RIKEN QUANTUM COMPUTING

Quantum computation and its physical realization by superconducting quantum circuits

Eisuke Abe

RIKEN Center for Quantum Computing

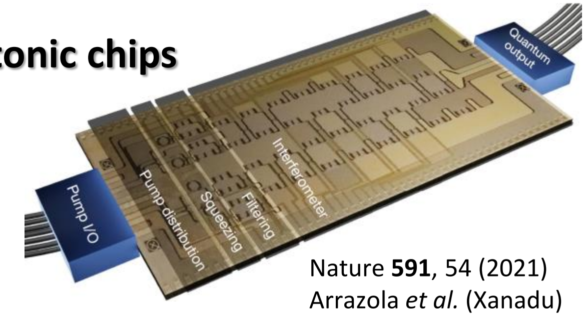
August 30, 2022 (Online)

2022 RIKEN-UCHU Summer School

Quantum Research on Nanomaterials with Optoelectronic Analysis

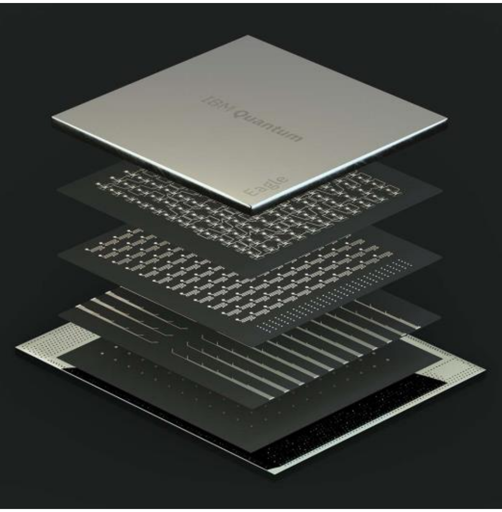
Platforms of quantum computers

Photonic chips

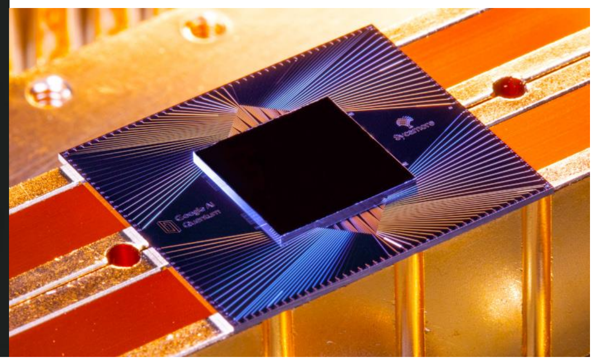


Nature **591**, 54 (2021)
Arrazola *et al.* (Xanadu)

