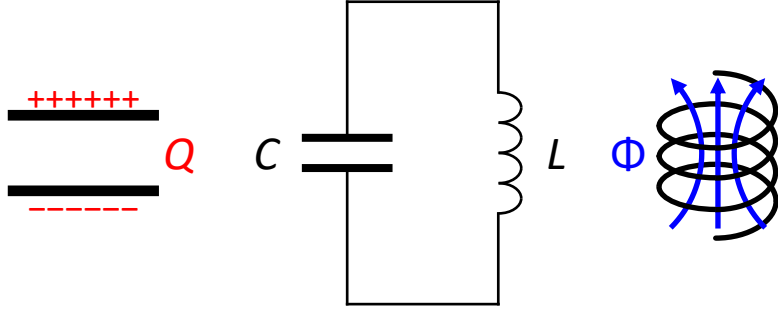
Energy levels

$$E_n = \hbar\omega_0 \left(\frac{1}{2} + n \right) \quad \text{with} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Harmonic oscillator



Quantum LC circuit



Hamiltonian

$$H_0 = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

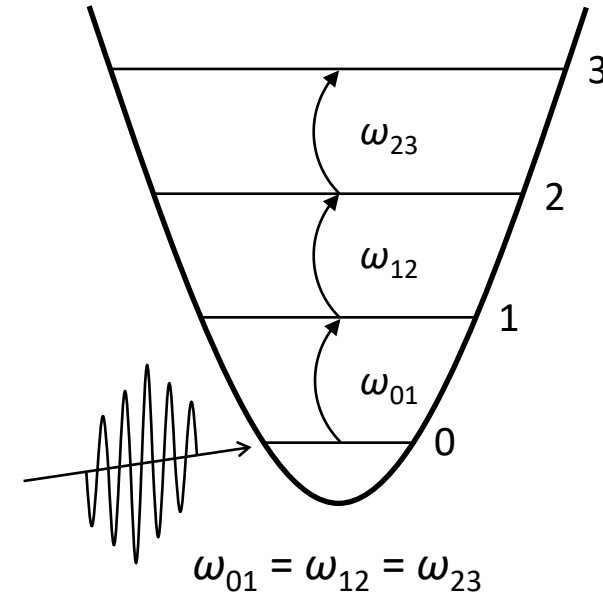
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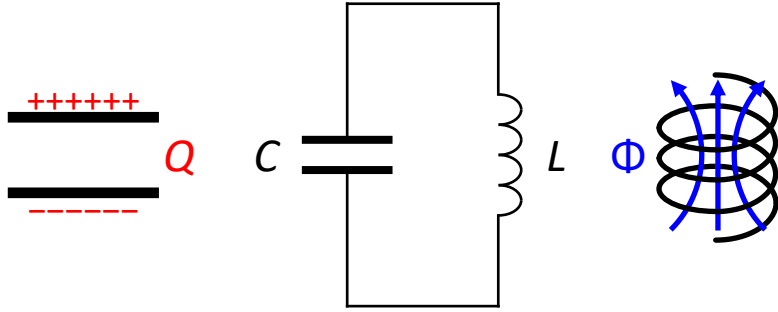
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Harmonic oscillator



Quantum LC circuit



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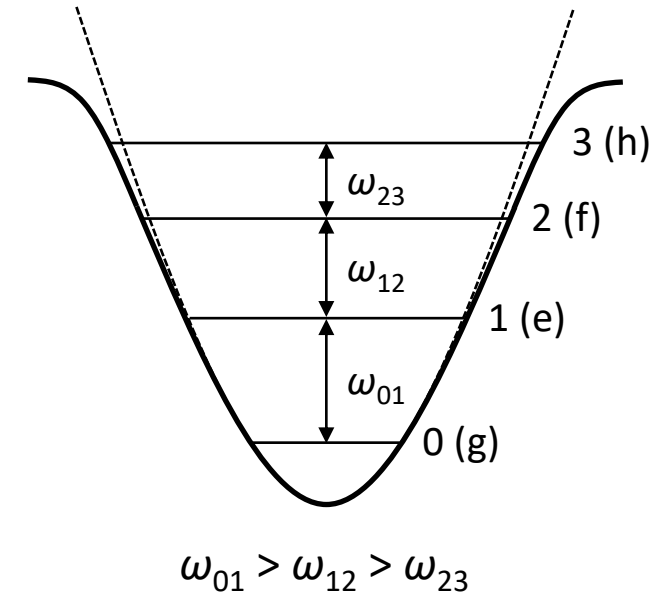
Commutation relation

$$[Q, \Phi] = i\hbar$$

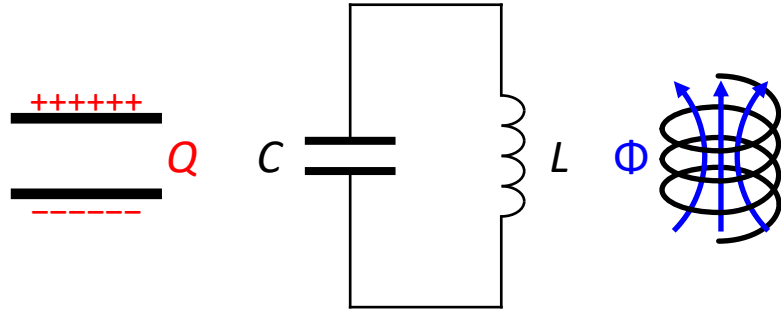
Energy levels

$$E_n = \hbar\omega_0 \left(\frac{1}{2} + n \right) \quad \text{with} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Anharmonic oscillator



Quantum LC circuit



Hamiltonian

$$H_0 = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

Commutation relation

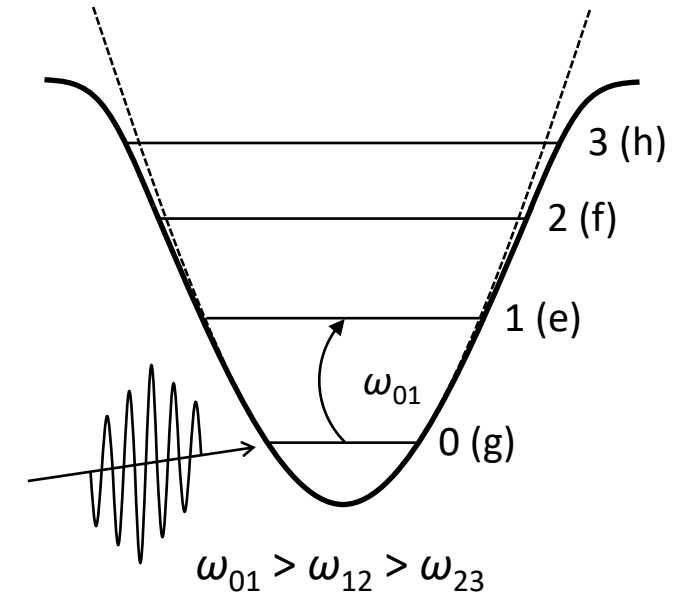
$$[Q, \Phi] = i\hbar$$

Energy levels

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Anharmonic oscillator

→ Qubit

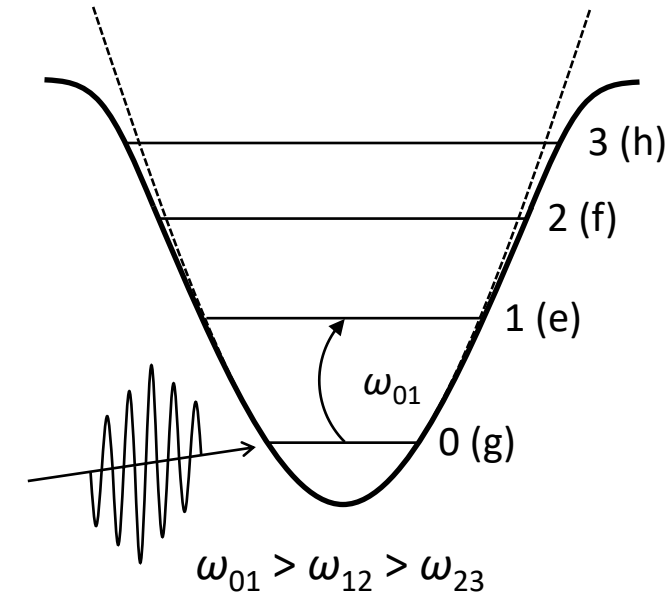


Quantum LC circuit

Anharmonic oscillator

→ Qubit

- We need nonlinearity to selectively address the lowest two levels.
- There are various ways to realize qubits, but in all cases the **nonlinearity is provided by Josephson junctions**.
- This example is known as a **transmon**. A **current-biased Josephson junction** is another example we've already seen. Recall that Leggett originally proposed a **double-well potential** to observe MQC.



Definitions and numbers

Characteristic impedance of the circuit: $Z_0 = \sqrt{\frac{L}{C}}$

Elementary charge: $e = 1.602176634 \times 10^{-19} \text{ C}$ (exact)

Planck constant: $h = \hbar \times 2\pi = 6.62607015 \times 10^{-34} \text{ Js}$ (exact)

Magnetic flux quantum: $\Phi_0 = \frac{1}{R_J} = \frac{h}{2e} \approx 2.067833848 \times 10^{-15} \text{ Wb}$

Quantum resistance: $R_Q = \frac{h}{(2e)^2} \approx 6.45 \text{ k}\Omega$ cf. $R_K = \frac{h}{e^2} \approx 25.8 \text{ k}\Omega$

Vacuum permittivity: $\epsilon_0 = 8.8541878188(14) \times 10^{-12} \text{ F/m}$

Vacuum permeability: $\mu_0 = 1.25663706127(20) \times 10^{-6} \text{ N/A}^2$

Characteristic impedance of vacuum: $Z_{\text{vac}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377 \text{ }\Omega$

Fluctuation and noise in an electrical circuit

Fluctuations of Q and Φ : equipartition theorem

$$\frac{(\delta Q)^2}{2C} \sim \frac{(\delta \Phi)^2}{2L} \sim \frac{1}{4} \hbar \omega_0$$

$$R_Q = \frac{h}{(2e)^2}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\longrightarrow \frac{(\delta Q)^2}{2C} = \frac{1}{4} \times \frac{h}{2\pi} \times \frac{1}{\sqrt{LC}} \longrightarrow \frac{(\delta Q)^2}{(2e)^2} = \frac{1}{(2e)^2} \times \frac{h}{4\pi} \times \sqrt{\frac{C}{L}} \longrightarrow \delta \bar{Q} \equiv \frac{\delta Q}{2e} = \sqrt{\frac{R_Q}{4\pi Z_0}}$$

$$\longrightarrow \frac{(\delta \Phi)^2}{2L} = \frac{1}{4} \times \frac{h}{2\pi} \times \frac{1}{\sqrt{LC}} \longrightarrow \frac{(\delta \Phi)^2}{(\Phi_0)^2} = \frac{(2e)^2}{h^2} \times \frac{h}{4\pi} \times \sqrt{\frac{L}{C}} \longrightarrow \delta \bar{\Phi} \equiv \frac{\delta \Phi}{\Phi_0} = \sqrt{\frac{Z_0}{4\pi R_Q}}$$

$$\Phi_0 = \frac{h}{2e}$$

Protection from noise

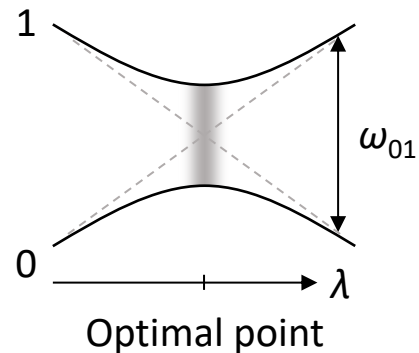
$$\delta\bar{Q} \equiv \frac{\delta Q}{2e} = \sqrt{\frac{R_Q}{4\pi Z_0}} \quad \delta\bar{\Phi} \equiv \frac{\delta\Phi}{\Phi_0} = \sqrt{\frac{Z_0}{4\pi R_Q}} \quad R_Q \approx 6.45 \text{ k}\Omega \quad Z_{\text{vac}} \approx 377 \Omega$$

$Z_0 > R_Q$: high impedance circuit

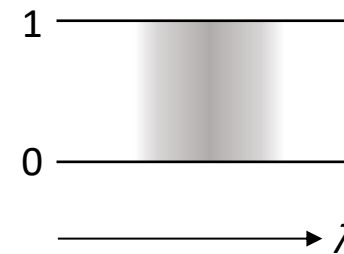
$Z_0 < R_Q$: low impedance circuit

Impedance of an ordinary electromagnetic environment is low,
meaning that the charge noise is much larger than the magnetic noise

Protection from the magnetic noise



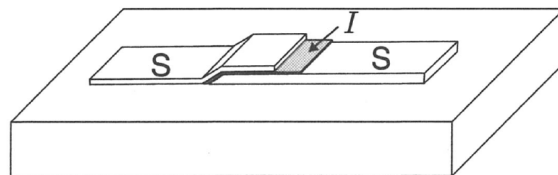
Protection from the charge noise



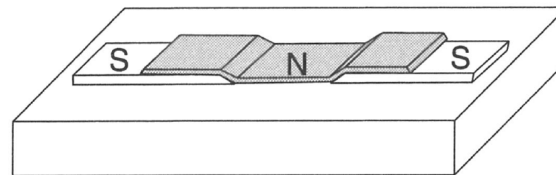
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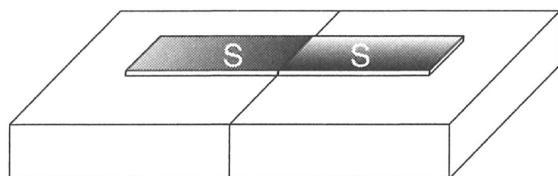
Josephson junction



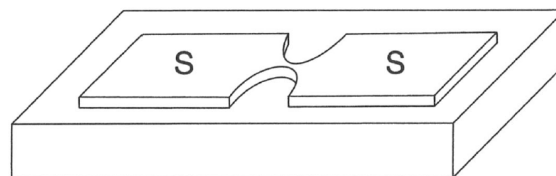
S-I-S junction



S-N-S junction



Grain boundary

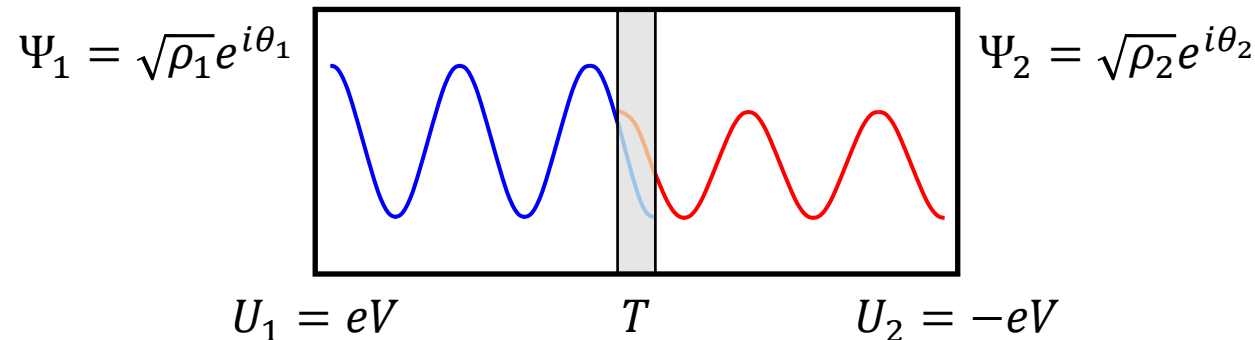


Microbridge

「超伝導」家泰弘 (朝倉書店)

Schrödinger eq. for coupled superconductors

$$\begin{cases} i\hbar\dot{\Psi}_1 = U_1\Psi_1 + T\Psi_2 \\ i\hbar\dot{\Psi}_2 = U_2\Psi_2 + T\Psi_1 \end{cases}$$



Josephson junction

Time derivative of $\Psi_1 = \sqrt{\rho_1} e^{i\theta_1}$

$$\dot{\Psi}_1 = \frac{\dot{\rho}_1}{2\sqrt{\rho_1}} e^{i\theta_1} + i\sqrt{\rho_1} \dot{\theta}_1 e^{i\theta_1}$$

Insert the above into $i\hbar\dot{\Psi}_1 = eV\Psi_1 + T\Psi_2$

$$i\hbar \left(\frac{\dot{\rho}_1}{2\sqrt{\rho_1}} e^{i\theta_1} + i\sqrt{\rho_1} \dot{\theta}_1 e^{i\theta_1} \right) = eV\sqrt{\rho_1} e^{i\theta_1} + T\sqrt{\rho_2} e^{i\theta_2}$$

$$\longrightarrow \quad \frac{\dot{\rho}_1}{2} + i\rho_1 \dot{\theta}_1 = \frac{eV}{i\hbar} \rho_1 + \frac{T}{i\hbar} \sqrt{\rho_1 \rho_2} e^{i\varphi}$$

Add c.c.

$$\dot{\rho}_1 = \frac{2T}{\hbar} \sqrt{\rho_1 \rho_2} \sin \varphi$$

$$\varphi \equiv \theta_2 - \theta_1$$

$$e^{i\varphi} - e^{-i\varphi} = 2i \sin \varphi$$

$$\downarrow$$

$$-i \frac{\dot{\rho}_1}{2\rho_1} + \dot{\theta}_1 = -\frac{eV}{\hbar} - \frac{T}{\hbar} \sqrt{\frac{\rho_2}{\rho_1}} e^{i\varphi}$$

Add c.c.

$$\dot{\theta}_1 = -\frac{eV}{\hbar} - \frac{T}{\hbar} \sqrt{\frac{\rho_2}{\rho_1}} \cos \varphi$$

$$e^{i\varphi} + e^{-i\varphi} = 2 \cos \varphi$$

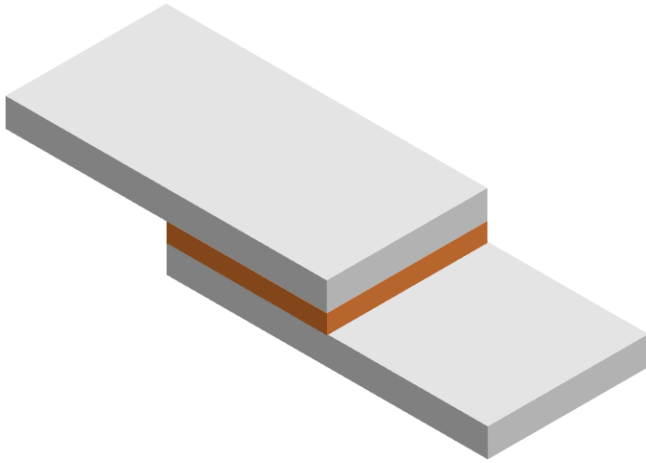
Josephson junction

$$\begin{array}{lcl}
 \dot{\rho}_1 = \frac{2T}{\hbar} \sqrt{\rho_1 \rho_2} \sin \varphi & \left. \vphantom{\begin{array}{l} \dot{\rho}_1 \\ \dot{\rho}_2 \end{array}} \right\} \longrightarrow & \frac{2T}{\hbar} \sqrt{\rho_1 \rho_2} \approx \frac{2T}{\hbar} \rho_0 \approx I_c \longrightarrow I = I_c \sin \varphi \\
 \dot{\rho}_2 = -\frac{2T}{\hbar} \sqrt{\rho_1 \rho_2} \sin \varphi = -\dot{\rho}_1 & & \longrightarrow \frac{dI}{dt} = I_c \cos \varphi \frac{d\varphi}{dt} \\
 \\
 \dot{\theta}_1 = -\frac{eV}{\hbar} - \frac{T}{\hbar} \sqrt{\frac{\rho_2}{\rho_1}} \cos \varphi & \left. \vphantom{\begin{array}{l} \dot{\theta}_1 \\ \dot{\theta}_2 \end{array}} \right\} \longrightarrow & \dot{\varphi} = \frac{2eV}{\hbar} \longrightarrow V = \frac{\hbar}{2e} \frac{d\varphi}{dt} \\
 \dot{\theta}_2 = \frac{eV}{\hbar} - \frac{T}{\hbar} \sqrt{\frac{\rho_2}{\rho_1}} \cos \varphi & \longrightarrow & V = \underbrace{\frac{\hbar}{2eI_c \cos \varphi} \frac{dI}{dt}}_{L_J} \approx \frac{\hbar}{2eI_c} \frac{1}{\sqrt{1 - (I/I_c)^2}} \frac{dI}{dt}
 \end{array}$$

Nonlinear inductor

$$L_J(0) = \frac{\hbar}{2eI_c} = 330 \text{ nH} \cdot \text{nA} \times I_c^{-1}$$

Josephson junction



JJ as a capacitor

$$C_J = \epsilon_0 \epsilon_r \frac{S_J}{d_J}$$

← Junction area
 ← Junction thickness

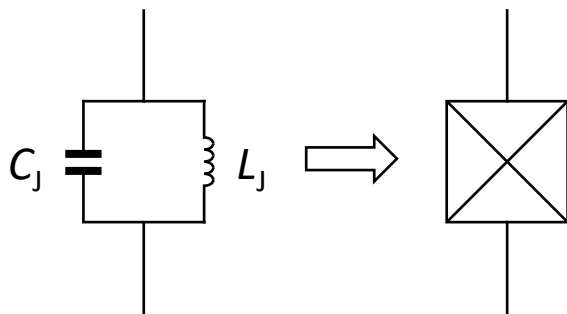
Plasma frequency

$$\frac{\omega_p}{2\pi} = \frac{1}{\sqrt{L_J(0)C_J}} = 9.4 \text{ GHz}$$

Josephson energy

$$\frac{E_J}{h} \approx 5 \text{ THz}$$

$$E_J = \frac{\hbar I_c}{2e}$$



e.g.

$$\epsilon_r = 10$$

$$d_J = 1 \text{ nm}$$

$$S_J = 10 \times 10 \text{ } \mu\text{m}^2$$

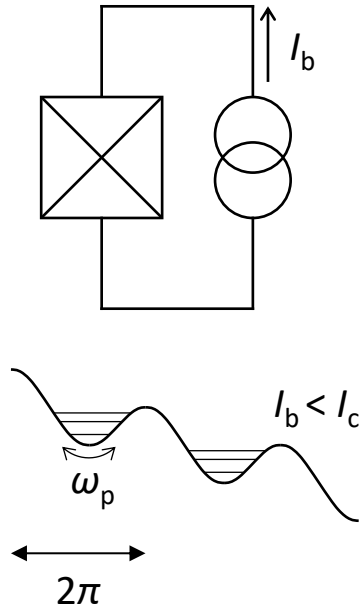
$$I_c = 10 \text{ } \mu\text{A}$$

$$L_J(0) = 33 \text{ pH}$$

$$C_J = 8.9 \text{ pF}$$

→ 1000+ levels in a single potential well

Current-biased phase qubit



$$H_1 = 4E_C n^2 - E_J \cos \varphi - \frac{\hbar}{2e} I_b \varphi$$

$\frac{e^2}{2C_\Sigma}$ (Charging energy)
 $[n, \varphi] = i\hbar$ (Particle number passing through the junction)

$$H_0 = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$$\frac{E_J}{E_C} > 10^4$$

$$\frac{\omega_p}{2\pi} < 5 \text{ GHz @ } I_b = 0.97 \times I_c$$

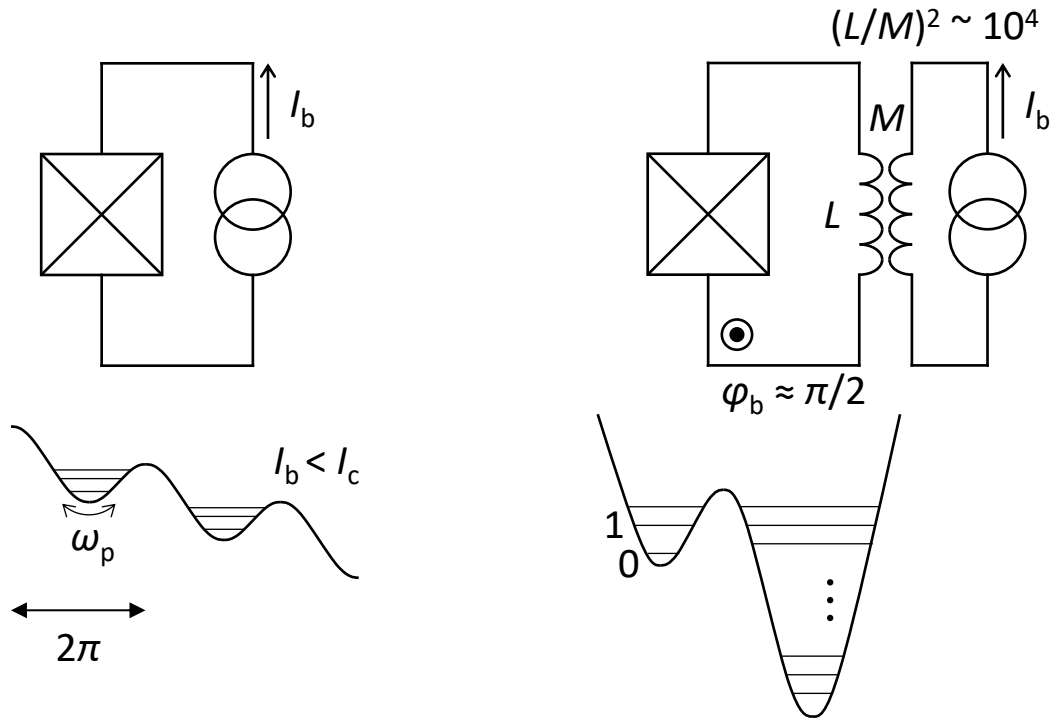
Is this a good qubit?

- The current source (50 Ω) directly connected to the junction is a noise source

Solution

- Flux-biasing via impedance transformation

Flux-biased phase qubit



$$H_2 = 4E_C n^2 - E_J \cos \left(\varphi + 2\pi \frac{LI_b}{\Phi_0} \right) + \frac{1}{2} E_L \varphi^2$$

Magnetic energy

$$\frac{1}{L} \left(\frac{\hbar}{2e} \right)^2$$

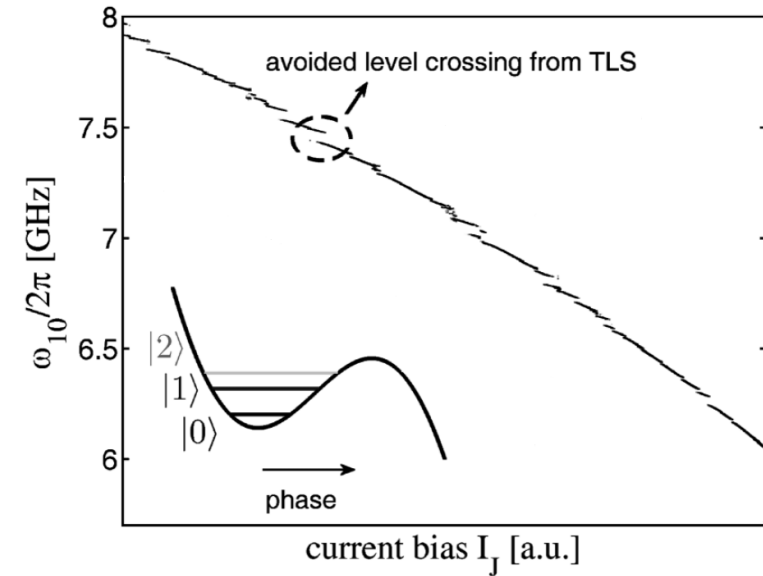
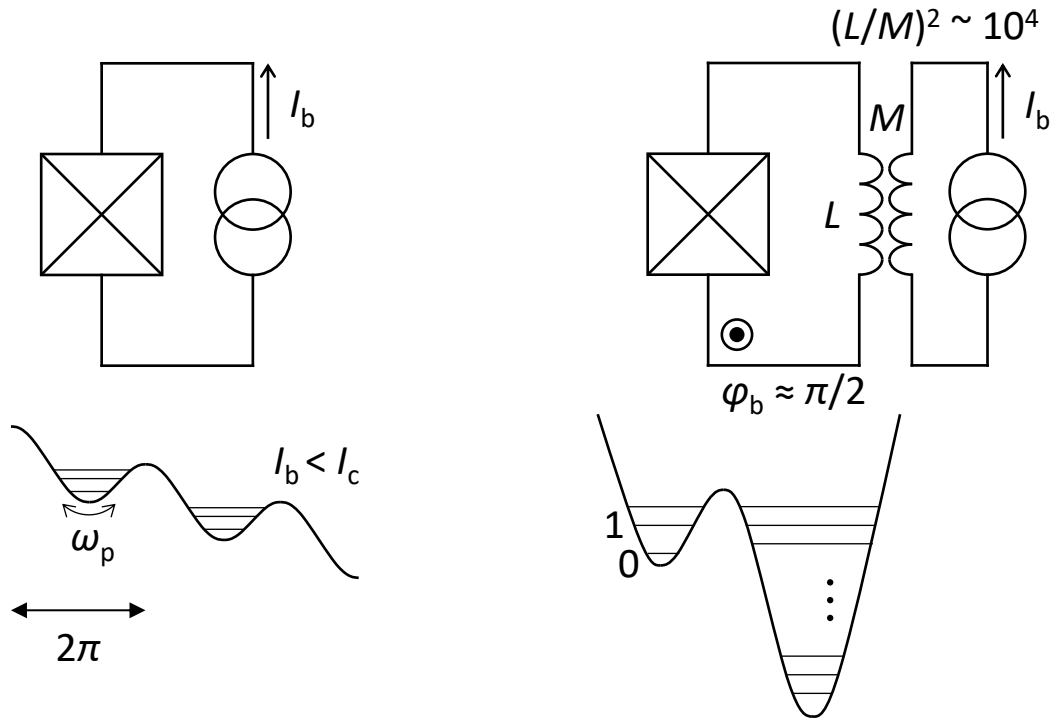
Is this a good qubit?

- The current source (50 Ω) directly connected to the junction is a noise source

Solution

- Flux-biasing via impedance transformation

Flux-biased phase qubit



Phys. Rev. Lett. **97**, 050502 (2006) M. Steffen *et al.*

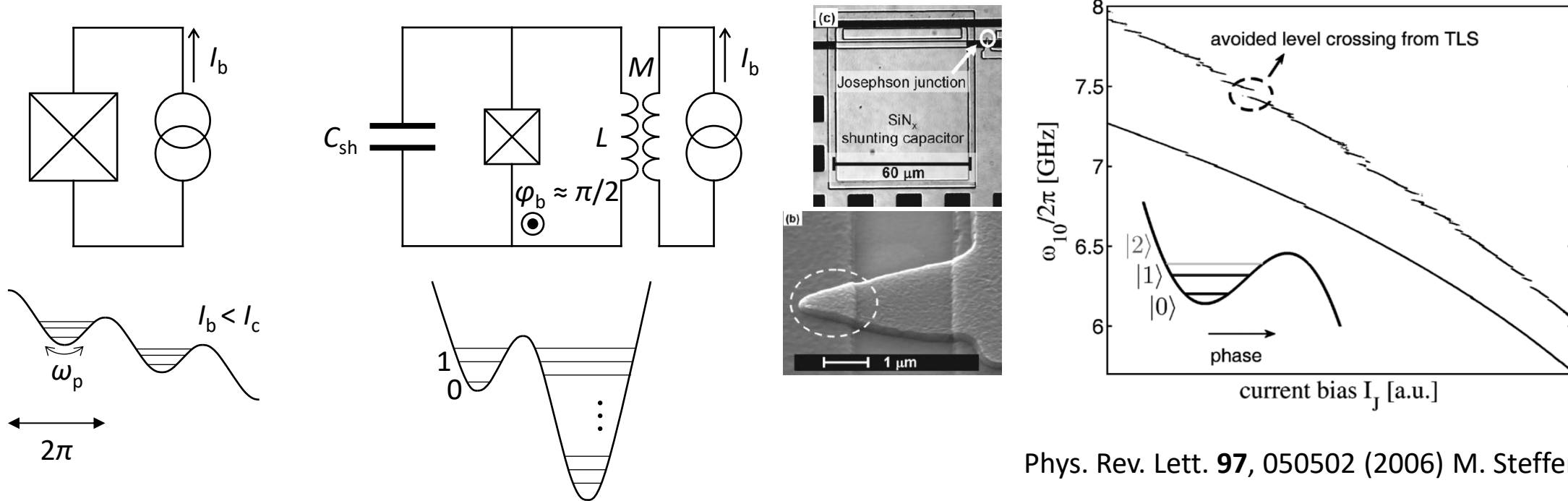
Is this a good qubit?

- The current source (50 Ω) directly connected to the junction is a noise source
- A large junction has a lot of defects, known as two-level systems (TLSs)

Solution

- Flux-biasing via impedance transformation

Flux-biased C-shunted phase qubit



Phys. Rev. Lett. **97**, 050502 (2006) M. Steffen *et al.*

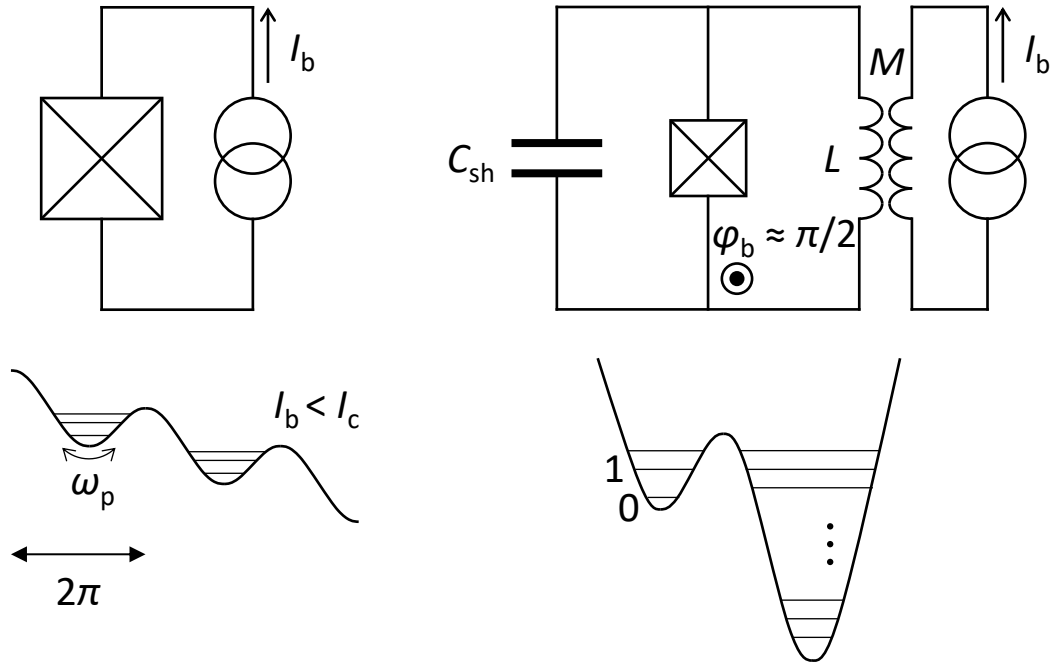
Is this a good qubit?

- The current source ($50\ \Omega$) directly connected to the junction is a noise source
- A large junction has a lot of defects, known as two-level systems (TLSs)

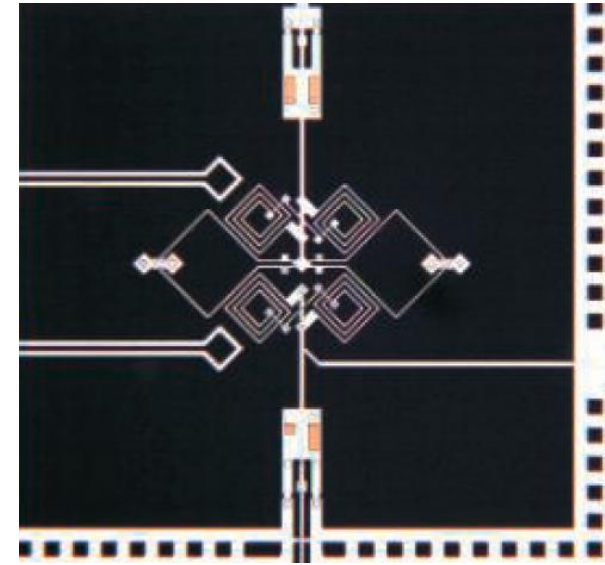
Solution

- Flux-biasing via impedance transformation
- A large shunt-capacitance and a smaller junction

Flux-biased C-shunted phase qubit



$$T_{1,2} < 1 \mu s$$



Science **307**, 1299 (2005) R. McDermott *et al.*

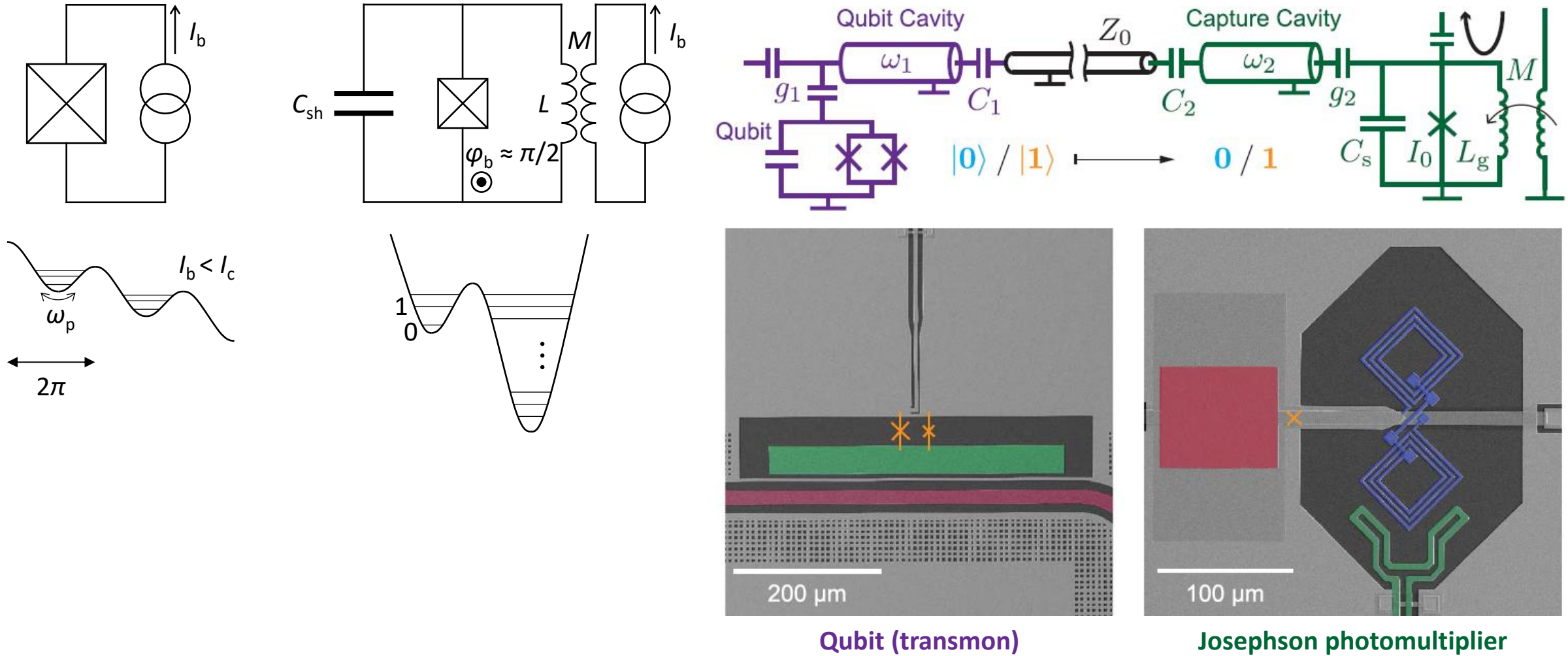
Is this a good qubit?

- The current source (50 Ω) directly connected to the junction is a noise source
- A large junction has a lot of defects, known as two-level systems (TLSs)

Solution

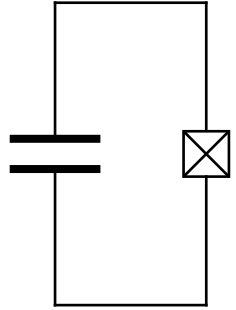
- Flux-biasing via impedance transformation
- A large shunt-capacitance and a smaller junction

Flux-biased C-shunted phase qubit as JPM



Transmon

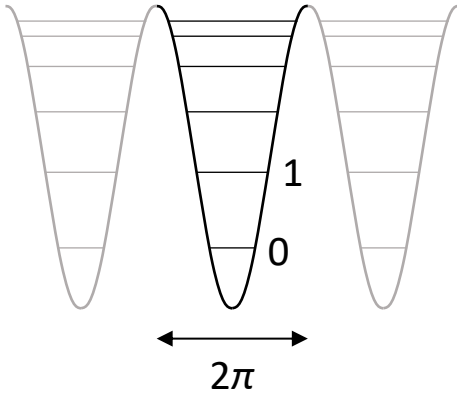
Transmission line shunted plasma oscillation qubit



$$H_3 = 4E_C(n - n_g)^2 - E_J \cos \theta$$

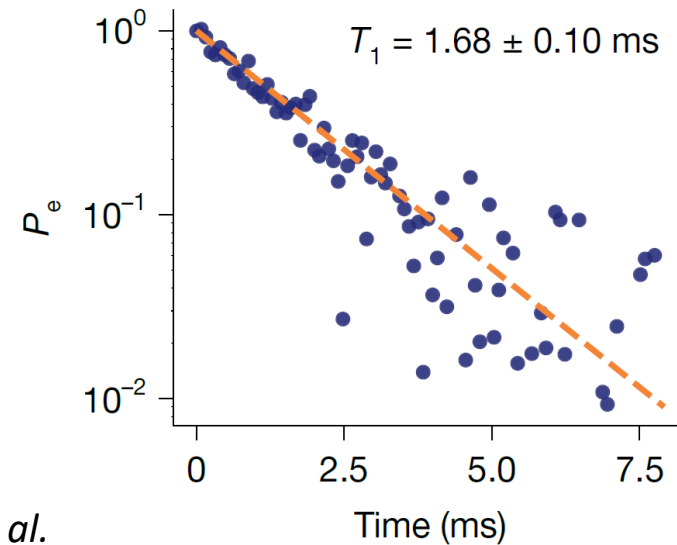
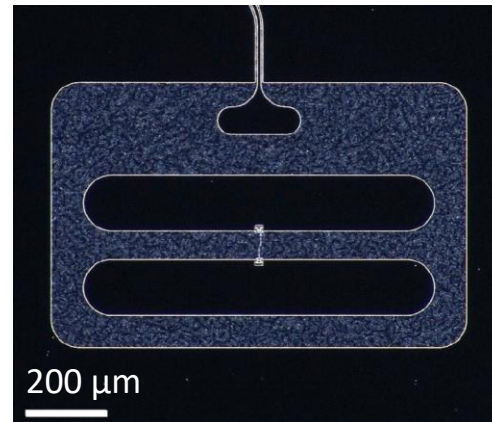
$\uparrow$
 2π -periodic

$$\frac{E_J}{E_C} \sim 50$$



J. Supercond. Nov. Magn. **34**, 1633(2021) M. H. Devoret

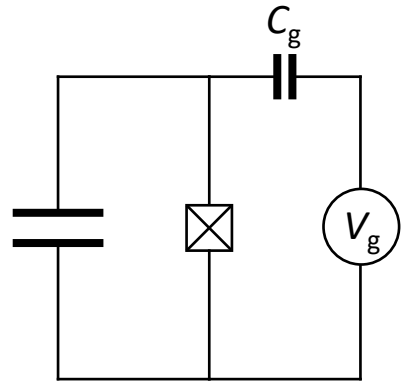
“Does Brian Josephson’s Gauge-Invariant Phase Difference Live on a Line or a Circle?”