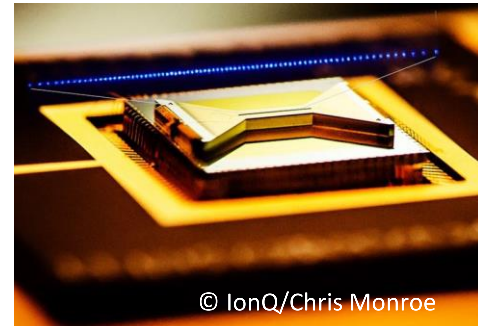
Superconducting quantum circuits

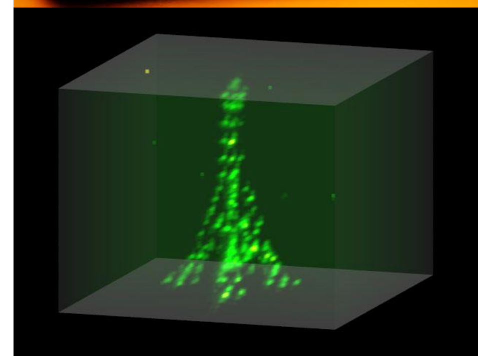


“Eagle” ©IBM



“Sycamore” ©Google

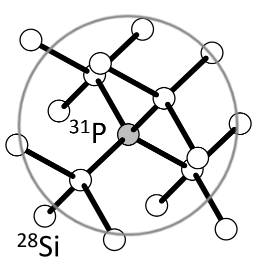


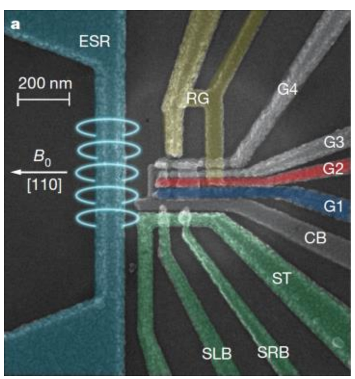


Nature **561**, 79 (2018)
Barredo *et al.*

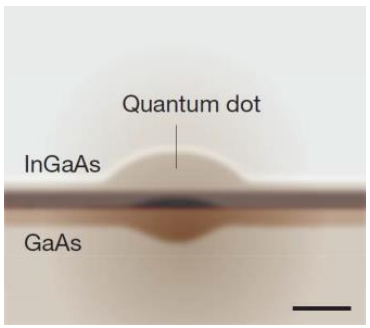
Trapped ions/Cold atoms

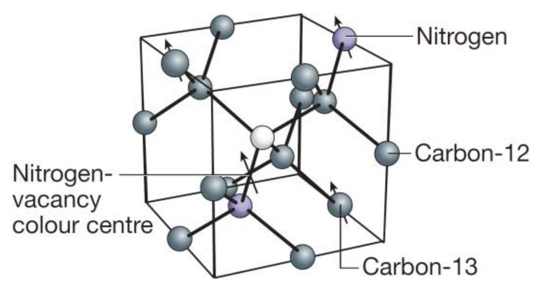
Semiconductors





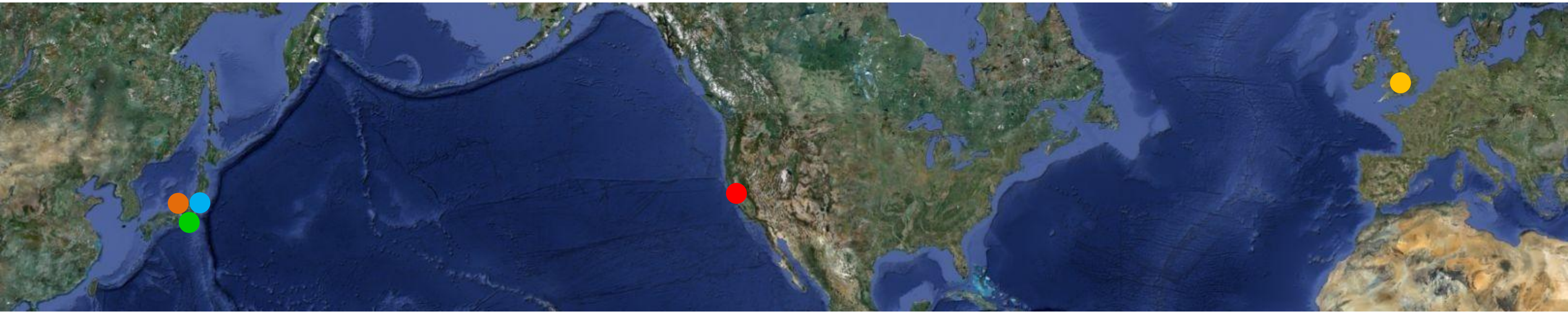
Nature **569**, 532 (2019) Huang *et al.*