Nature **647**, 343 (2025) M. P. Bland *et al.*

Transmon

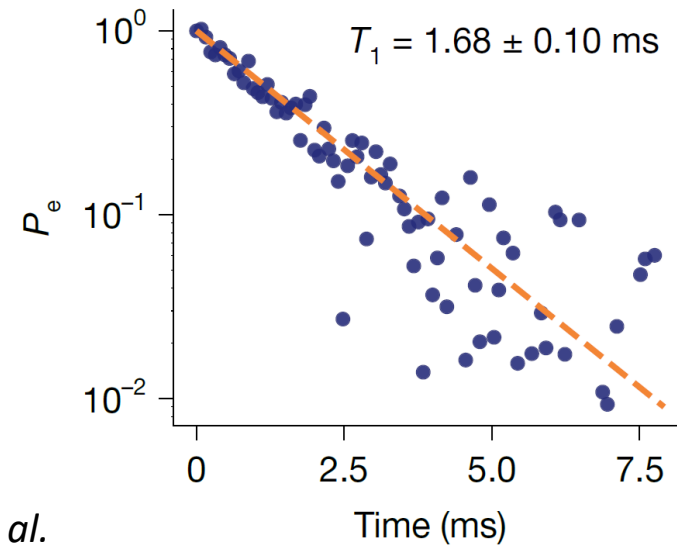
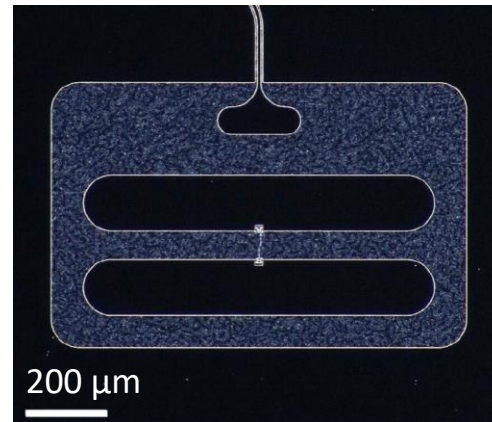
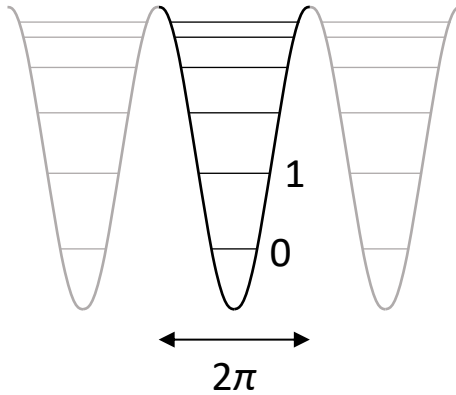
Transmission line shunted plasma oscillation qubit



$$H_3 = 4E_C(n - n_g)^2 - E_J \cos \theta$$

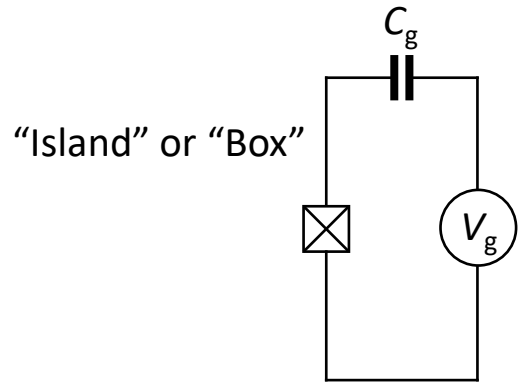
↑
Charge offset

$$\frac{E_J}{E_C} \sim 50$$



Nature **647**, 343 (2025) M. P. Bland *et al.*

Charge qubit

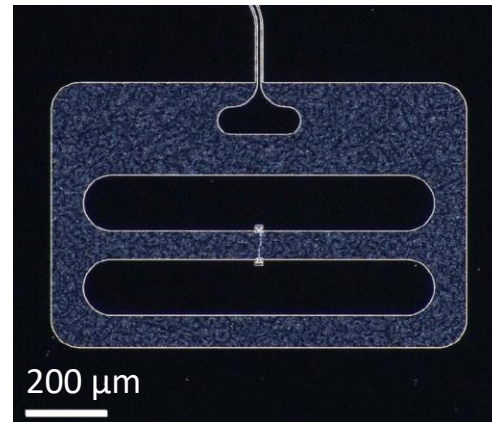
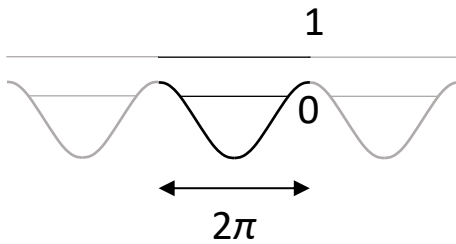


$$H_3 = 4E_C(n - n_g)^2 - E_J \cos \theta$$

↑
Charge offset

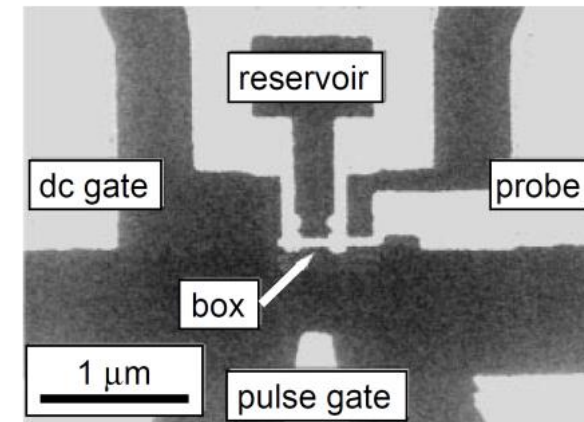
$$\frac{E_J}{E_C} < 1$$

$$n_g = 0.5$$



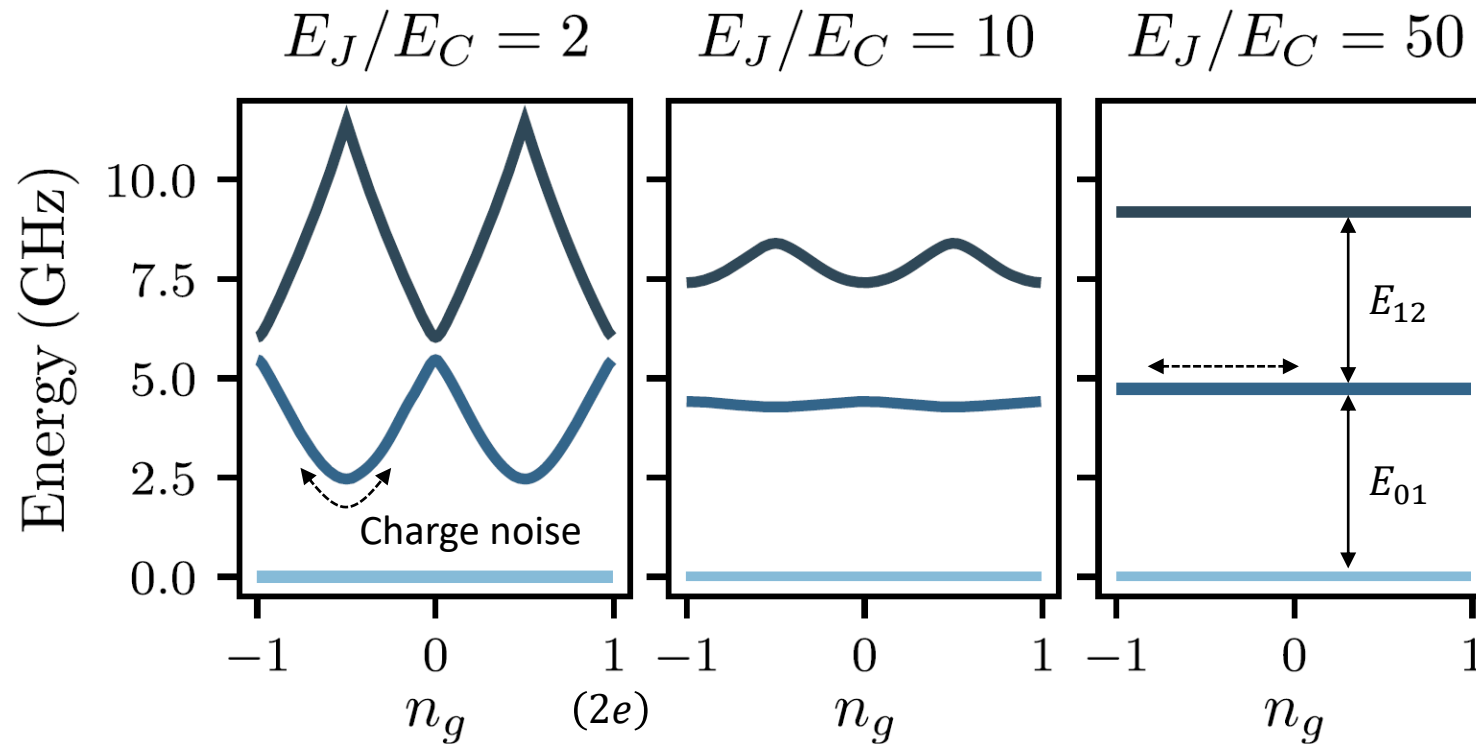
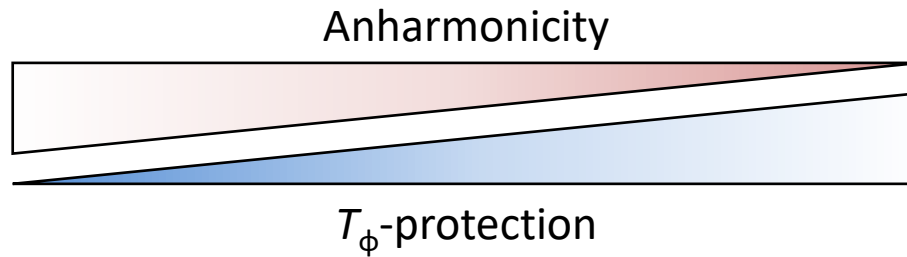
Nature **647**, 343 (2025) M. P. Bland *et al.*

$$T_2 \approx 1 \text{ ns}$$



Nature **398**, 786 (1999) Y. Nakamura *et al.*

From charge qubit to transmon



$$\frac{1}{T_\phi} \propto \left| \frac{\partial \omega_{01}}{\partial \lambda} \right|^2 S_\lambda(\omega \rightarrow 0)$$

$$\frac{1}{T_1} \propto |\langle 0|O|1 \rangle|^2 S_\lambda(\omega_{01})$$

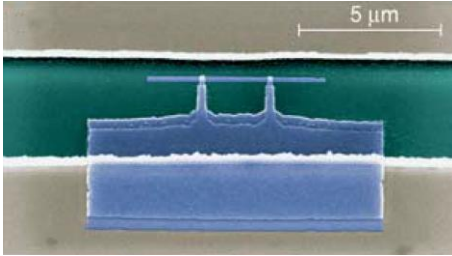
Not protected from T_1 process
→ Materials matters

$$\alpha \equiv E_{12} - E_{01} = -E_C$$

$$E_{01} = \sqrt{8E_C E_J} - E_C$$

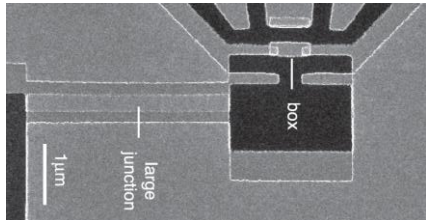
Improving T_1 and T_2

Yale QPB in Circuit QED



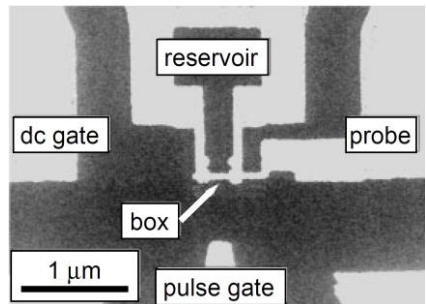
Nature **431**, 162 (2004) Wallraff *et al.*
Phys. Rev. Lett. **95**, 060501 (2005) Wallraff *et al.*

Saclay Quantronium

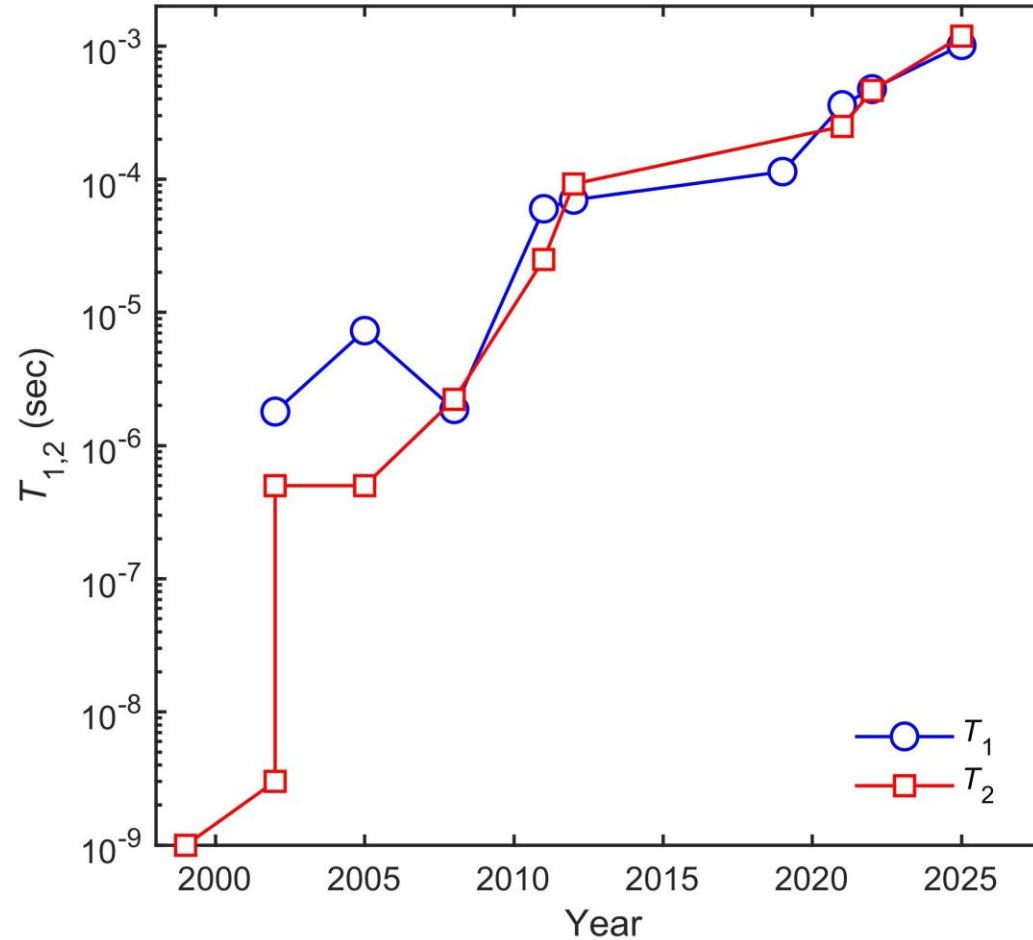


Science **296**, 886 (2002) Vion *et al.*

NEC Cooper Pair Box



Nature **398**, 786 (1999) Nakamura *et al.*
Phys. Rev. Lett. **88**, 047901 (2002) Nakamura *et al.*



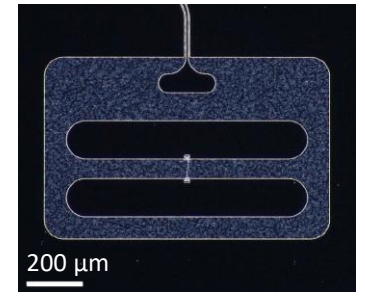
IBM 3D: Phys. Rev. B **86**, 100506 (2012) Rigetti *et al.*

Rigetti 2D: IEEE IEDM (2019)/arXiv:1901.08042 Nersisyan *et al.*

Princeton 2D Ta-based: Nature Commun. **12**, 1779 (2021) Place *et al.*

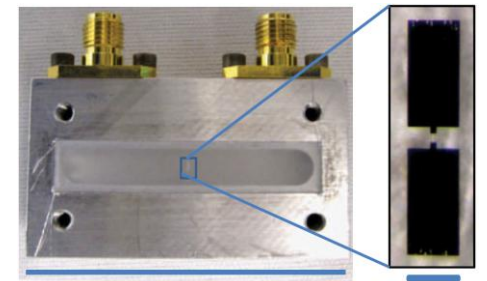
Beijing Academy of QIS 2D: npj Quantum Info. **8**, 3 (2022) Wang *et al.*

Princeton 2D Ta-based



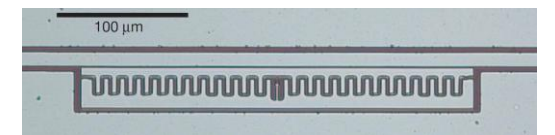
Nature **647**, 343 (2025) Bland *et al.*

Yale 3D Transmon



Phys. Rev. Lett. **107**, 240501 (2011) Paik *et al.*

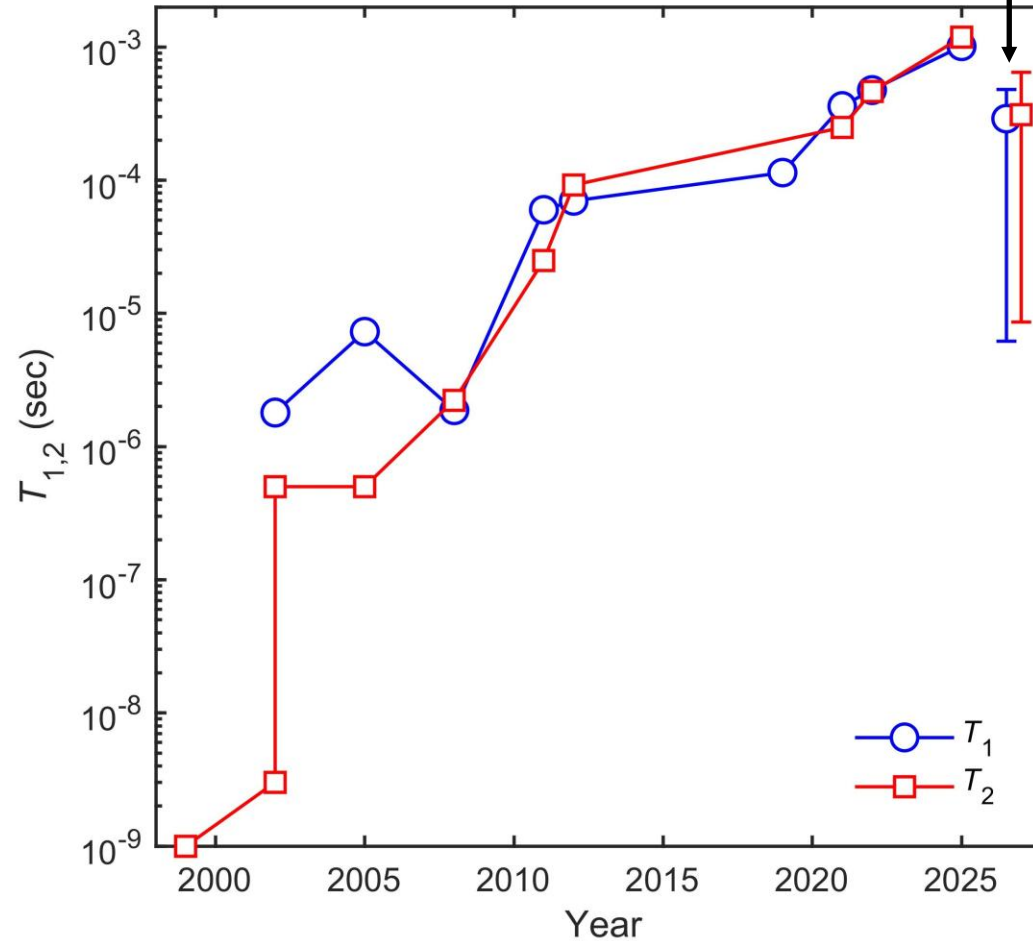
Yale 2D Transmon



Nature **449**, 328 (2007) Houck *et al.*
Phys. Rev. B **77**, 180502 (2008) Schreier *et al.*

Improving T_1 and T_2

IBM 156Q Heron r3 “ibm_pittsburgh” (Median)



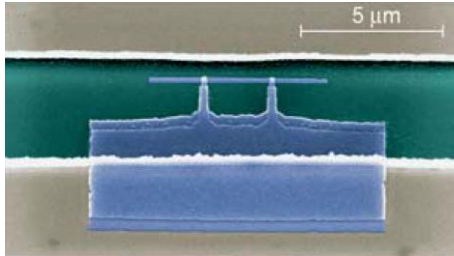
IBM 3D: Phys. Rev. B **86**, 100506 (2012) Rigetti *et al.*

Rigetti 2D: IEEE IEDM (2019)/arXiv:1901.08042 Nersisyan *et al.*

Princeton 2D Ta-based: Nature Commun. **12**, 1779 (2021) Place *et al.*

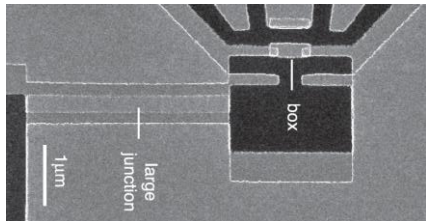
Beijing Academy of QIS 2D: npj Quantum Info. **8**, 3 (2022) Wang *et al.*

Yale QPB in Circuit QED



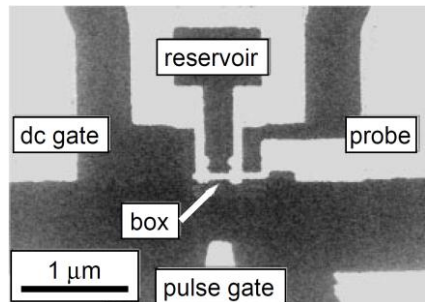
Nature **431**, 162 (2004) Wallraff *et al.*
Phys. Rev. Lett. **95**, 060501 (2005) Wallraff *et al.*

Saclay Quantronium



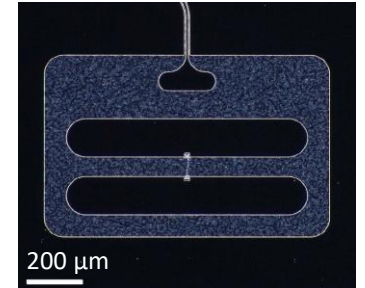
Science **296**, 886 (2002) Vion *et al.*

NEC Cooper Pair Box



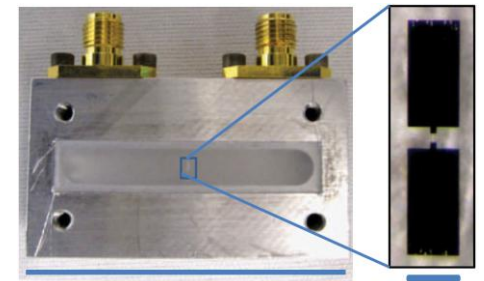
Nature **398**, 786 (1999) Nakamura *et al.*
Phys. Rev. Lett. **88**, 047901 (2002) Nakamura *et al.*

Princeton 2D Ta-based



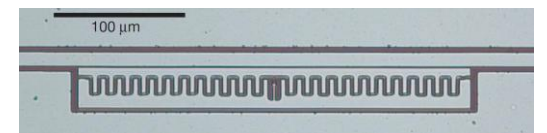
Nature **647**, 343 (2025) Bland *et al.*

Yale 3D Transmon



Phys. Rev. Lett. **107**, 240501 (2011) Paik *et al.*

Yale 2D Transmon



Nature **449**, 328 (2007) Houck *et al.*
Phys. Rev. B **77**, 180502 (2008) Schreier *et al.*

Dielectric loss

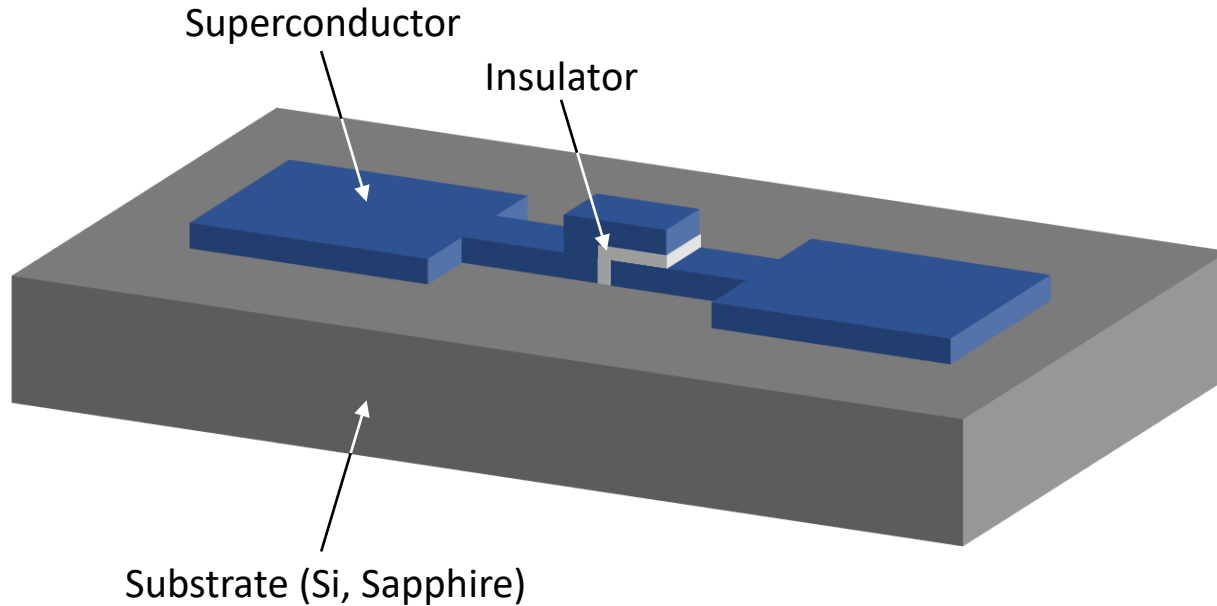
Q factor

$$\frac{1}{Q} = \frac{1}{\omega_q T_1} = \sum_i \frac{p_i}{Q_i} = \sum_i p_i \tan \delta_i$$

Participation ratio Loss tangent

Dielectric loss @low- T (+ single-photon level)

- Superconducting film (Al, Nb, TiN, NbN, Ta...) $< 10^{-5}$
- Substrate (Si, Sapphire) $< 10^{-6}$
- Amorphous material (SiO_x , AlO_x , SiN_x) $< 10^{-3}$



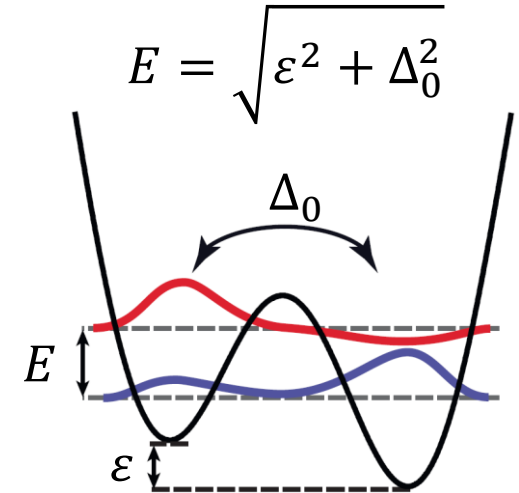
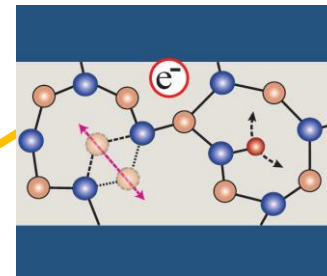
Dielectric loss

Q factor

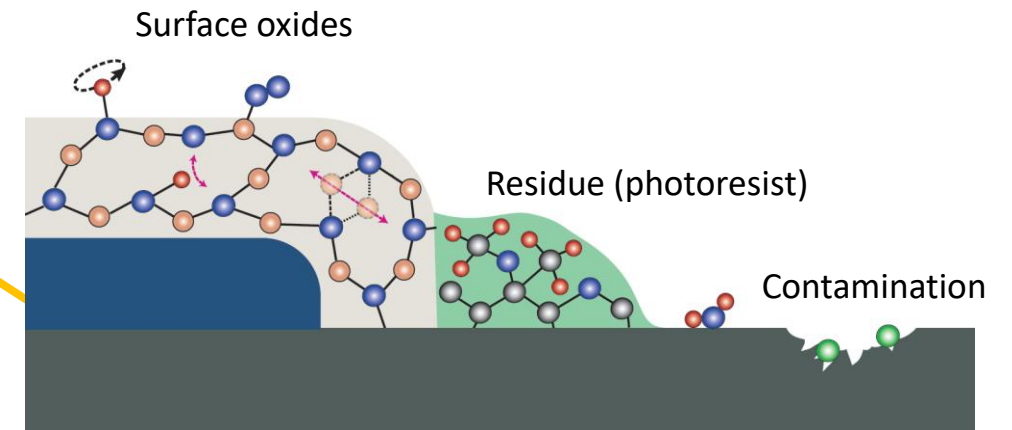
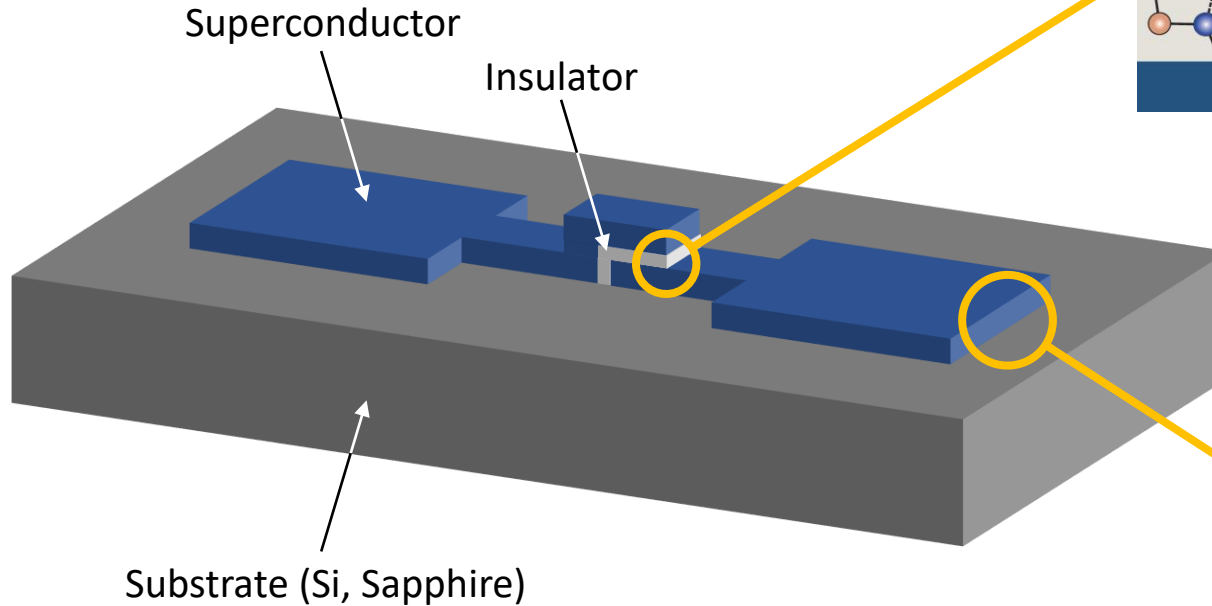
$$\frac{1}{Q} = \frac{1}{\omega_q T_1} = \sum_i \frac{p_i}{Q_i} = \sum_i p_i \tan \delta_i$$

Participation ratio
Loss tangent

Amorphous layer



Rep. Prog. Phys. **82**, 124501 (2019) C. Müller *et al.*



npj Quantum Inf. **5**, 105 (2019) J. Lisenfeld *et al.*

Dielectric loss

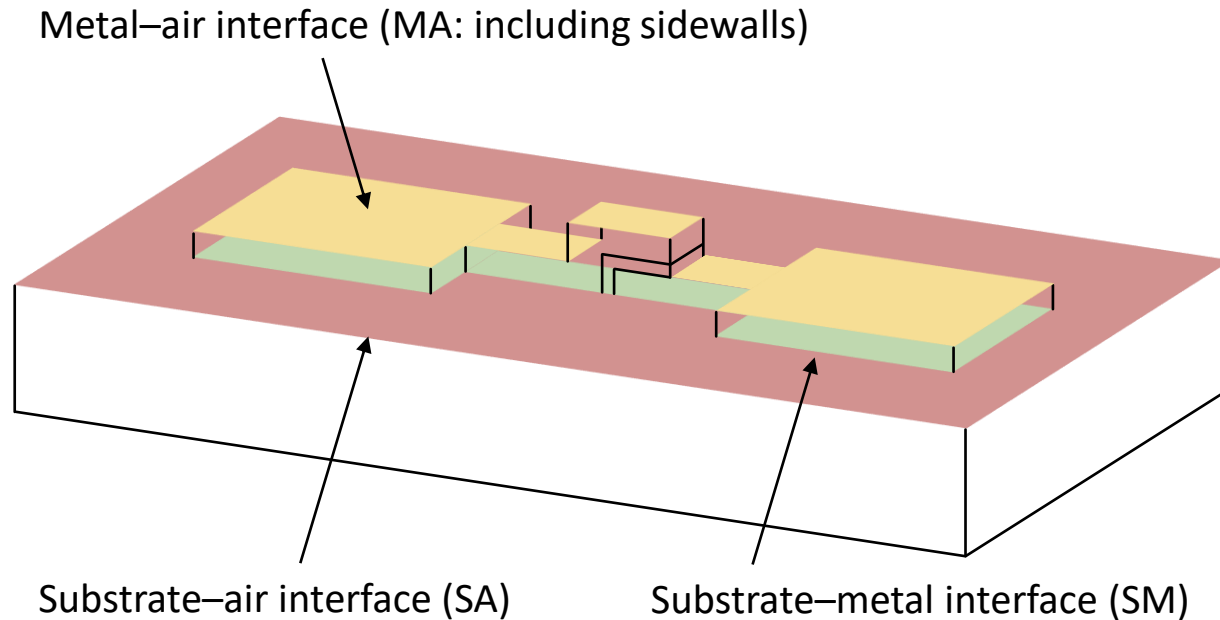
Q factor

$$\frac{1}{Q} = \frac{1}{\omega_q T_1} = \sum_i \frac{p_i}{Q_i} = \sum_i p_i \tan \delta_i$$

Participation ratio Loss tangent

Dielectric loss @low- T (+ single-photon level)

- Superconducting film (Al, Nb, TiN, NbN, Ta...) $< 10^{-5}$
- Substrate (Si, Sapphire) $< 10^{-6}$
- Amorphous material (SiO_x , AlO_x , SiN_x) $< 10^{-3}$



Loss tangent at the interface

$$p_i = \frac{\epsilon_i t_i}{2U_{\text{tot}}} \int_{S_i} |E_i|^2 ds$$

Dielectric loss

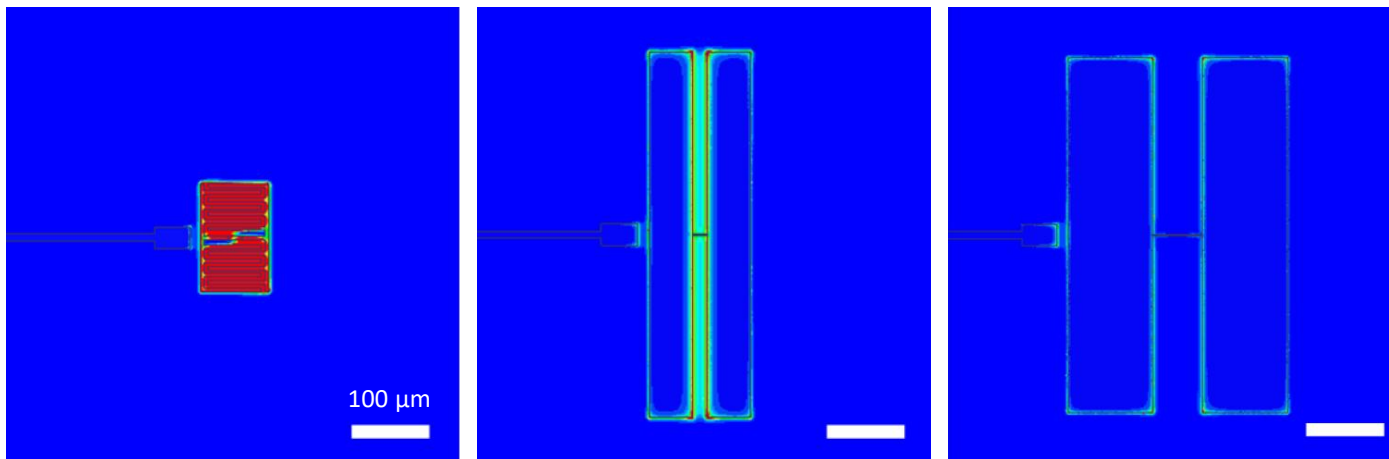
Q factor

$$\frac{1}{Q} = \frac{1}{\omega_q T_1} = \sum_i \frac{p_i}{Q_i} = \sum_i \underbrace{p_i}_{\text{Participation ratio}} \underbrace{\tan \delta_i}_{\text{Loss tangent}}$$

Dielectric loss @low- T (+ single-photon level)

- Superconducting film (Al, Nb, TiN, NbN, Ta...) $< 10^{-5}$
- Substrate (Si, Sapphire) $< 10^{-6}$
- Amorphous material (SiO_x , AlO_x , SiN_x) $< 10^{-3}$

E-field distributions in 3 designs sharing the same C



IDC gap: 5 μm

Paddle gap: 20 μm

Paddle gap: 70 μm

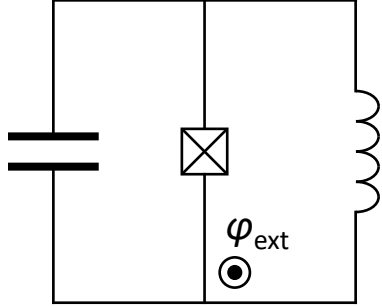
Loss tangent at the interface

$$p_i = \frac{\epsilon_i t_i}{2U_{\text{tot}}} \int_{S_i} |E_i|^2 ds$$

Contents

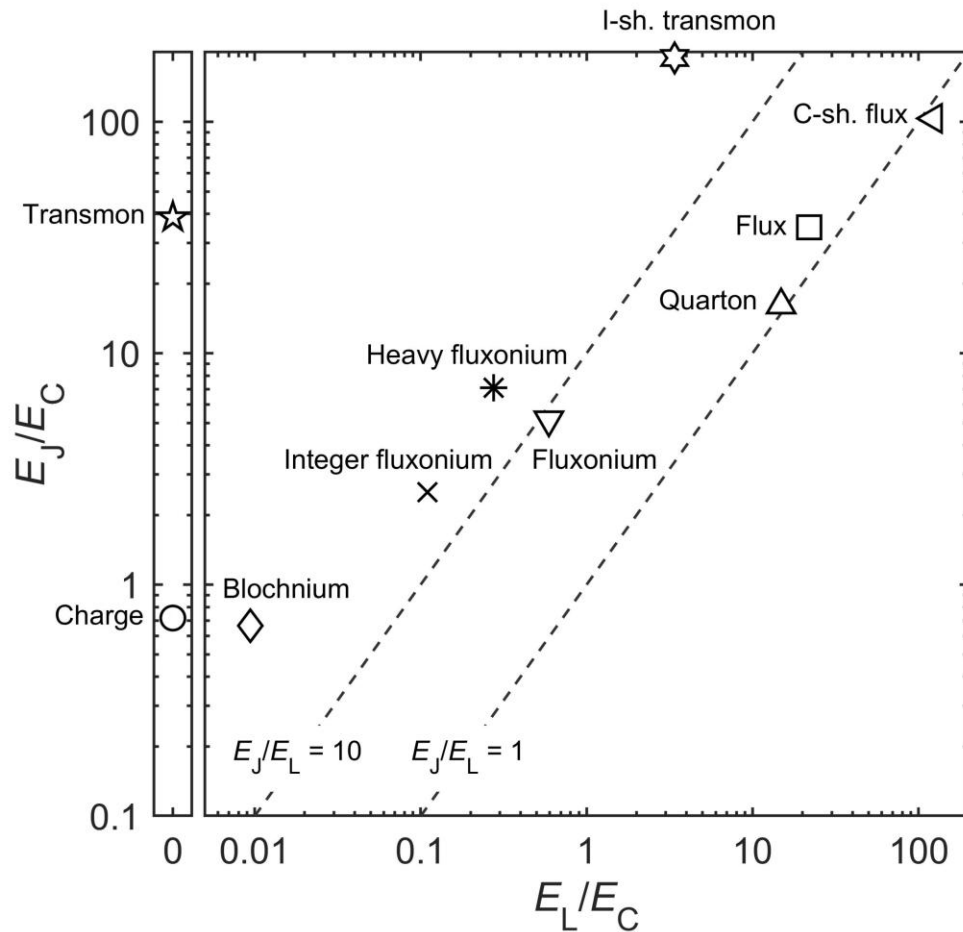
- Brief history: from discovery of superconductivity to dawn of quantum era
- **Superconducting qubits**
 - Qubit, coherence time, and protection from noise
 - Josephson junction
 - Plasma oscillation qubit: phase qubit and transmon (derived from charge qubit)
 - **Loop-based qubit: flux qubit, fluxonium, and Blochonium (dual of transmon)**
 - Protected qubit: $0-\pi$ qubit
- Quantum operations with transmons
 - Circuit QED
 - Dispersive readout
 - Single- and two-qubit gates
- Building quantum computers (→ Symposium talk)

Loop-based qubit



$$H_4 = 4E_C n^2 - E_J \cos(\varphi + \varphi_{\text{ext}}) + \frac{1}{2} E_L \varphi^2$$

Device parameters of superconducting qubits



(Source)

Transmon: Nature **647**, 343 (2025) M. P. Bland *et al.* (Device 17)

Charge: Nature **398**, 786 (1999) Y. Nakamura *et al.*

C-sh. flux: Nature Commun. **7**, 12964 (2016) F. Yan *et al.* (Device B)

Flux: Science **299**, 1869 (2003) I. Chiorescu *et al.*

Qarton: arXiv:2006.04130 F. Yan *et al.* (Device C)

I-sh. transmon: Nature Commun. **14**, 3968 (2023) F. Hassani *et al.* (Device B)

Fluxonium: Phys. Rev. Lett. **130**, 267001 (2023) A. Somoroff *et al.*

Heavy fluxonium: Phys. Rev. X **11**, 011010 (2021) H. Zhang *et al.*

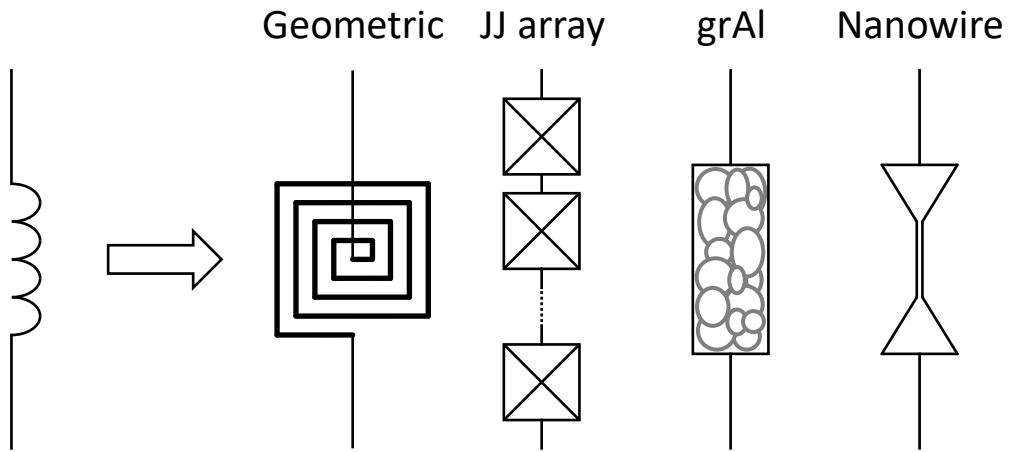
Integer fluxonium : PRX Quantum **5**, 040318 (2024) R. A. Mencia *et al.* (Device A)

Blochonium: Nature **585**, 368 (2020) I. V. Pechenezhskiy *et al.*

* Plotted are representative values. The numbers will vary with devices & designs

** Phase qubit is omitted from the plot because of extremely large $E_J/E_C \sim 10^4$

Superinductor



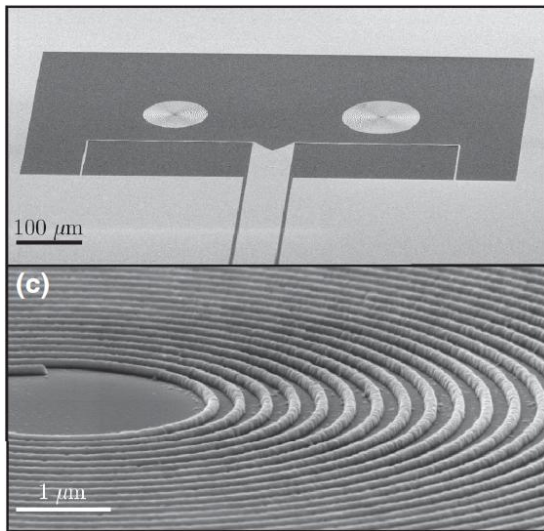
High impedance circuit up to 10 GHz

$$Z_0 = \sqrt{\frac{L}{C_s}} \geq R_Q \approx 6.45 \text{ k}\Omega$$

$$\omega_0 = \frac{1}{\sqrt{LC_s}} \leq 2\pi \times 10 \text{ GHz}$$

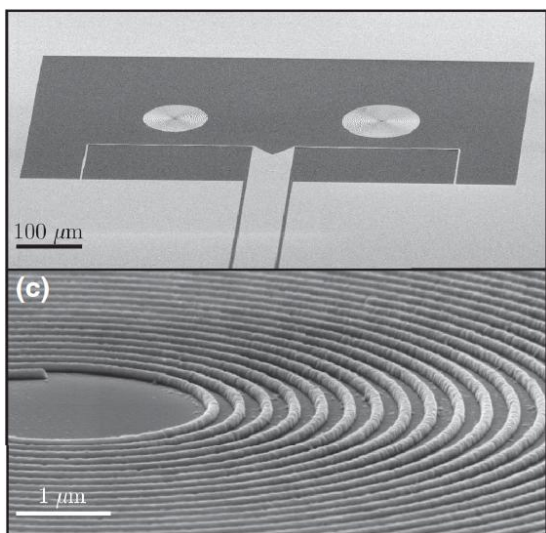
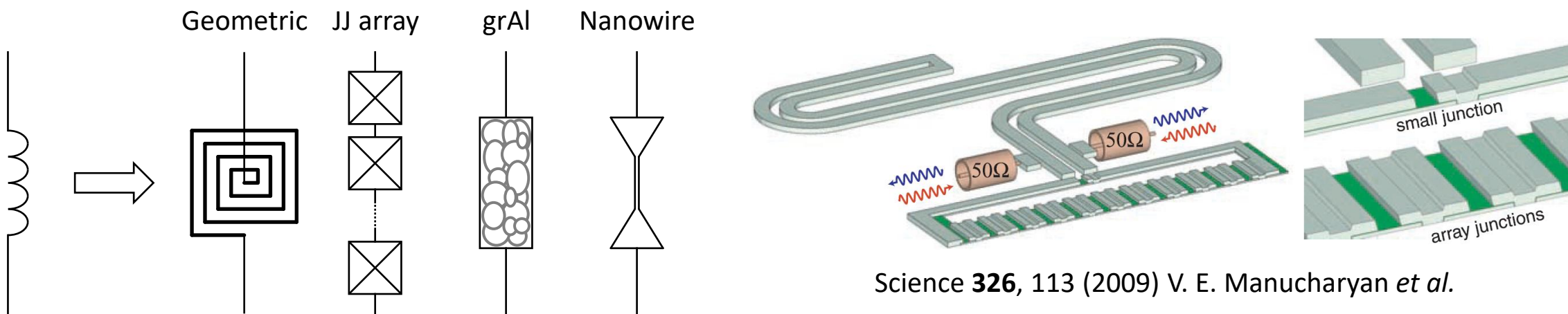
$$\longrightarrow L \geq \frac{6.45 \text{ k}\Omega}{2\pi \times 10 \text{ GHz}} \sim 100 \text{ nH}$$

In spiral inductors, C_s (stray capacitance) becomes larger as the winding number increases, reducing both Z_0 and ω_0