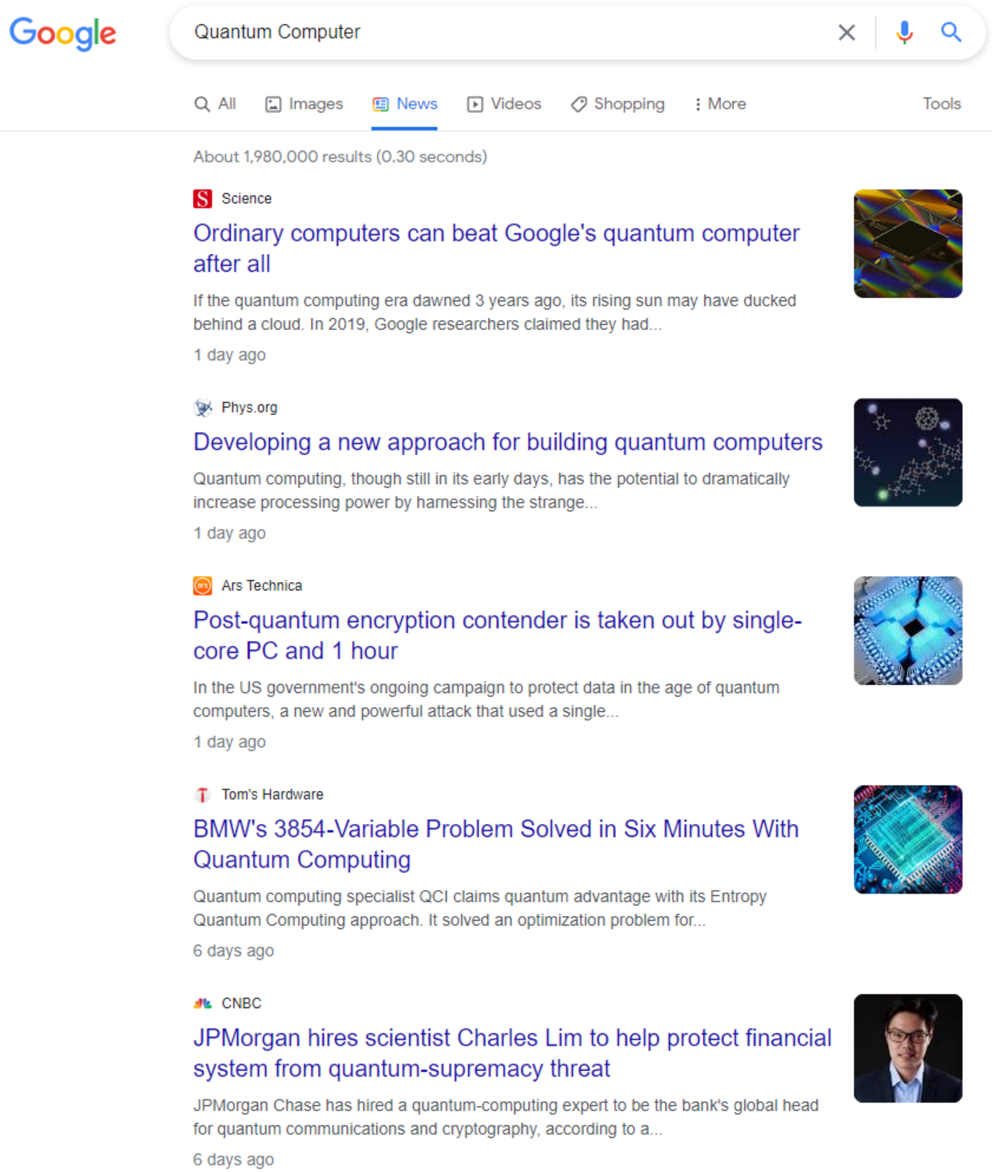
Nature **464**, 45 (2010) Ladd *et al.*

My CV



- **2001.4 – 2006.3 (Keio)** → Quantum computing (silicon donors)
- **2006.4 – 2009.12 (ISSP, Tokyo)** → Quantum transport (GaAs QDs, Al SET)
- **2010.1 – 2011.6 (Oxford)** → Hybrid system (spin–cavity coupling)
- **2011.7 – 2015.3 (Stanford)** → Quantum network (InAs QDs)
- **2015.4 – 2019.1 (Keio)** → Quantum sensing (diamond NV centers)
- **2019.2 – Present (RIKEN)** → Quantum computing (Superconducting quantum circuits)

“Quantum Computer” in the news



Searched on August 4, around 2 pm

“Wow, there are lots of

- developments
- interests (both scientific & business)
- debates
- hypes

about quantum computer!”

“Wait, what really is it?”

“Well, it uses quantum effects for computation blah blah blah”

“Hum… it sounds like tautology”

Contents

- **Quantum computation**
  - From an electron in a double-well potential to qubit
  - Quantum gates
  - Deutsch–Jozsa algorithm

- **Quantum error correction**
  - DiVincenzo's criteria and the need of QEC
  - Spin, spin resonance, and spin relaxation
  - Basics of quantum error correction

- **Superconducting quantum circuits**
  - Circuit QED and transmon
  - Quantum control
  - Recent experiments by Google and ETH

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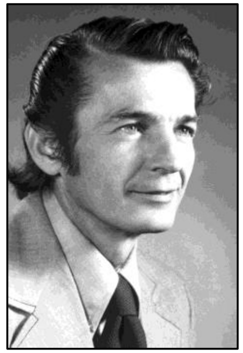
Nobel Prize in Physics, 1973

Leo Esaki
(1925–)



Ivar Giaever
(1929–)

Brian Josephson
(1940–)

"for their experimental discoveries regarding **tunneling phenomena** in semiconductors and superconductors, respectively"

"for his theoretical predictions of the properties of a supercurrent through a tunnel barrier, in particular those phenomena which are generally known as the **Josephson effects**"

Nobel Prize in Physics, 1973

Leo Esaki
(1925–)

New Phenomenon in Narrow Germanium *p-n* Junctions

LEO ESAKI
Tokyo Tsushin Kogyo, Limited, Shinagawa, Tokyo, Japan
(Received October 11, 1957)

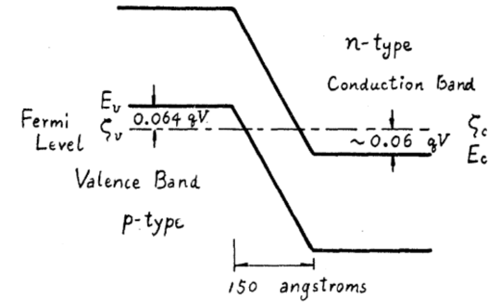


FIG. 2. Energy diagram of the *p-n* junction at 300°K and no bias voltage.

"for their experimental discoveries regarding **tunneling phenomena** in semiconductors and superconductors, respectively"

- Tunnel (Esaki) diode
  → Electron is a **WAVE**

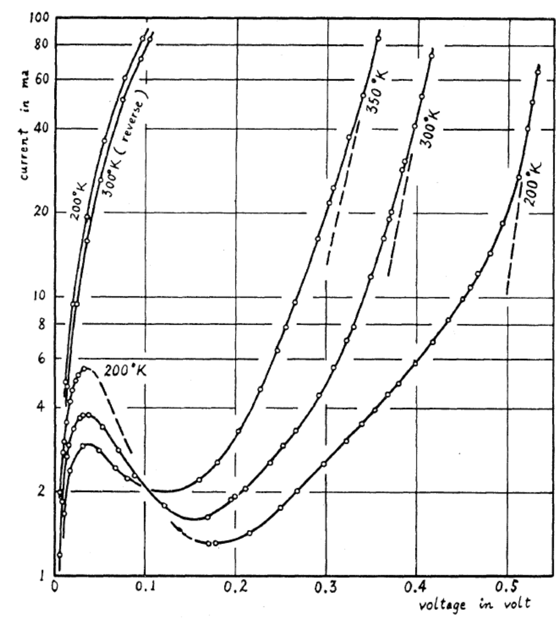
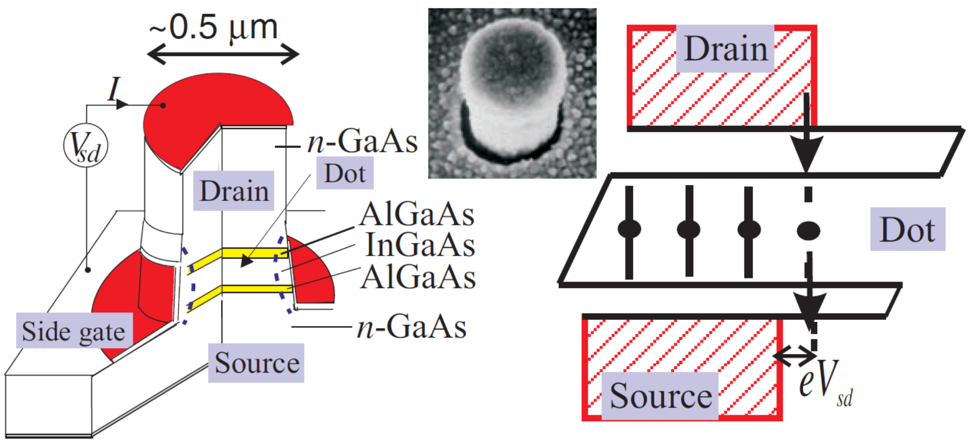


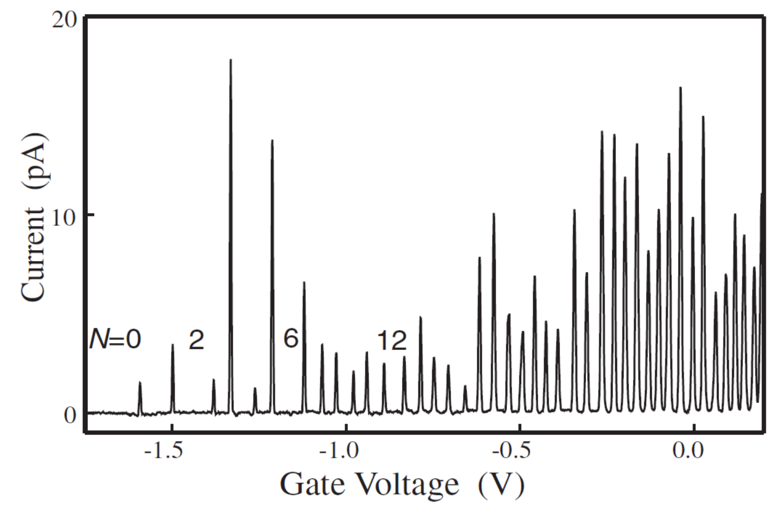
FIG. 1. Semilog plots of the measured current-voltage characteristic at 200°K, 300°K, and 350°K.

Phys. Rev. **109**, 603 (1958) Esaki

From tunnel diode to quantum dot

Resonant tunneling diode → Vertical QD (1996)



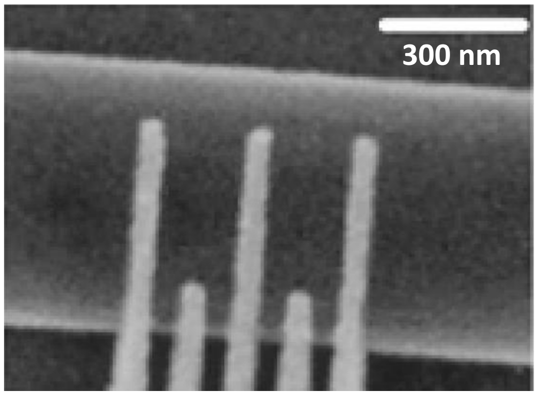


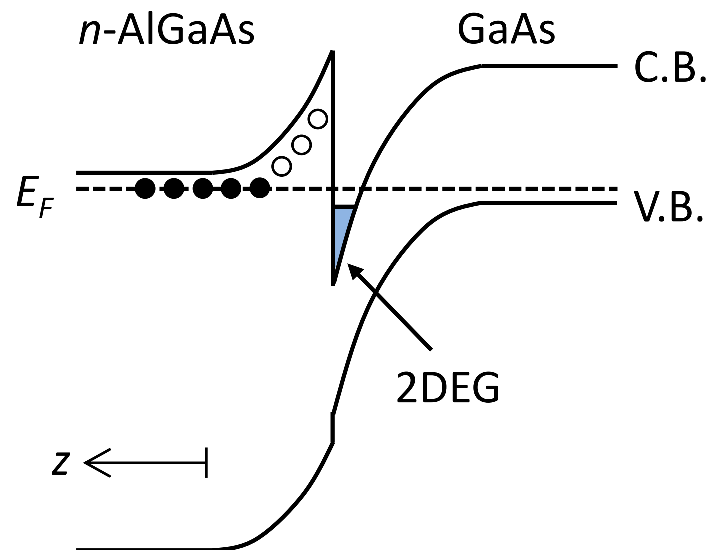
- Tunnel (Esaki) diode
  → Electron is a **WAVE**