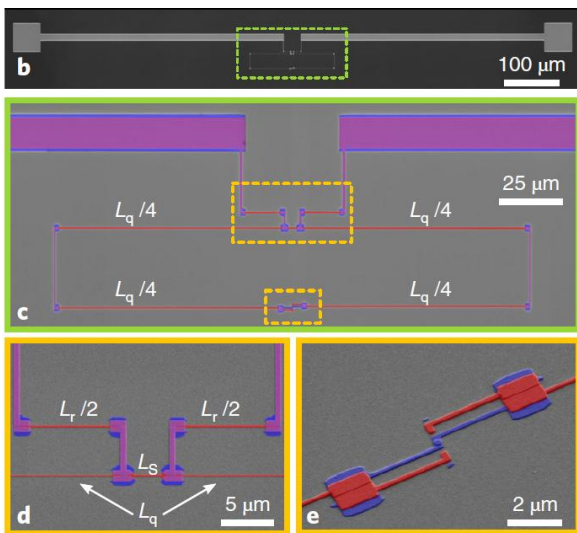


Phys. Rev. Appl. **14**, 044055 (2020)
M. Peruzzo *et al.*

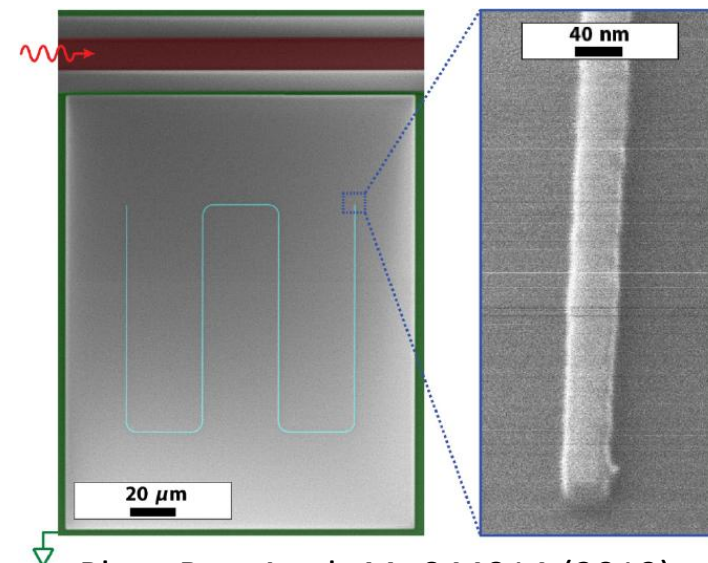
Superinductor



Phys. Rev. Appl. **14**, 044055 (2020)
M. Peruzzo *et al.*

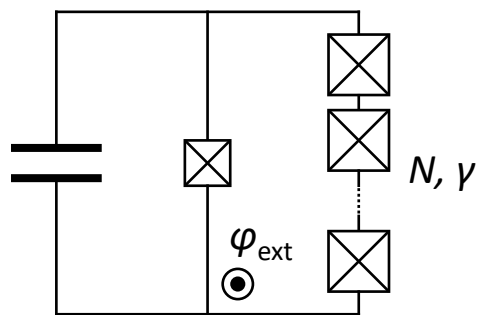
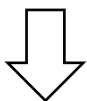
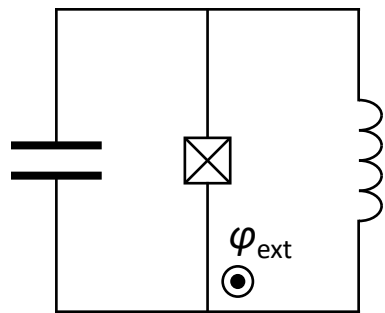


Nature Mater. **18**, 816 (2019)
L. Grünhaupt *et al.*



Phys. Rev. Appl. **11**, 044014 (2019)
D. Niepce *et al.*

Multi-junction qubit



$$H_4 = 4E_C n^2 - E_J \cos(\varphi + \varphi_{\text{ext}}) + \frac{1}{2} E_L \varphi^2$$

$$H_5 = 4E_C n^2 - E_J \cos(\varphi + \varphi_{\text{ext}}) - \gamma N E_J \cos\left(\frac{\varphi}{N}\right)$$

@ $\varphi_{\text{ext}} = \pi$

$$\approx 4E_C n^2 + E_J \left(\frac{\gamma/N - 1}{2} \varphi^2 + \frac{1}{24} \varphi^4 \right)$$

$$E_L \approx E_J \frac{\gamma}{N}$$

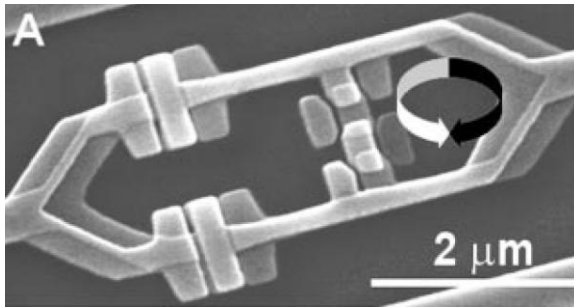
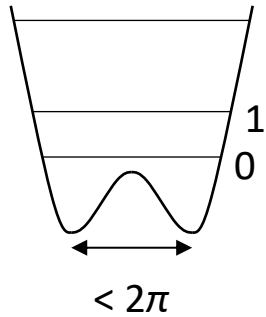
Size factor ($\gamma > 1$)

Phase is assumed to be equally distributed across the junction array

Flux qubit and its variants

Flux qubit

$$\gamma/N < 1$$



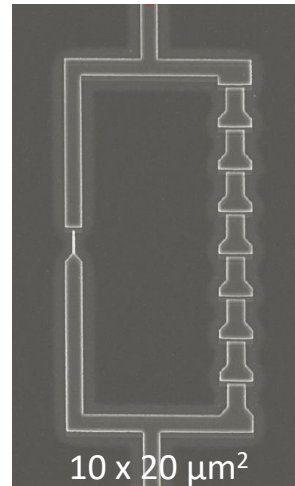
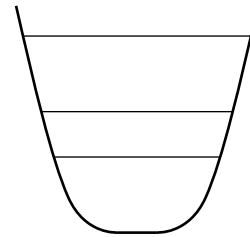
Science **299**, 1869 (2003) I. Chiorescu *et al.*

C-sh. flux (low impedance)

$T_1 \approx 40\ \mu\text{s}$, $T_2 \approx 2T_1$, $\alpha \approx 500\text{--}900\ \text{MHz}$

Quarton

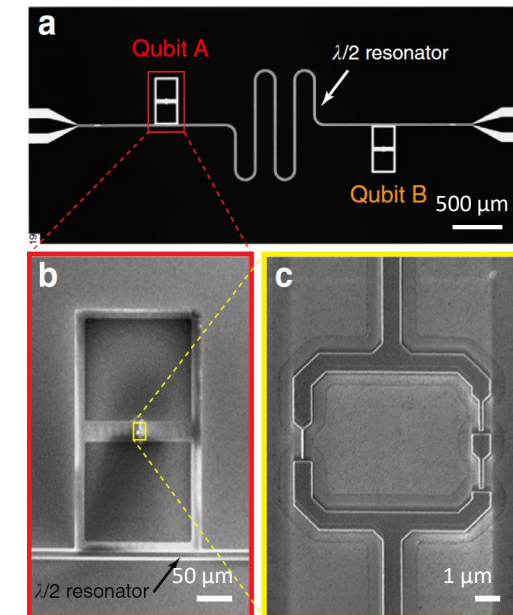
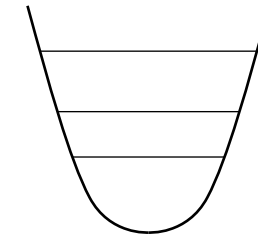
$$\gamma/N = 1$$



arXiv:2006.04130 F. Yan *et al.*

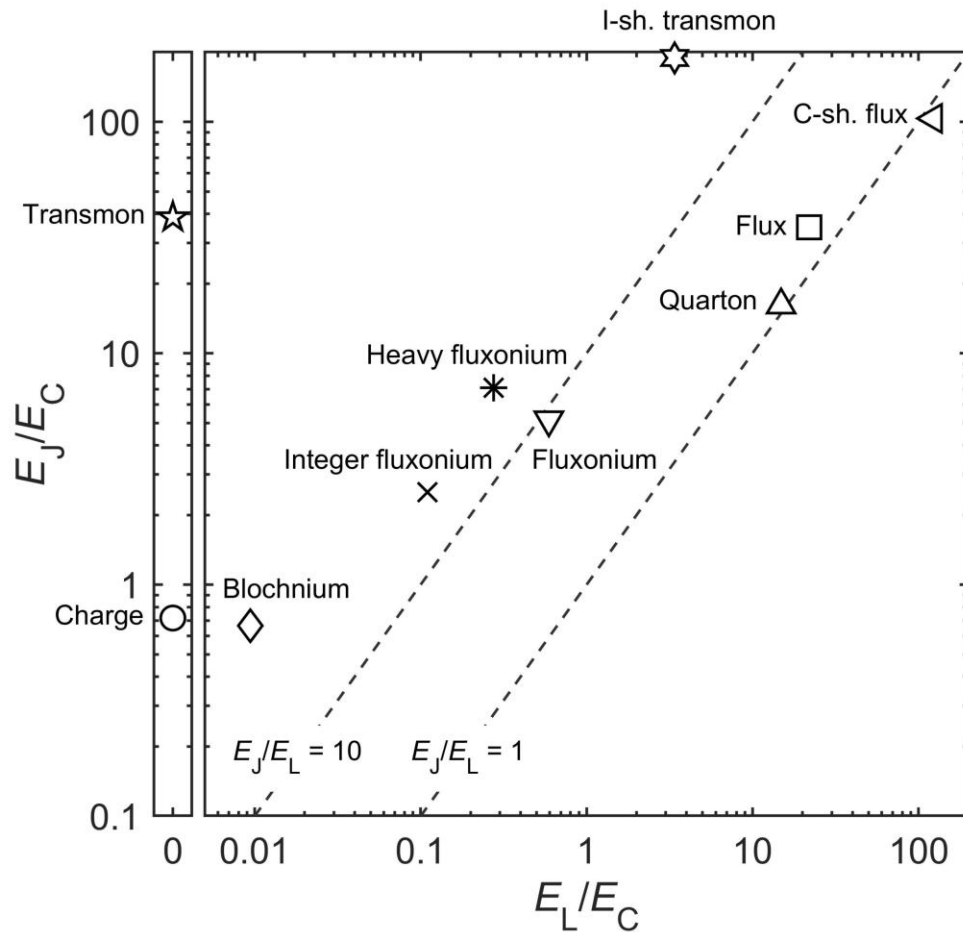
C-shunted flux qubit

$$\gamma/N > 1$$



Nature Commun. **7**, 12964 (2016) F. Yan *et al.*

Device parameters of superconducting qubits



(Source)

Transmon: Nature **647**, 343 (2025) M. P. Bland *et al.* (Device 17)

Charge: Nature **398**, 786 (1999) Y. Nakamura *et al.*

C-sh. flux: Nature Commun. **7**, 12964 (2016) F. Yan *et al.* (Device B)

Flux: Science **299**, 1869 (2003) I. Chiorescu *et al.*

Quarton: arXiv:2006.04130 F. Yan *et al.* (Device C)

I-sh. transmon: Nature Commun. **14**, 3968 (2023) F. Hassani *et al.* (Device B)

Fluxonium: Phys. Rev. Lett. **130**, 267001 (2023) A. Somoroff *et al.*

Heavy fluxonium: Phys. Rev. X **11**, 011010 (2021) H. Zhang *et al.*

Integer fluxonium: PRX Quantum **5**, 040318 (2024) R. A. Mencia *et al.* (Device A)

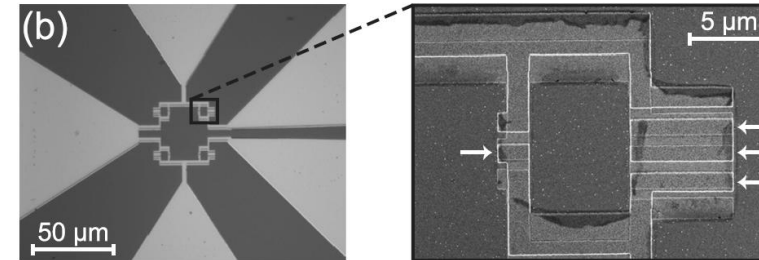
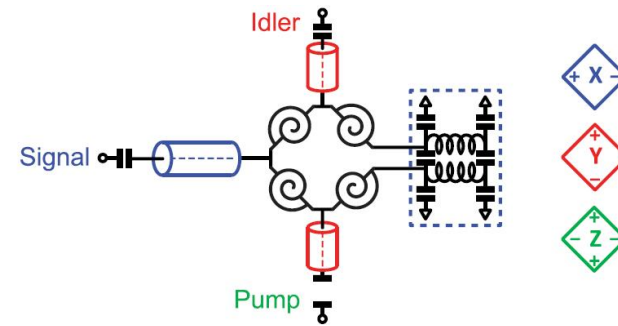
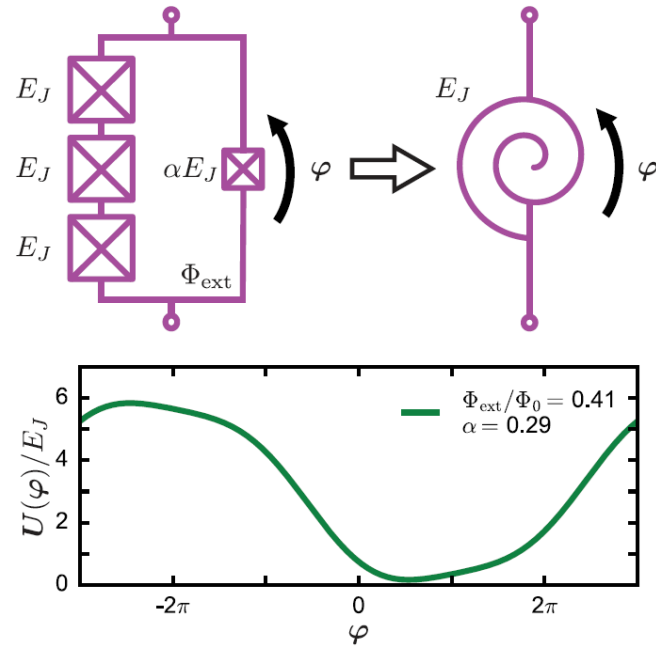
Blochonium: Nature **585**, 368 (2020) I. V. Pechenezhskiy *et al.*

* Plotted are representative values. The numbers will vary with devices & designs

** Phase qubit is omitted from the plot because of extremely large $E_J/E_C \sim 10^4$

SNAIL

Superconducting **N**onlinear **A**symmetric Inductive **e**lement



Appl. Phys. Lett. **110**, 222603 (2017) N. E. Frattini *et al.*

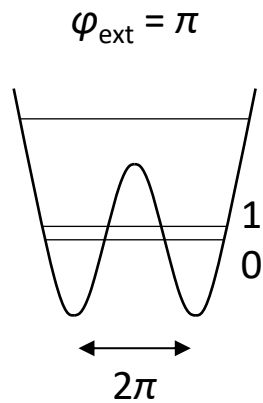


M. H. Devoret

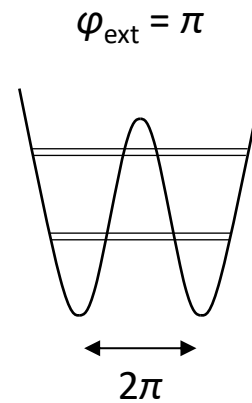
I recently nicknamed one of the devices we invented in my group. I called it the “SNAIL” because I was envious of the SQUID and the SLUG, both John Clarke’s creatures, and we thought we needed another mollusk in the zoo of superconducting circuits (...) so we invented the SNAIL to complete a trio.

Fluxonium and its variants

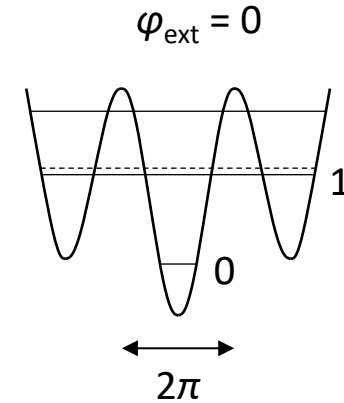
Fluxonium



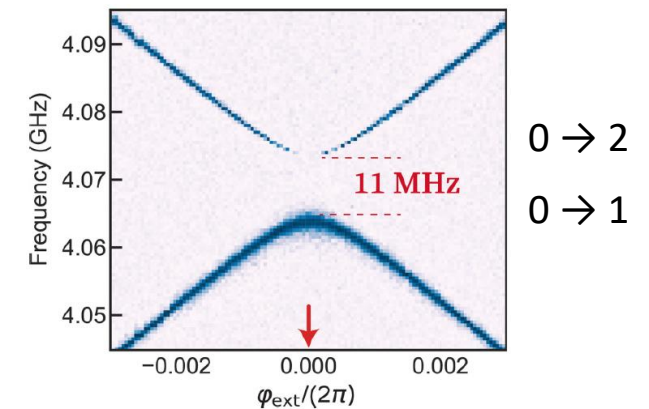
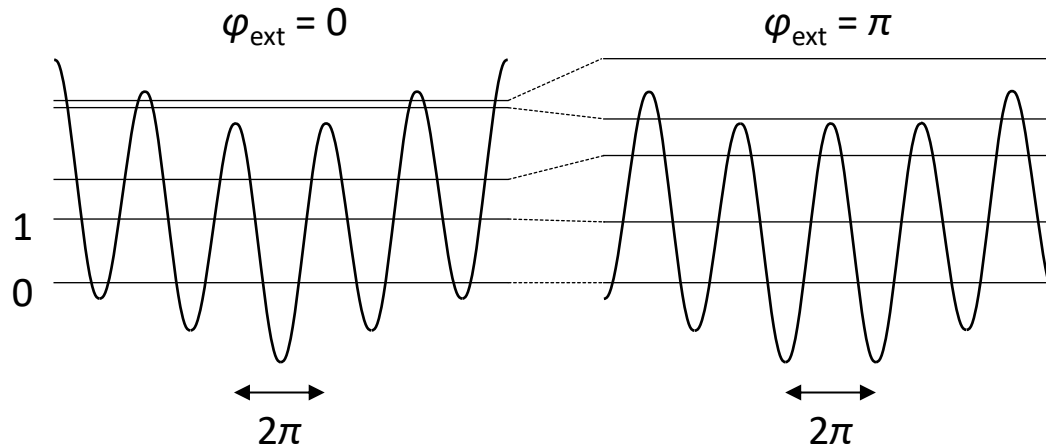
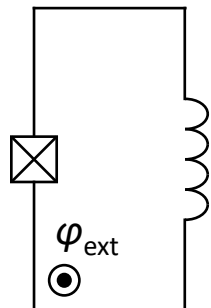
Heavy fluxonium



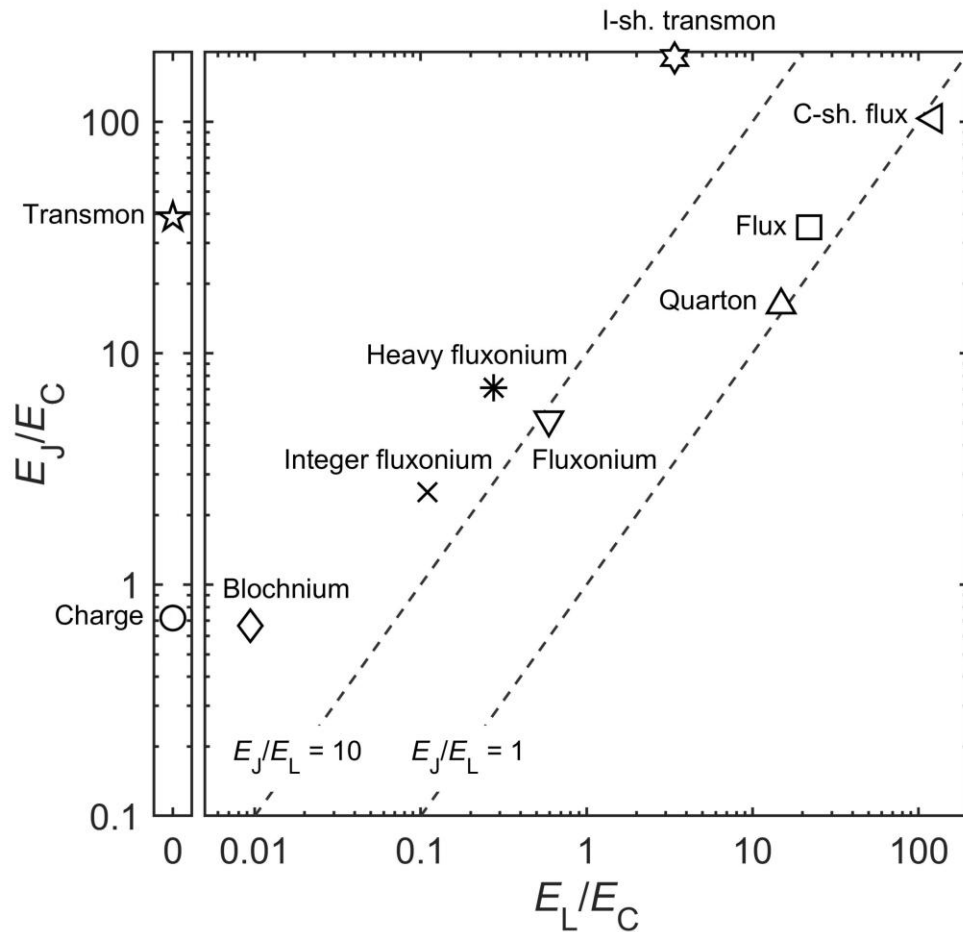
Integer fluxonium



Blochnium



Device parameters of superconducting qubits



(Source)

Transmon: Nature **647**, 343 (2025) M. P. Bland *et al.* (Device 17)

Charge: Nature **398**, 786 (1999) Y. Nakamura *et al.*

C-sh. flux: Nature Commun. **7**, 12964 (2016) F. Yan *et al.* (Device B)

Flux: Science **299**, 1869 (2003) I. Chiorescu *et al.*

Qarton: arXiv:2006.04130 F. Yan *et al.* (Device C)

I-sh. transmon: Nature Commun. **14**, 3968 (2023) F. Hassani *et al.* (Device B)

Fluxonium: Phys. Rev. Lett. **130**, 267001 (2023) A. Somoroff *et al.*

Heavy fluxonium: Phys. Rev. X **11**, 011010 (2021) H. Zhang *et al.*

Integer fluxonium : PRX Quantum **5**, 040318 (2024) R. A. Mencia *et al.* (Device A)

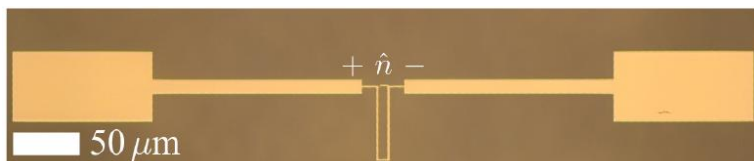
Blochonium: Nature **585**, 368 (2020) I. V. Pechenezhskiy *et al.*

* Plotted are representative values. The numbers will vary with devices & designs

** Phase qubit is omitted from the plot because of extremely large $E_J/E_C \sim 10^4$

Fluxonium and its variants

Fluxonium



Phys. Rev. Lett. **130**, 267001 (2023)
A. Somoroff *et al.*

$$T_1 = 1.2\text{ ms}, T_2^* \approx 1.16\text{ ms}$$
$$\omega_q = 163\text{ MHz}$$

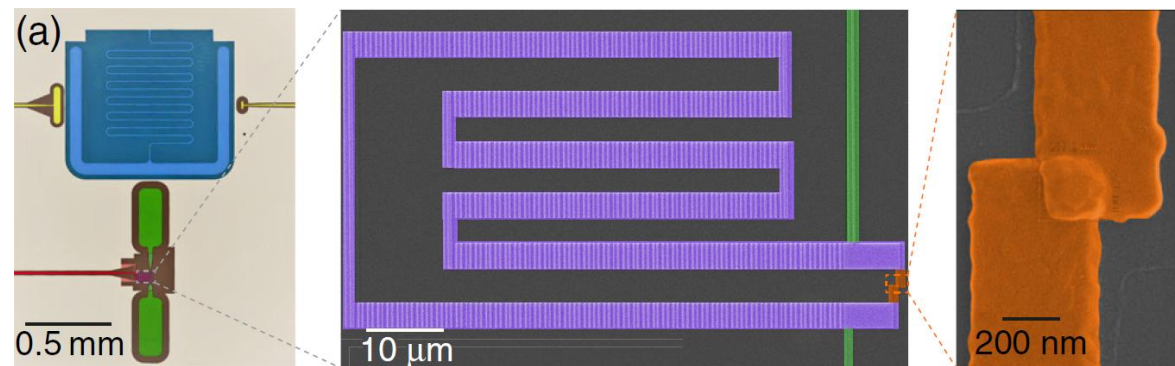


Integer fluxonium



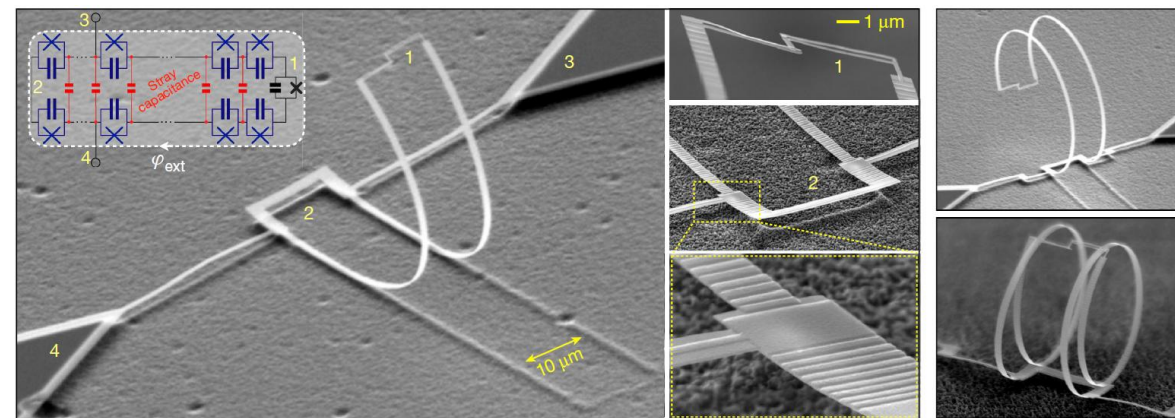
PRX Quantum **5**, 040318 (2024) R. A. Mencia *et al.*

Heavy fluxonium



Phys. Rev. X **11**, 011010 (2021) H. Zhang *et al.*

Blochnium



Nature **585**, 368 (2020) I. V. Pechenezhskiy *et al.*

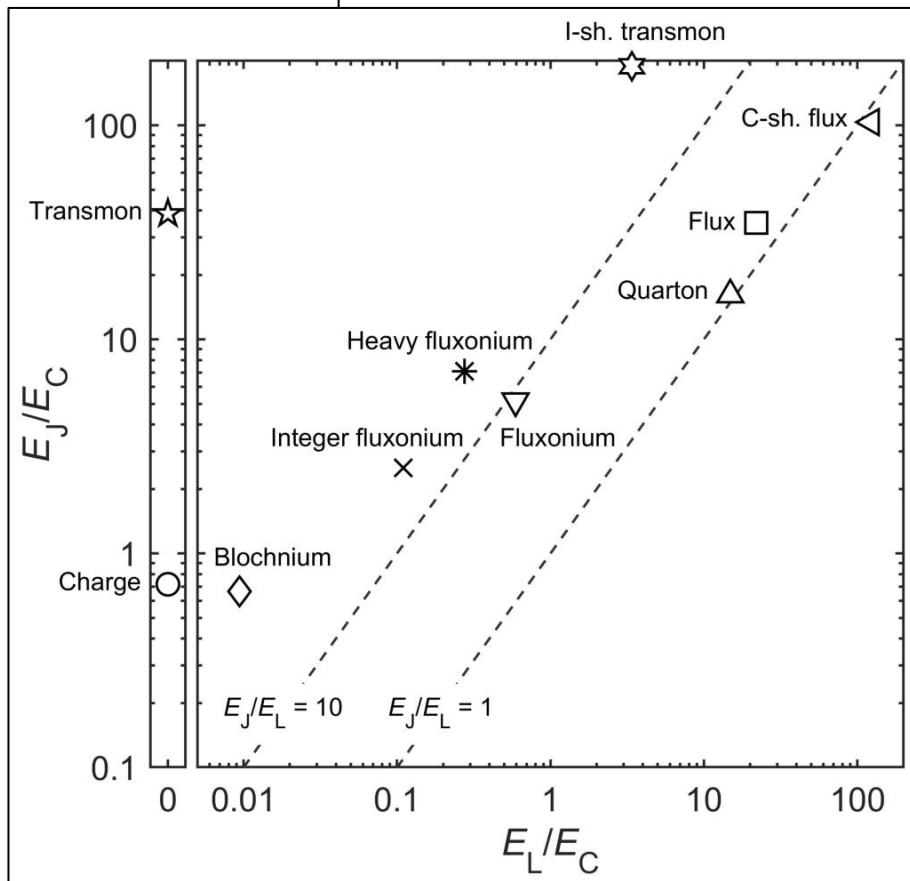
Blochnium as dual of transmon

Article

The superconducting quasicharge qubit

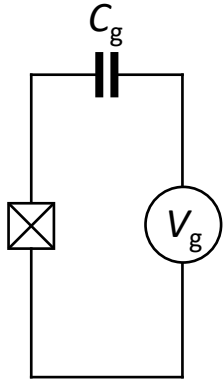
Ivan V. Pechenezhskiy^{1,3}, Raymond A. Mencia^{1,3}, Long B. Nguyen¹, Yen-Hsiang Lin^{1,2} & Vladimir E. Manucharyan¹✉

The non-dissipative nonlinearity of Josephson junctions¹ converts macroscopic superconducting circuits into artificial atoms², enabling some of the best-controlled qubits today^{3,4}. Three fundamental types of superconducting qubit are known⁵, each reflecting a distinct behaviour of quantum fluctuations in a Cooper pair condensate: single-charge tunnelling (charge qubit^{6,7}), single-flux tunnelling (flux qubit⁸) and phase oscillations (phase qubit⁹ or transmon¹⁰). Yet, the dual nature of charge and flux suggests that circuit atoms must come in pairs. Here we introduce the missing superconducting qubit, 'blochnium', which exploits a coherent insulating response of a single Josephson junction that emerges from the extension of phase fluctuations beyond 2π (refs. ^{11–14}).



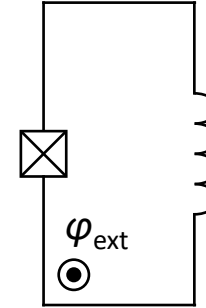
Nature **585**, 368 (2020) I. V. Pechenezhskiy *et al.*

“Quasicharge”



$$H_3 = 4E_C(n - n_g)^2 - E_J \cos \theta$$

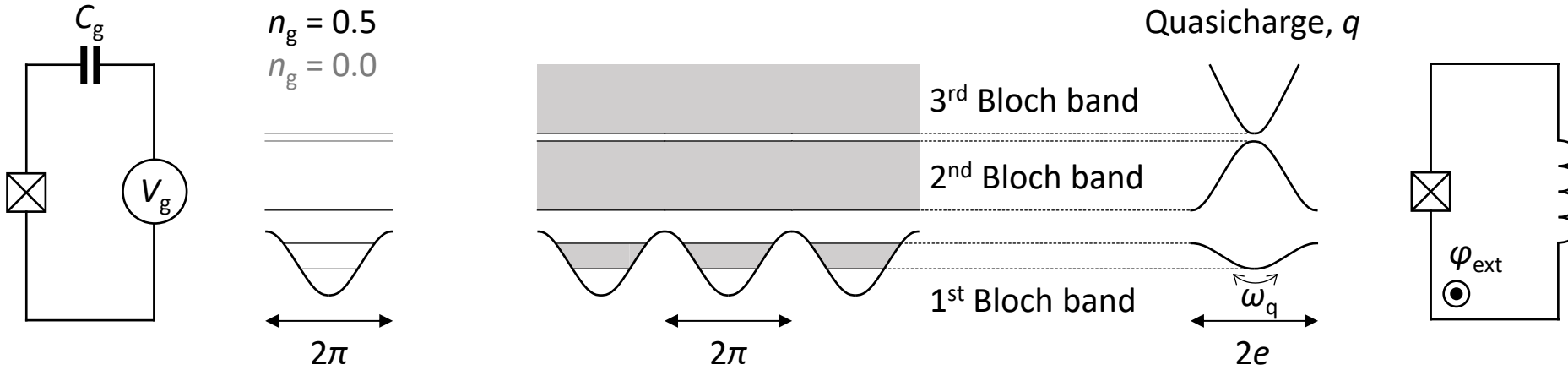
$$H_4 = 4E_C n^2 - E_J \cos(\varphi + \varphi_{\text{ext}}) + \frac{1}{2} E_L \varphi^2$$



Suppose E_L is very small (i.e., L is very large)

The potential is almost cosine, but the variable is still φ
(To phrase paradoxically, an infinitely large L is equivalent to non-existence)

“Quasicharge”



Bloch's theorem in 1D solid of lattice constant a

$$\Psi_k(x) = \psi_k(x)e^{ikx} \quad \text{with } \psi_k(x) = \psi_k(x + a)$$

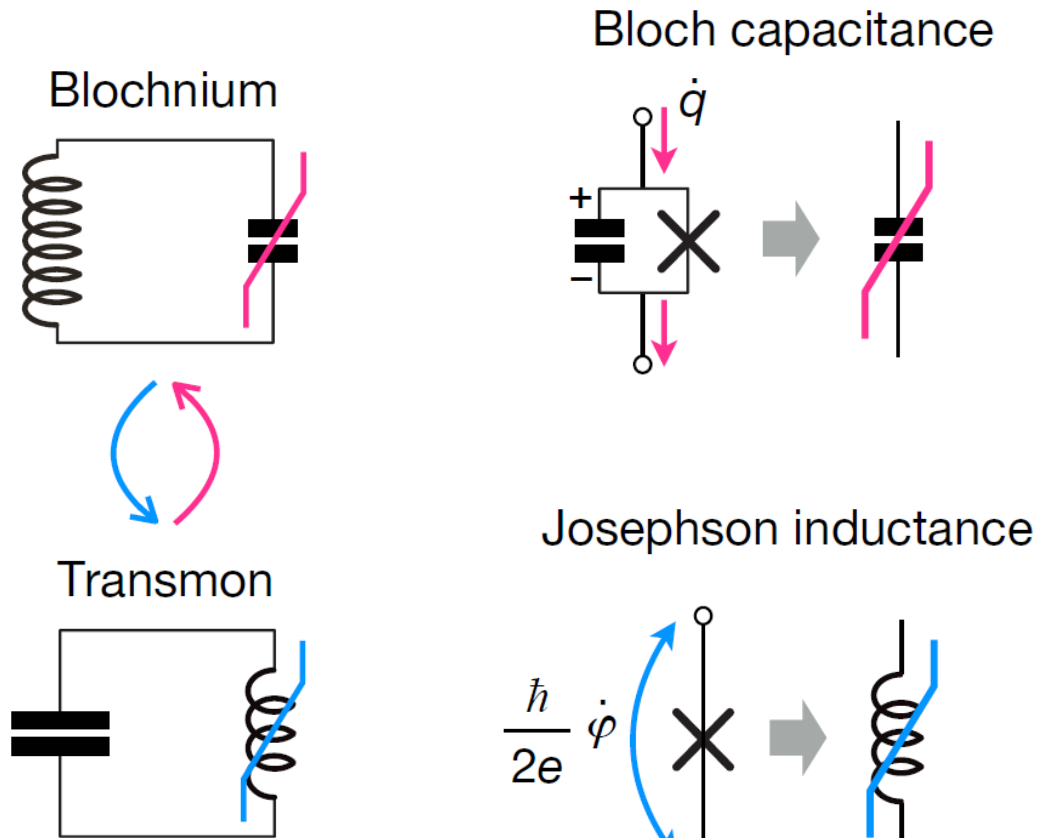
Crystal momentum or quasimomentum with period $2\pi/a$

Josephson junction of 2π -periodic phase

$$\Psi_q(\varphi) = \psi_q(\varphi)e^{iq\varphi/2e} \quad \text{with } \psi_k(\varphi) = \psi_k(\varphi + 2\pi)$$

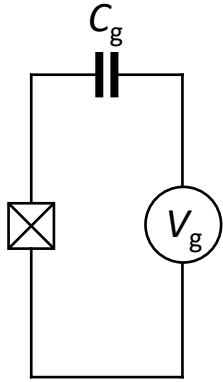
Quasicharge with period $2e$

Blochnium as dual of transmon



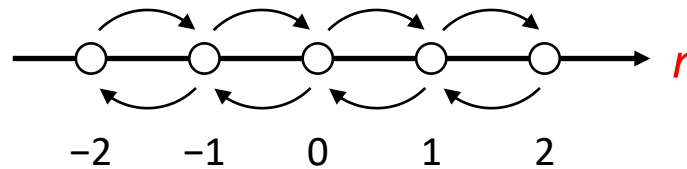
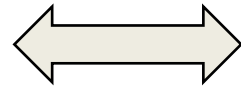
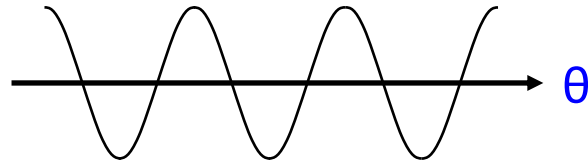
- Oscillation of q ($-e \leq q \leq e$): quasicharge oscillation
- T_ϕ -protection from flat dispersion in φ -space
- For JJ to work as Bloch C, very high impedance is required (*i.e.*, large E_L , as required to realize a cosine potential)
- Oscillation of θ ($-\pi \leq \theta \leq \pi$): plasma oscillation
- T_ϕ -protection from flat dispersion in n_g -space

Cooper-pair tunneling and 1D tight binding model



$$H_3 = 4E_C(n - n_g)^2 - E_J \cos \theta$$

$$= 4E_C(n - n_g)^2 \sum_{n=-\infty}^{\infty} |n\rangle\langle n| - \frac{1}{2}E_J \sum_{n=-\infty}^{\infty} (|n+1\rangle\langle n| + |n\rangle\langle n+1|)$$



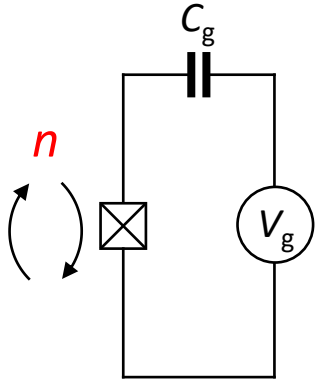
$$|\theta\rangle = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} e^{i\theta n} |n\rangle$$

$$\cos \theta |\theta\rangle = \left(\frac{e^{-i\theta} + e^{i\theta}}{2} \right) \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} e^{i\theta n} |n\rangle = \frac{1}{2\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} e^{i\theta(n-1)} |n\rangle + e^{i\theta(n+1)} |n\rangle$$

$$= \frac{1}{2\sqrt{2\pi}} \left(\sum_{n=-\infty}^{\infty} e^{i\theta n} |n+1\rangle + \sum_{n=-\infty}^{\infty} e^{i\theta n} |n-1\rangle \right) = \frac{1}{2\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \left(\sum_{n'=-\infty}^{\infty} |n'+1\rangle\langle n'| + |n'-1\rangle\langle n'| \right) e^{i\theta n} |n\rangle$$

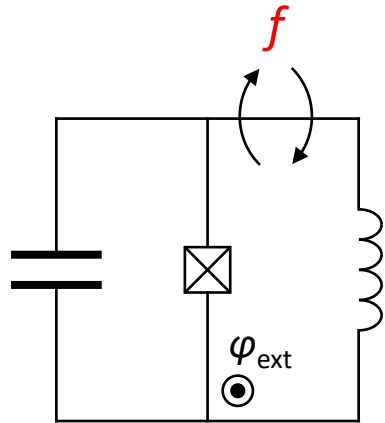
$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} (|n+1\rangle\langle n| + |n\rangle\langle n+1|) |\theta\rangle$$

Cooper-pair tunneling and fluxon tunneling



$$H_3 = 4E_C(n - n_g)^2 - E_J \cos \theta$$

$$= 4E_C(n - n_g)^2 \sum_{n=-\infty}^{\infty} |n\rangle\langle n| - \frac{1}{2}E_J \sum_{n=-\infty}^{\infty} (|n+1\rangle\langle n| + |n\rangle\langle n+1|)$$



$$H_4 = 4E_C n^2 - E_J \cos(\varphi + \varphi_{\text{ext}}) + \frac{1}{2}E_L \varphi^2$$

$$= \frac{1}{2}E_L(f - \Phi_{\text{ext}})^2 \sum_{f=-\infty}^{\infty} |f\rangle\langle f| - \frac{1}{2}E_S \sum_{f=-\infty}^{\infty} (|f+1\rangle\langle f| + |f\rangle\langle f+1|)$$

Fluxon tunneling (phase-slip) rate

Contents

- Brief history: from discovery of superconductivity to dawn of quantum era
- **Superconducting qubits**
 - Qubit, coherence time, and protection from noise
 - Josephson junction
 - Plasma oscillation qubit: phase qubit and transmon (derived from charge qubit)
 - Loop-based qubit: flux qubit, fluxonium, and Blochonium (dual of transmon)
 - Protected qubit: $0-\pi$ qubit
- Quantum operations with transmons
 - Circuit QED
 - Dispersive readout
 - Single- and two-qubit gates
- Building quantum computers (→ Symposium talk)

Protected qubit: 0- π qubit

Can a qubit be made protected both from T_1 and T_ϕ ?

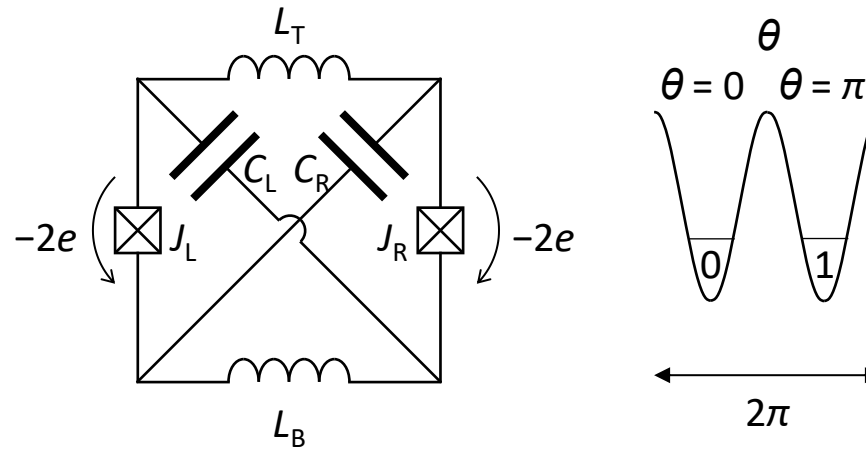
Transmon

$$\frac{1}{T_\phi} \propto \left| \frac{\partial \omega_{01}}{\partial \lambda} \right|^2 S_\lambda(\omega \rightarrow 0)$$

Fluxonium

$$\frac{1}{T_1} \propto |\langle 0 | O | 1 \rangle|^2 S_\lambda(\omega_{01})$$

→ Multi-mode



Double Cooper pair tunneling

$$\frac{1}{2} \sum_{n=-\infty}^{\infty} (|n+2\rangle\langle n| + |n\rangle\langle n+2|) = \cos 2\theta$$

Protected qubit: 0- π qubit

Can a qubit be made protected both from T_1 and T_ϕ ?