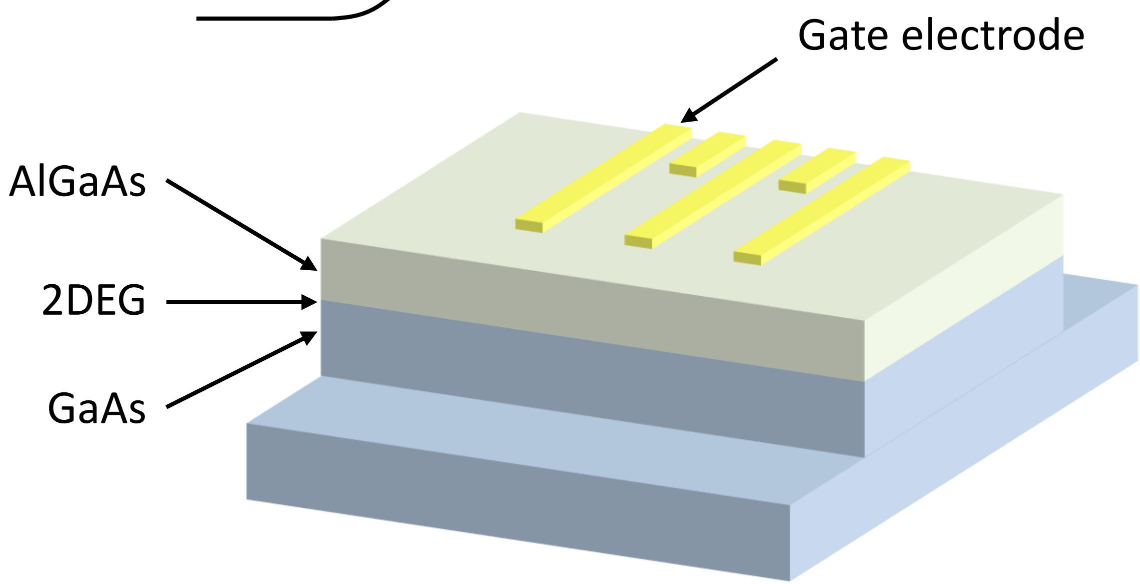
- Single electron transistor
  → Electron is a **PARTICLE**

Rep. Prog. Phys. **64**, 701 (2001) Kouwenhoven *et al.*

Lateral double quantum dot







Rep. Prog. Phys. **69**, 759 (2006) Fujisawa *et al.*

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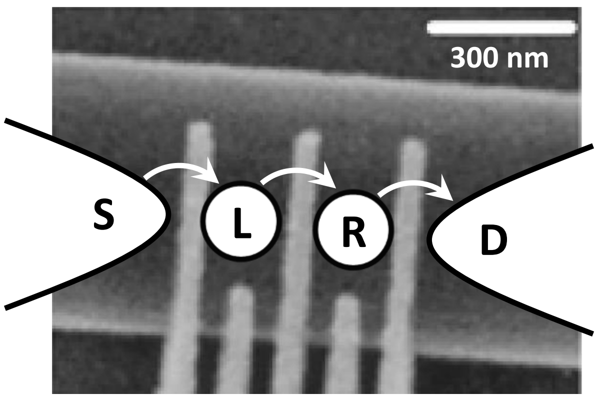
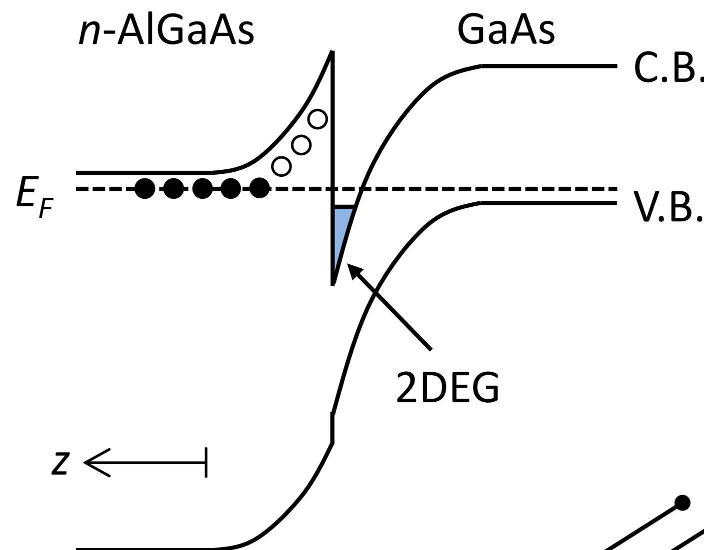
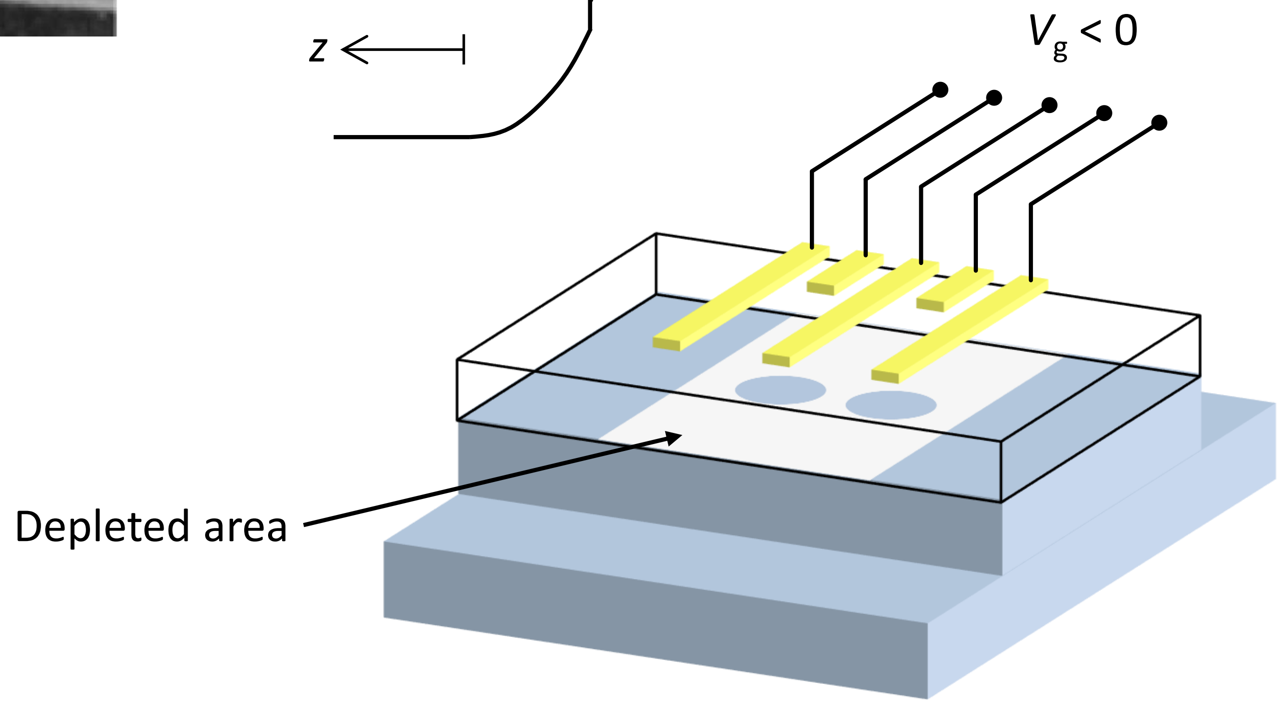
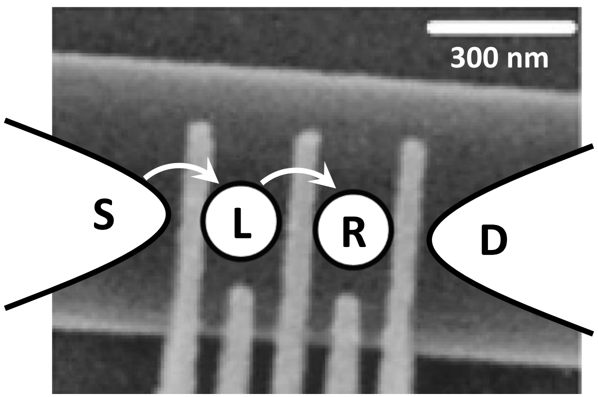