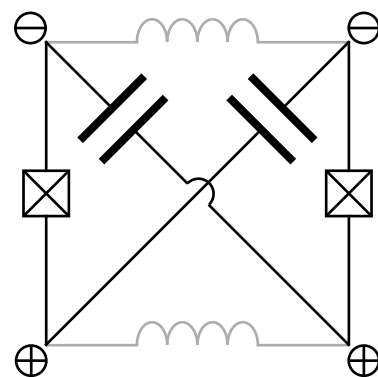
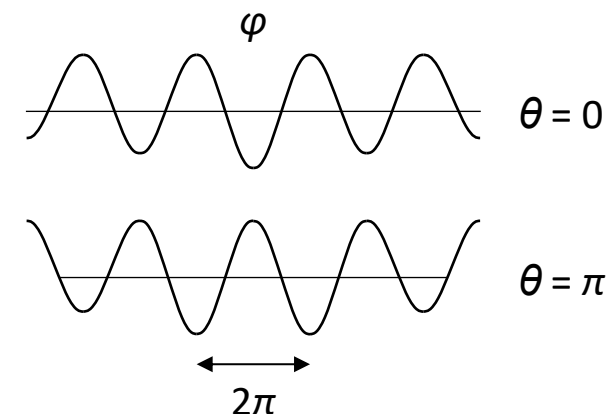
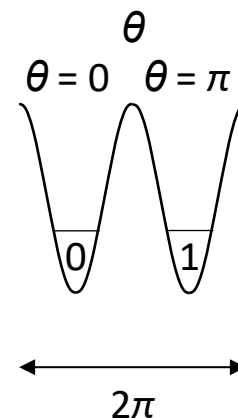
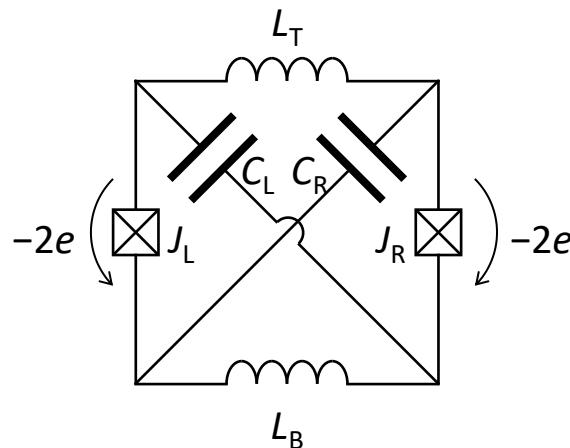
Transmon

$$\frac{1}{T_\phi} \propto \left| \frac{\partial \omega_{01}}{\partial \lambda} \right|^2 S_\lambda(\omega \rightarrow 0)$$

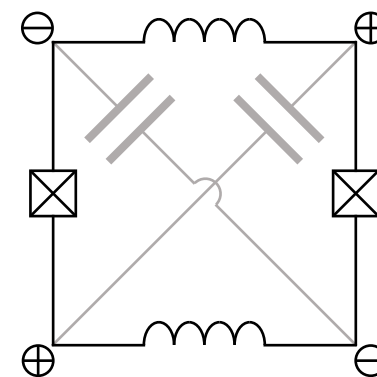
Fluxonium

$$\frac{1}{T_1} \propto |\langle 0 | O | 1 \rangle|^2 S_\lambda(\omega_{01})$$

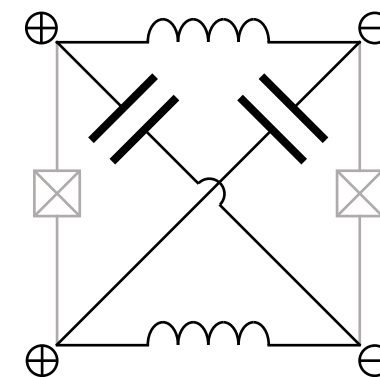
→ Multi-mode



Transmon-like heavy (θ) mode

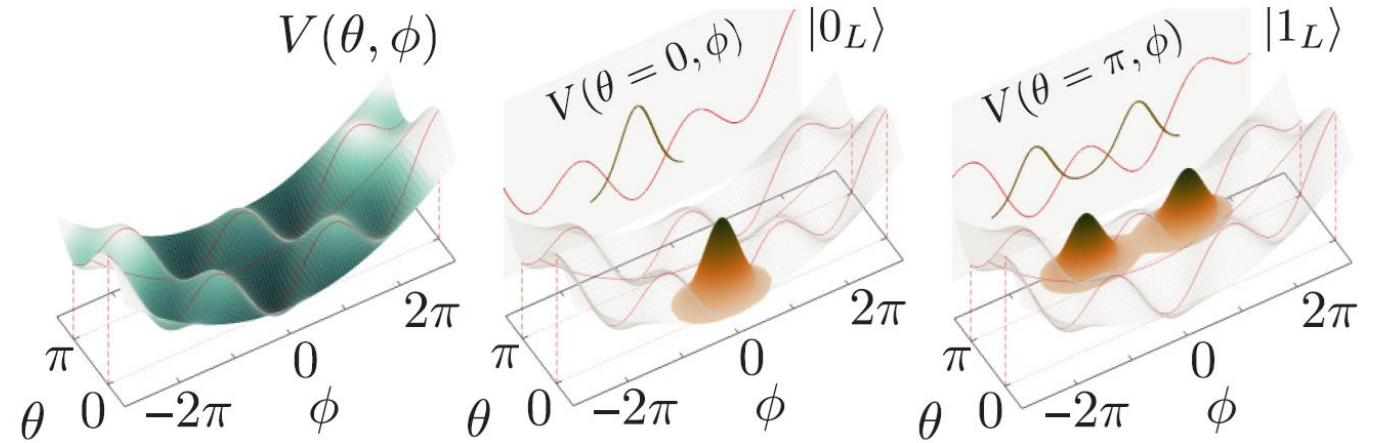
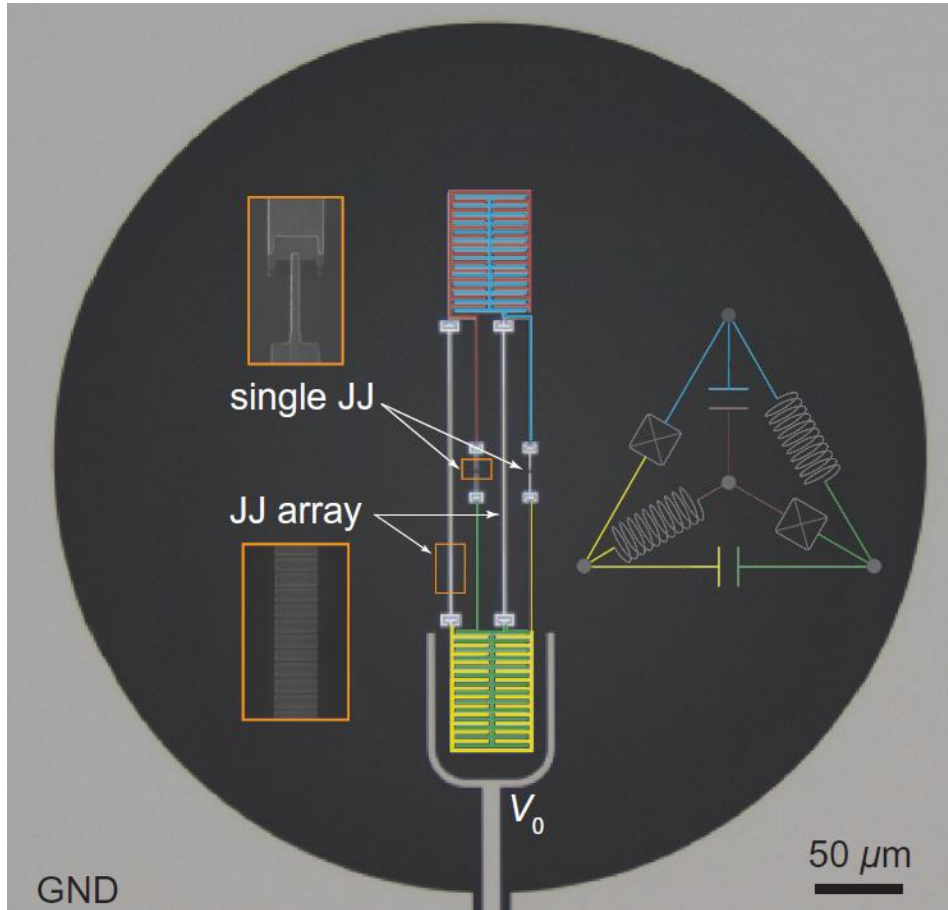


Fluxonium-like light (ϕ) mode



Cavity-like parasitic (ζ) mode

“Soft” 0- π qubit



Large C provided by IDC: **T_1 -protection by design**

$$T_1 = 1.6\ \text{ms},\ T_2 = 25\ \mu\text{s}$$

Contents

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- Building quantum computers (→ Symposium talk)

DiVincenzo's criteria

1. A scalable physical system with well characterized qubits
2. The ability to initialize the state of the qubits to a simple fiducial state, such as $|000\dots\rangle$
3. Long relevant decoherence times, much longer than the gate operation time
4. A universal set of quantum gates
5. A qubit-specific measurement capability

Fortschr. Phys. **48**, 771 (2000) D. P. DiVincenzo

DiVincenzo's criteria

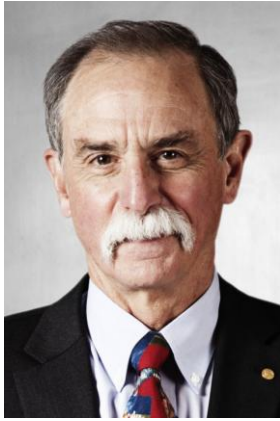
1. A scalable physical system with well characterized qubits
2. **The ability to initialize the state of the qubits to a simple fiducial state, such as $|000\dots\rangle$**
3. Long relevant decoherence times, much longer than the gate operation time
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5. **A qubit-specific measurement capability**

Fortschr. Phys. **48**, 771 (2000) D. P. DiVincenzo

Cavity QED and trapped ions



S. Haroche

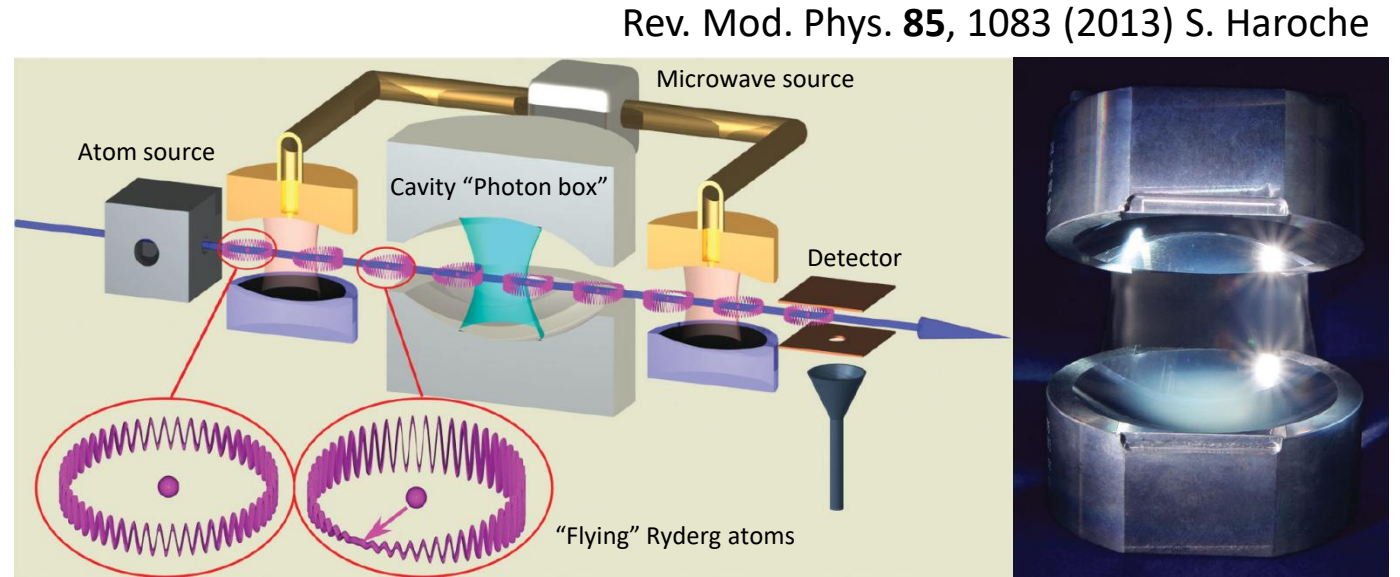


D. J. Winland



2012

© Nobel Foundation



Rev. Mod. Phys. **85**, 1083 (2013) S. Haroche

“for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems”

Physics Today **49 (8)**, 51 (1996) S. Haroche & J.-M. Raimond

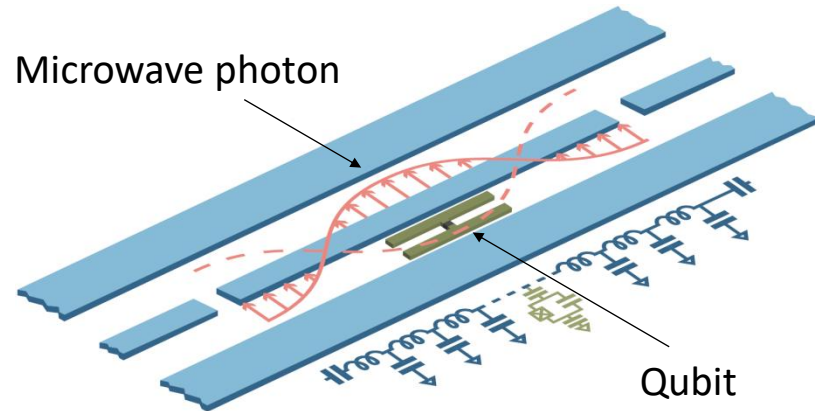
QUANTUM COMPUTING: DREAM OR NIGHTMARE?

Recent experiments have deepened our insight into the wonderfully counterintuitive quantum theory. But are they really harbingers of quantum computing? We doubt it.

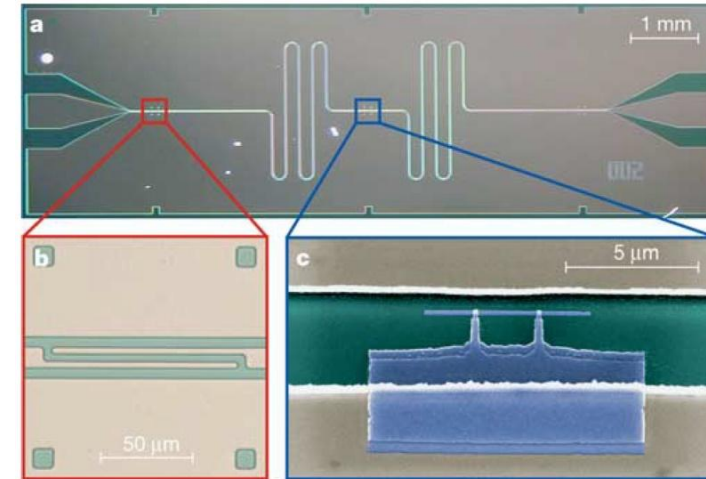
Serge Haroche and Jean-Michel Raimond

Circuit QED

Control of interaction between a qubit & a microwave photon in a superconducting quantum circuit



Rev. Mod. Phys. **93**, 025005 (2021) A. Blais *et al.*



Nature **431**, 162 (2004) A. Wallraff *et al.*

Robert Schoelkopf



<https://quantuminstitute.yale.edu/people/robert-schoelkopf>

Steven Girvin



<https://quantuminstitute.yale.edu/people/steven-girvin>

PHYSICAL REVIEW A **69**, 062320 (2004)

Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation

Alexandre Blais,¹ Ren-Shou Huang,^{1,2} Andreas Wallraff,¹ S. M. Girvin,¹ and R. J. Schoelkopf¹

¹*Departments of Physics and Applied Physics, Yale University, New Haven, Connecticut 06520, USA*

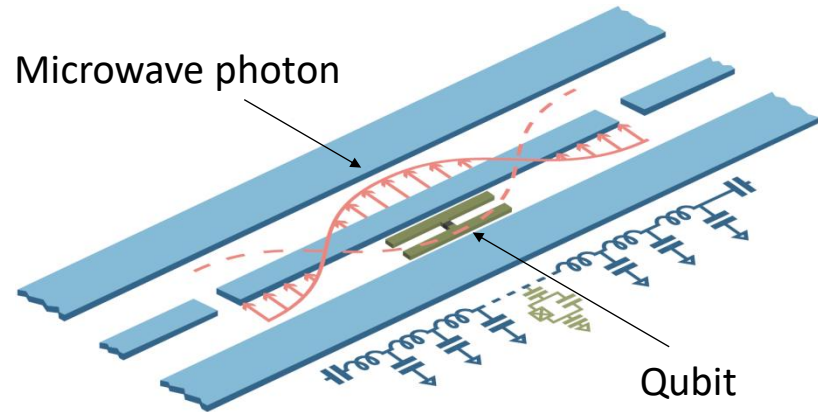
²*Department of Physics, Indiana University, Bloomington, Indiana 47405, USA*

(Received 7 February 2004; published 29 June 2004; corrected 23 July 2004)

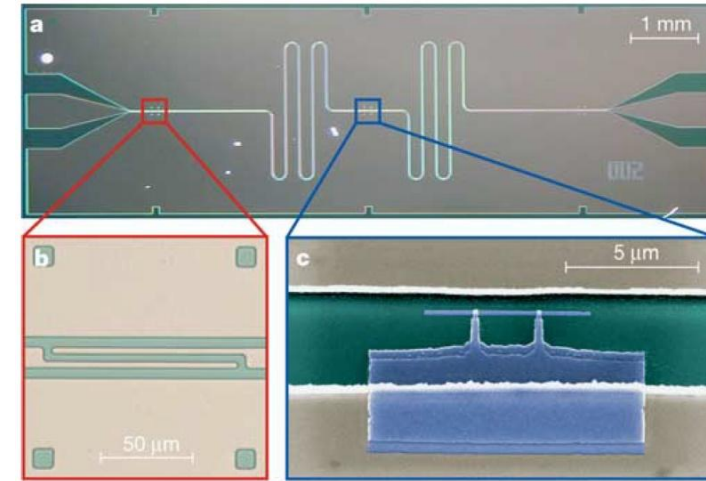
Phys. Rev. A **69**, 062320 (2004) A. Blais *et al.*

Circuit QED

Control of interaction between a qubit & a microwave photon in a superconducting quantum circuit



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Nature **431**, 162 (2004) A. Wallraff *et al.*

Robert Schoelkopf

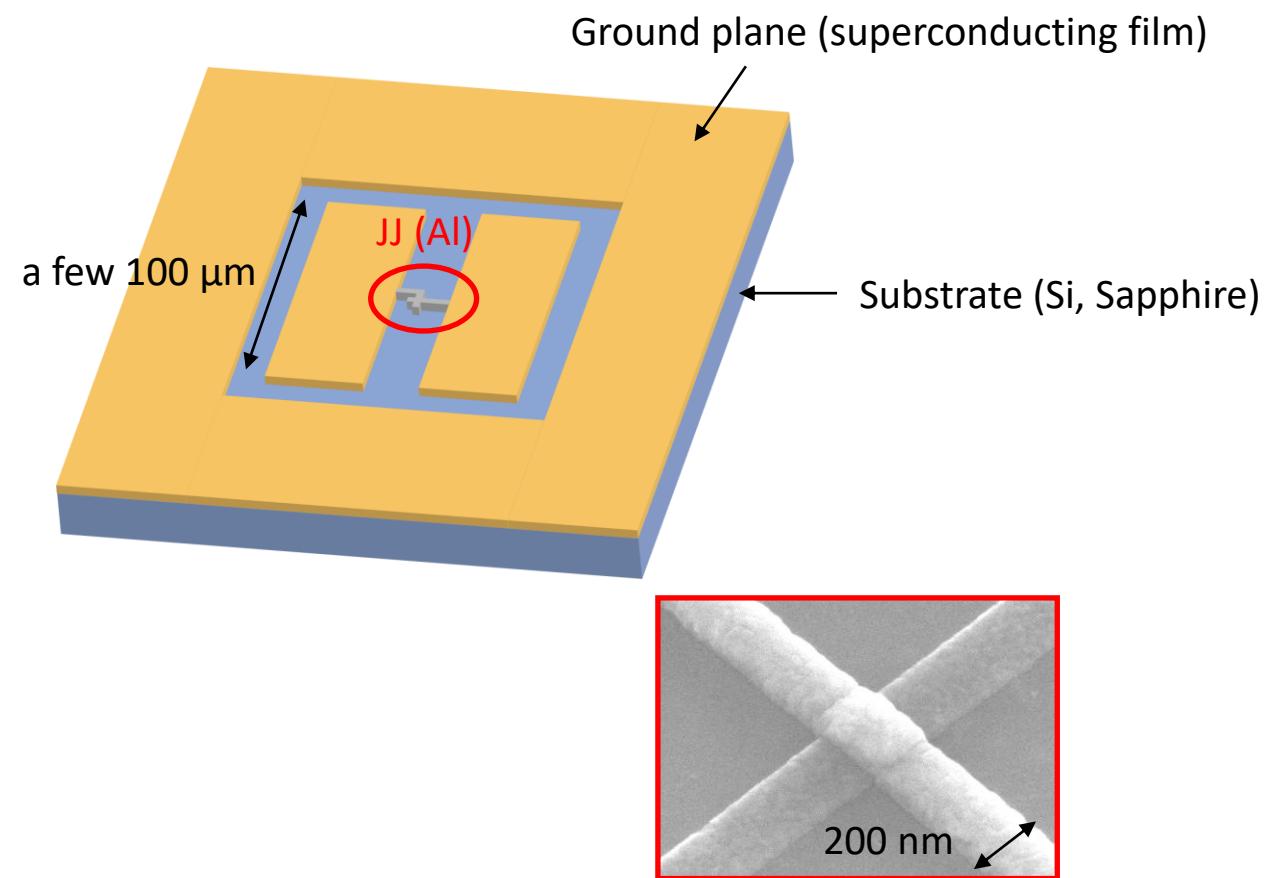


Steve Girvin came, I remember (...) us having our first face-to-face meeting. And he came for 45 minutes or an hour, sat in my office, and said (...) “We should talk about these Cooper pair boxes and microwaves, because I think it’s exactly like an ion trap system, where you have atoms coupled to bosonic modes. You know, phonons moving.” And I said, “Yes! I know.” And we kind of cartooned up circuit QED on the blackboard in that one meeting.

Steven Girvin



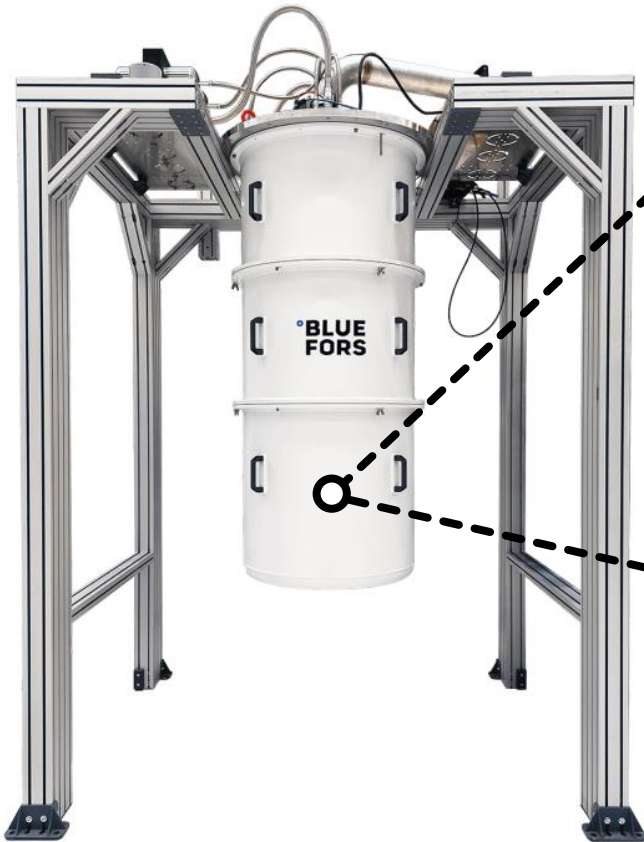
Qubit (transmon)