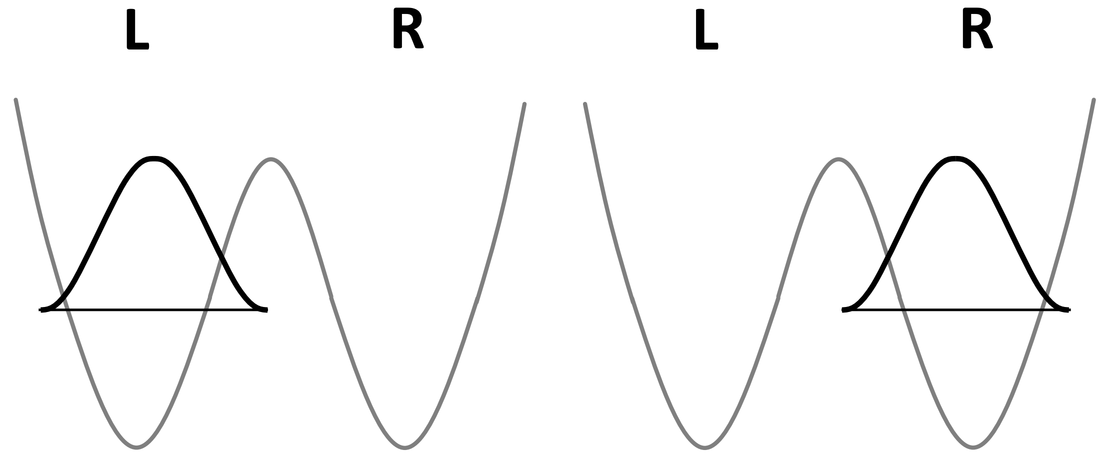




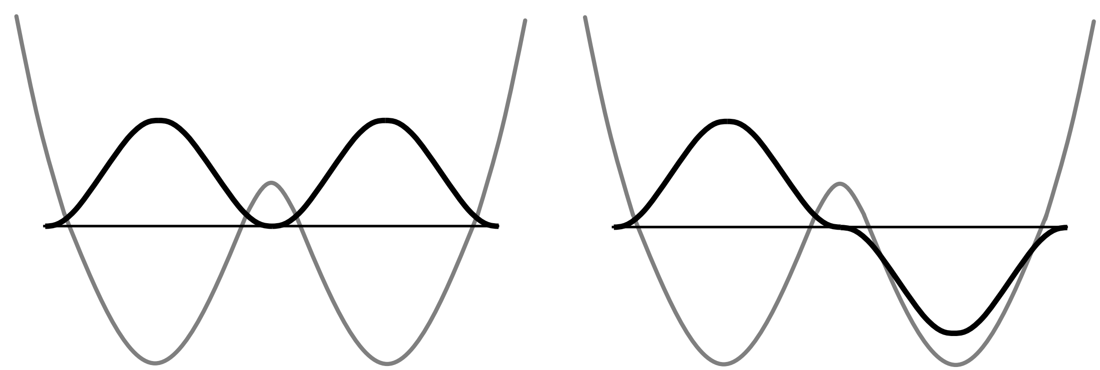
Rep. Prog. Phys. **69**, 759 (2006) Fujisawa *et al.*

Single electron in a double-well potential





A single electron (wavefunction) can spread over the two QDs (bonding & antibonding states)



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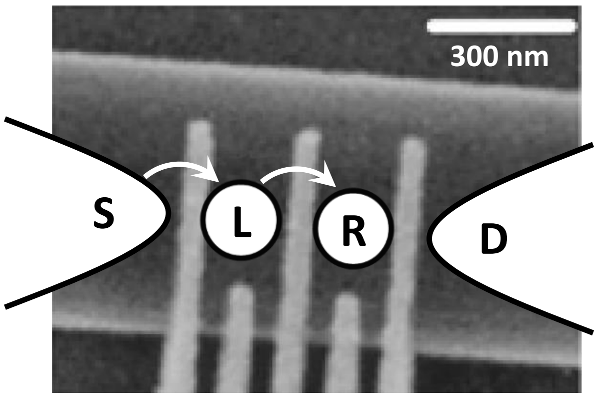
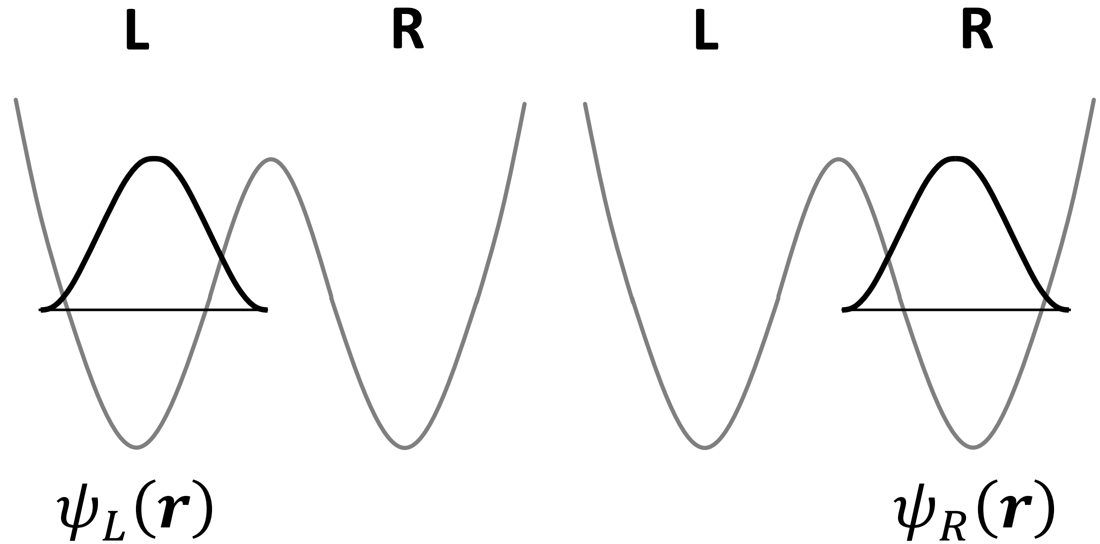




Normalization

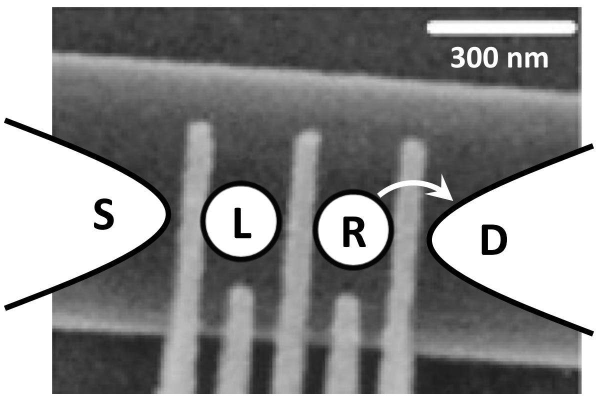
$$\int |\psi_L(\boldsymbol{r})|^2 d\boldsymbol{r} = \int |\psi_R(\boldsymbol{r})|^2 d\boldsymbol{r} = 1$$