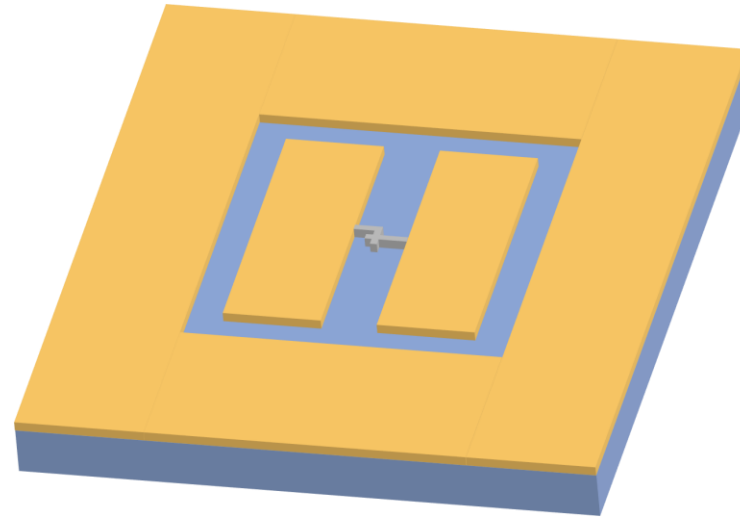
Initialization

Relaxation and active reset

A commercial $^3\text{He}/^4\text{He}$ dilution refrigerator such as this one can reach down to 10 mK, nearly automatically



$$E_{01}/h = 2\text{--}10 \text{ GHz } (h/k_B \approx 48 \text{ mK/GHz})$$

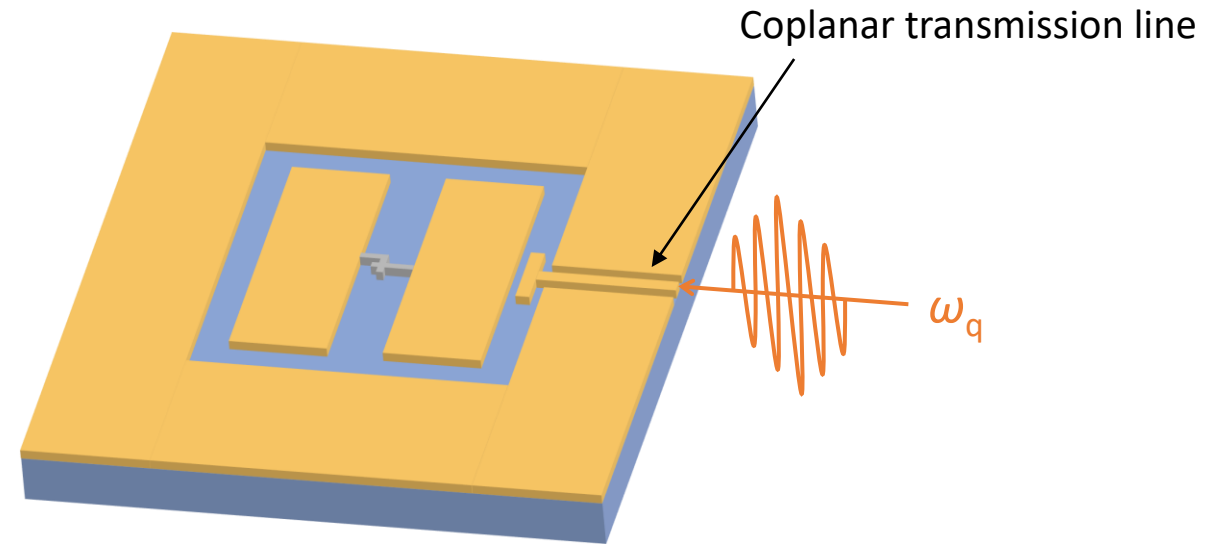
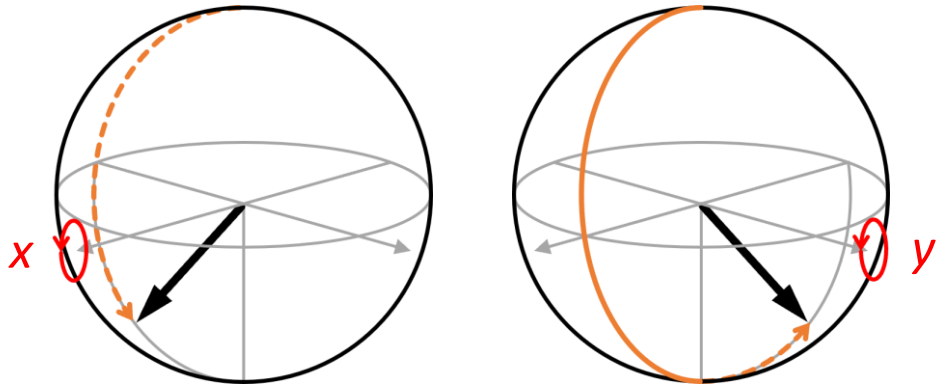


$$\exp\left(-\frac{h \times 5 \text{ GHz}}{k_B \times 10 \text{ mK}}\right) \approx 10^{-11}$$

Single-qubit gate

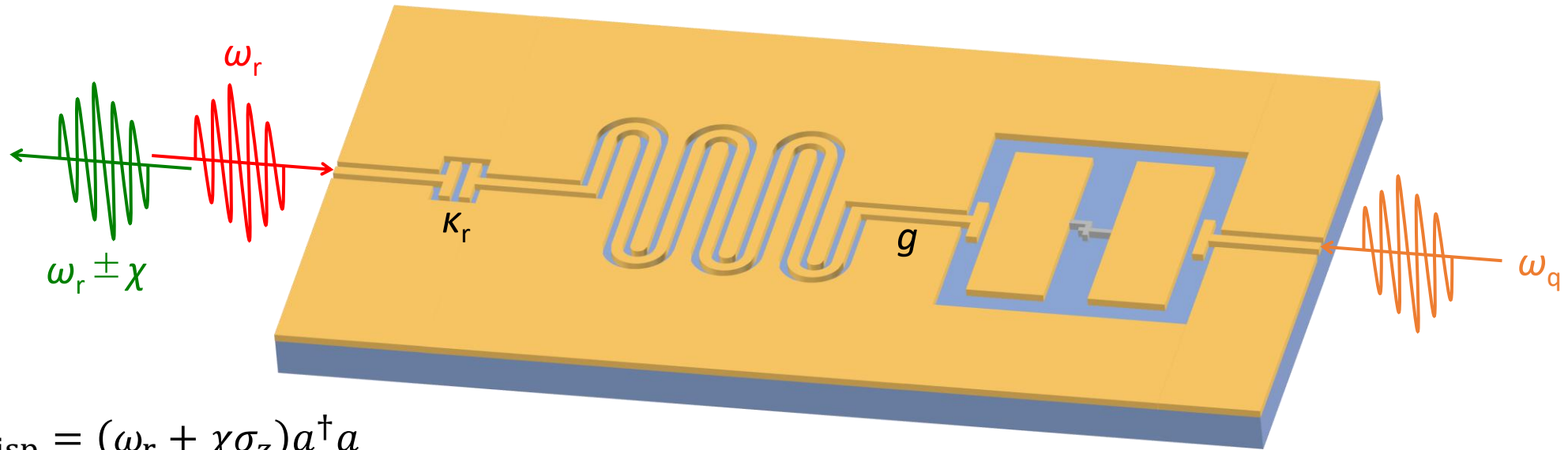
Rabi drive: microwave pulse on resonance with the qubit frequency

By controlling the phases of microwave pulses, rotations around x and y can be realized, sufficient for arbitrary single-qubit gates



Readout

Qubit-state-dependent frequency (or phase) shift (dispersive shift)

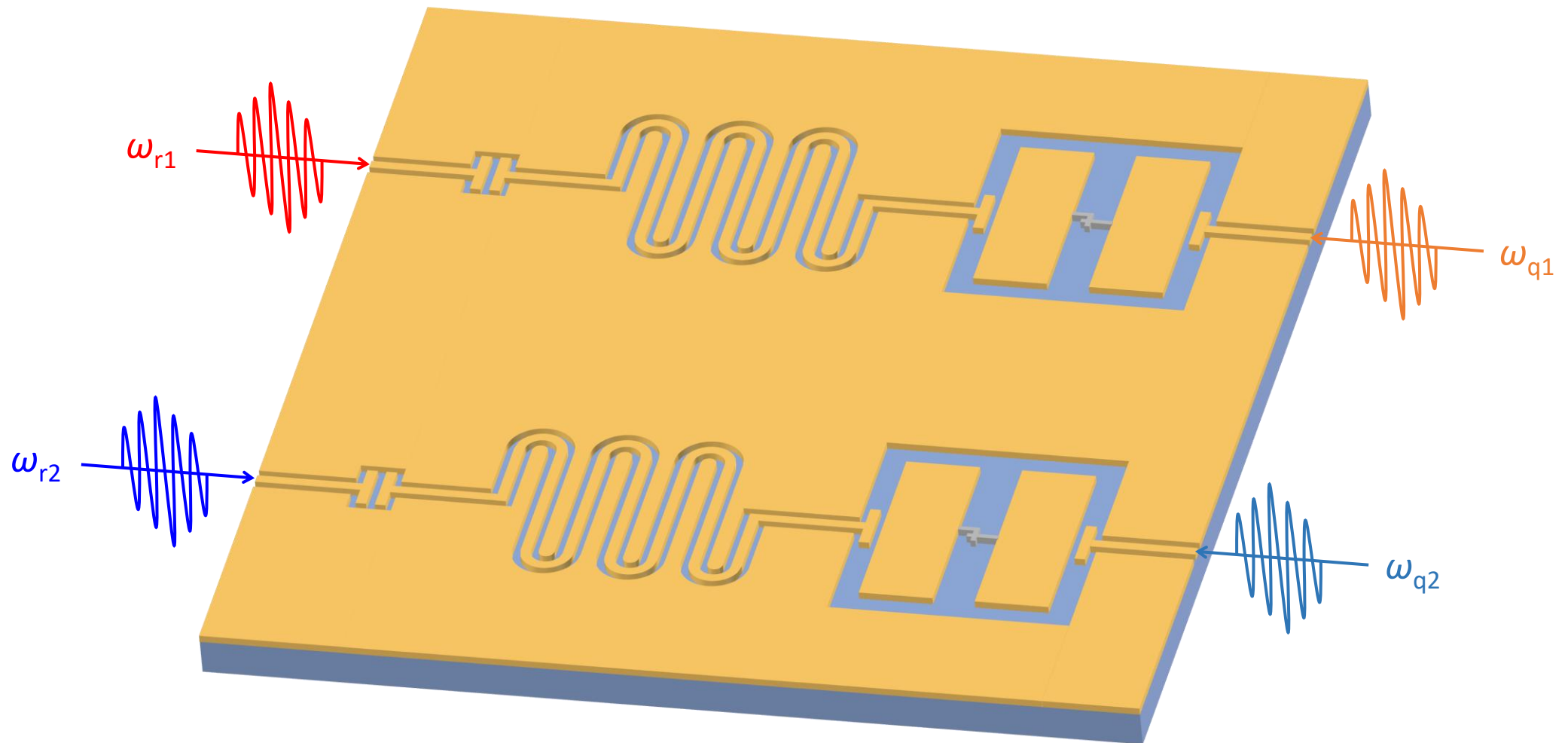


$$H_{\text{disp}} = (\omega_r + \chi \sigma_z) a^\dagger a$$

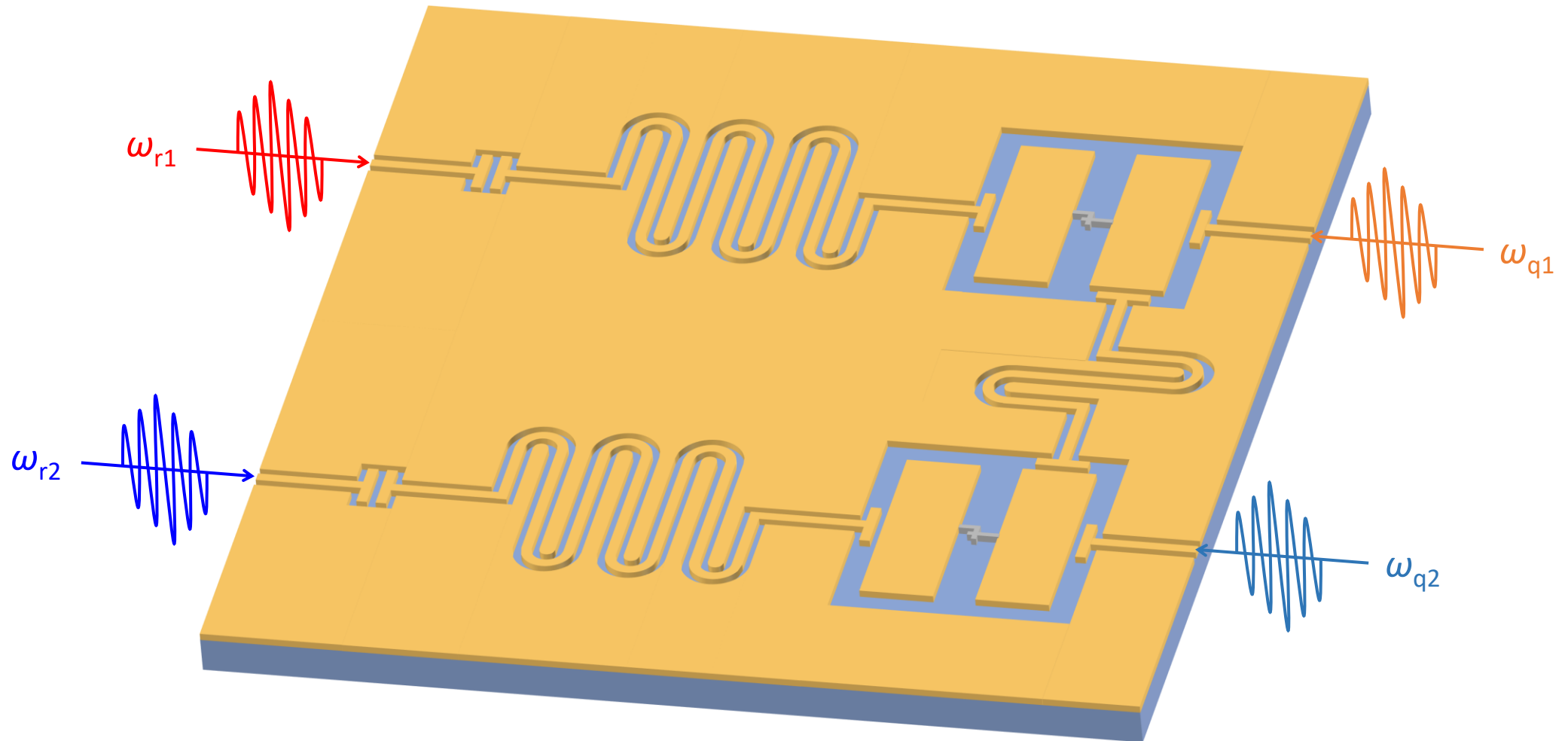
$$\chi = \frac{g^2}{|\Delta|}$$

$$|\Delta| = |\omega_q - \omega_r| \gg g, \kappa_r$$

Two transmon-circuits

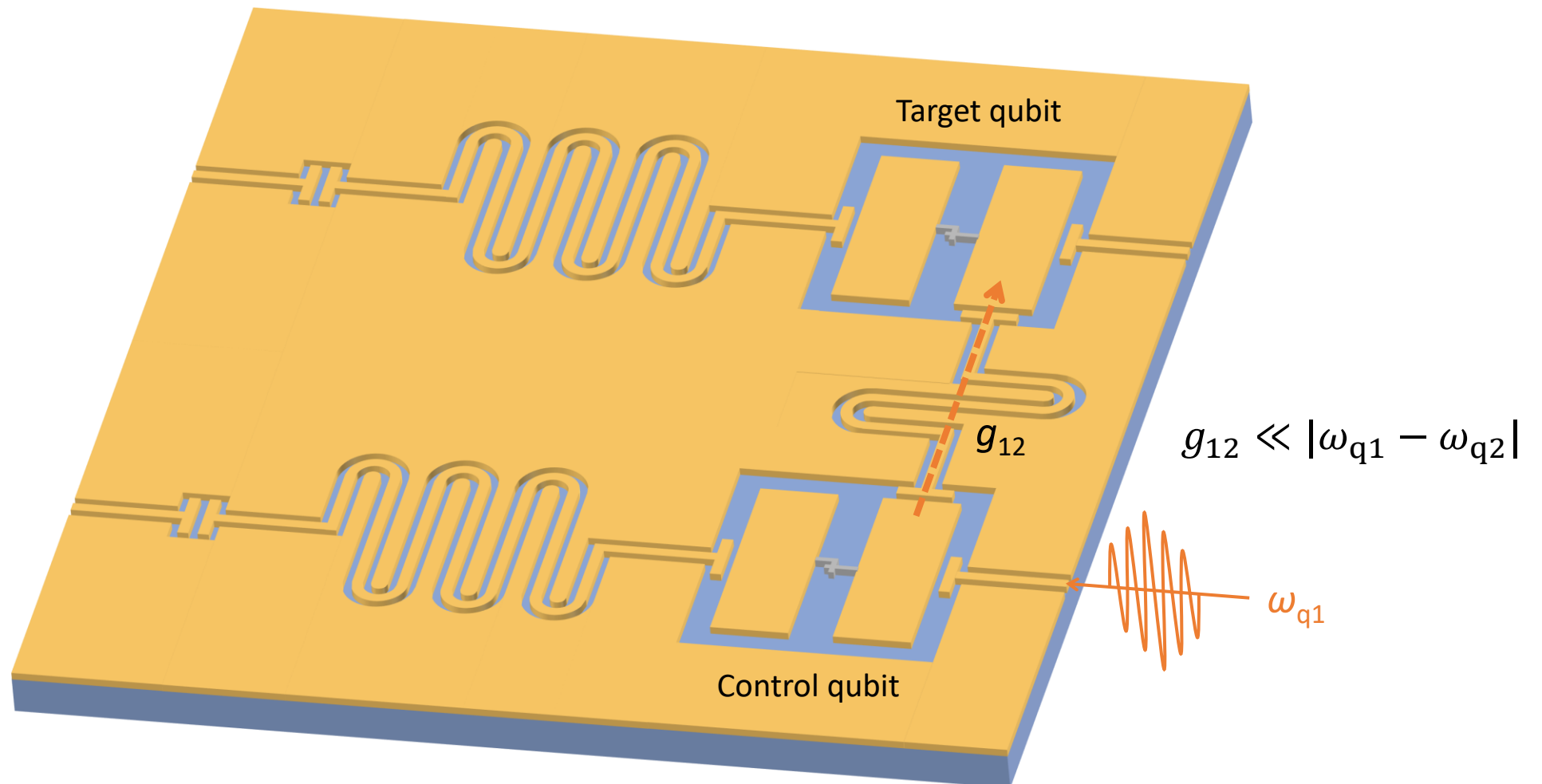


Qubit-qubit coupler



Two-qubit gate

Cross-resonance (CR) gate: rotation of target-q conditional on the state of control-q



Contents

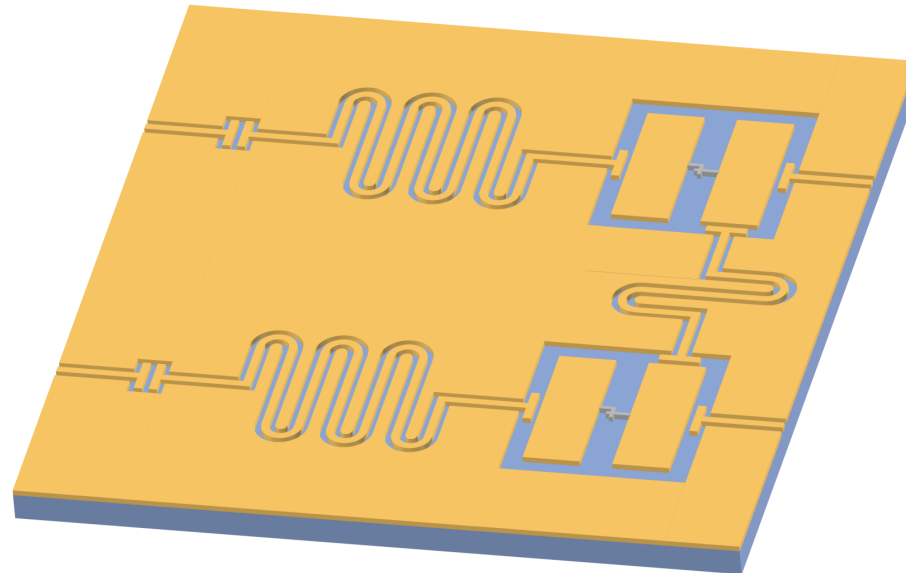
- Brief history: from discovery of superconductivity to dawn of quantum era
- Superconducting qubits
 - Qubit, coherence time, and protection from noise
 - Josephson junction
 - Plasma oscillation qubit: phase qubit and transmon (derived from charge qubit)
 - Loop-based qubit: flux qubit, fluxonium, and Blochonium (dual of transmon)
 - Protected qubit: $0-\pi$ qubit
- Quantum operations with transmons
 - Circuit QED
 - Dispersive readout
 - Single- and two-qubit gates
- **Building quantum computers (→ Symposium talk)**

DiVincenzo's criteria

1. A **scalable** physical system with well characterized qubits
2. The ability to initialize the state of the qubits to a simple fiducial state, such as $|000\dots\rangle$
3. Long relevant decoherence times, much longer than the gate operation time
4. A universal set of quantum gates
5. A qubit-specific measurement capability

Fortschr. Phys. **48**, 771 (2000) D. P. DiVincenzo

In this scalable?



Building quantum computers

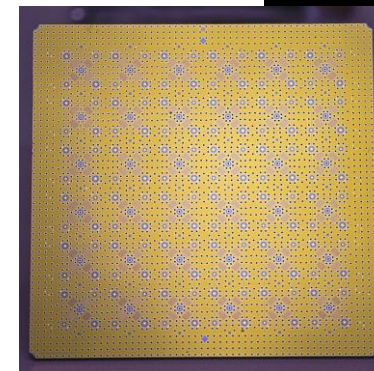
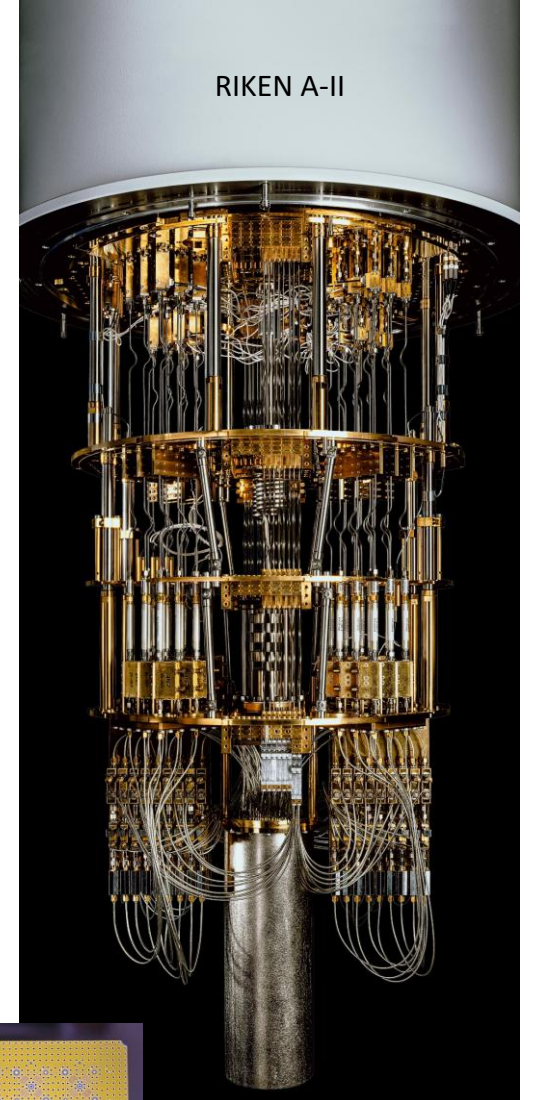
My symposium talk: 10:40–11:00 on Thursday, September 3

Scaling up superconducting quantum circuits with tileable qubit architecture

The development of quantum computers is now in full swing in both atomic and solid-state systems. In the case of superconducting system, the first coherent control of a so-called Cooper-pair box was demonstrated more than a quarter century ago, and consecutive efforts to improve design, materials, control theory since then have led to recent dramatic increases in the number of available qubits (more than a hundred), the qubit coherence (longer than a millisecond), and the control fidelity (better than 99.9% in two-qubit gates). To scale up a superconducting system, one of the key challenges is how to deliver microwave signals to the individual qubits that are just a few hundred microns apart in a two-dimensional chip. To address this issue, we adopt a tileable square-lattice qubit architecture with the coaxial cables addressing the backside of the chip from the vertical direction. In this talk, I will discuss the chip design, three-dimensional integrated wiring and performance of integrated chips of superconducting quantum computers being developed at RIKEN Center for Quantum Computing.

Thank you for your attention!

Looking forward to seeing you on Thursday!



RIKEN 144Q chip