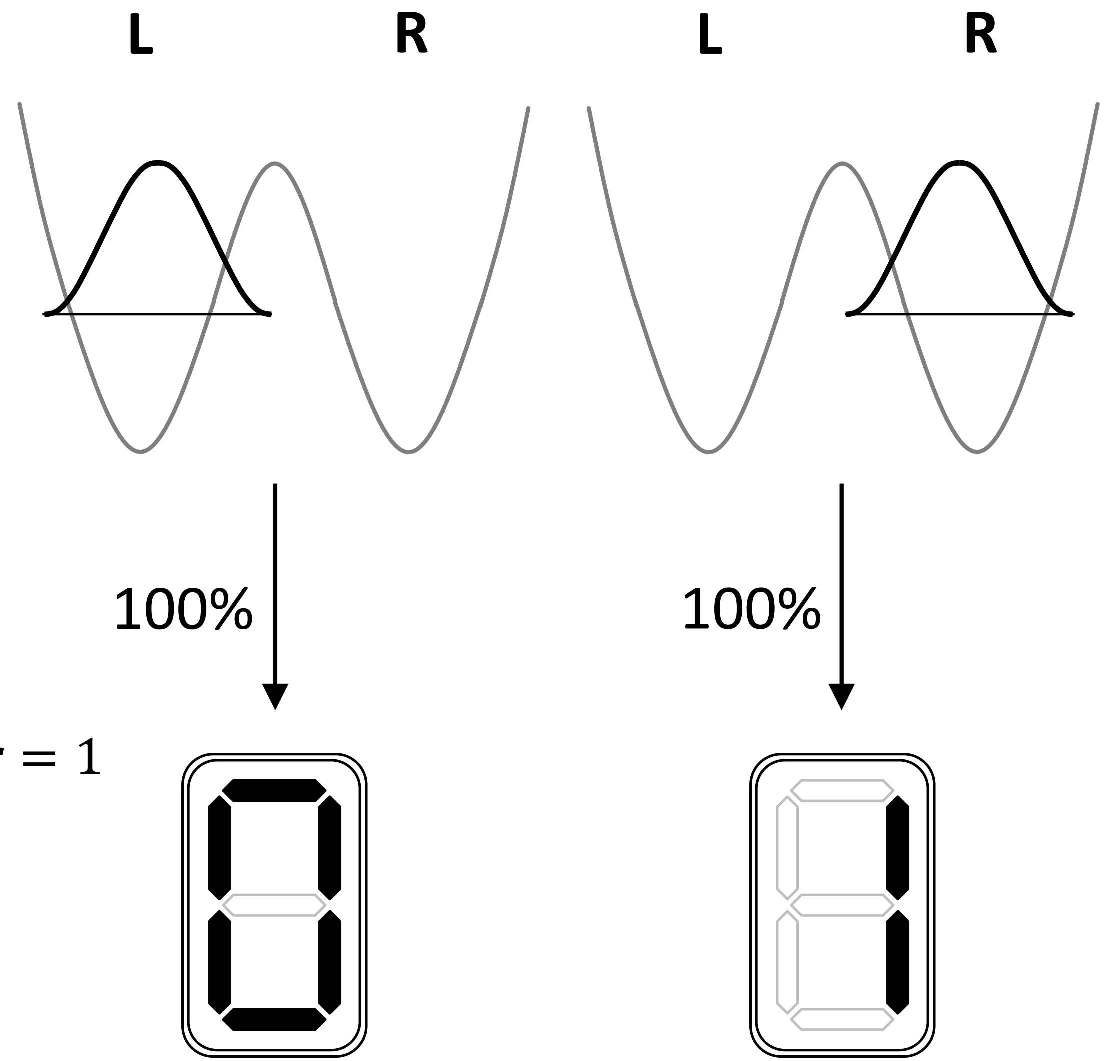
Orthogonality

$$\int \psi_R^*(\boldsymbol{r})\psi_L(\boldsymbol{r}) d\boldsymbol{r} = 0$$

Rep. Prog. Phys. **69**, 759 (2006) Fujisawa *et al.*

Measuring the location of an electron





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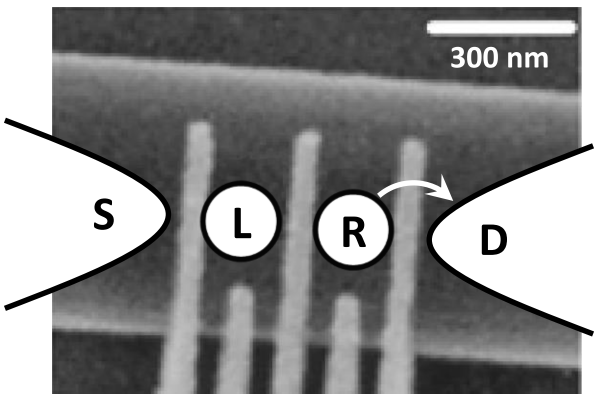

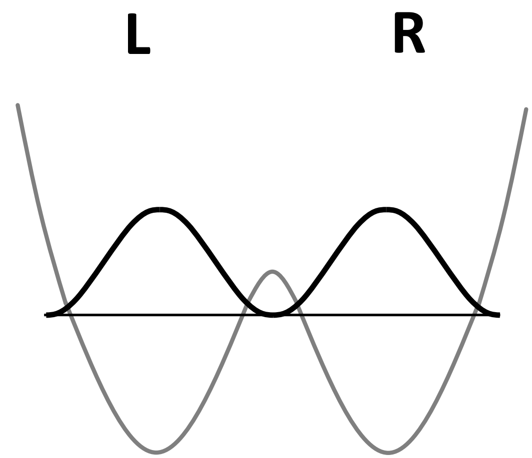

$$\psi_B(\boldsymbol{r}) = \frac{1}{\sqrt{2}}[\psi_L(\boldsymbol{r}) + \psi_R(\boldsymbol{r})]$$

Bonding state

Born rule

$$\left| \int \psi_B^*(\boldsymbol{r}) \psi_L(\boldsymbol{r}) d\boldsymbol{r} \right|^2 = 0.5$$

$$\left| \int \psi_B^*(\boldsymbol{r}) \psi_R(\boldsymbol{r}) d\boldsymbol{r} \right|^2 = 0.5$$

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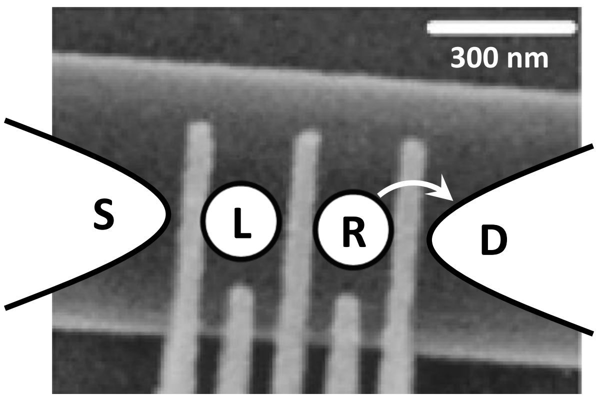

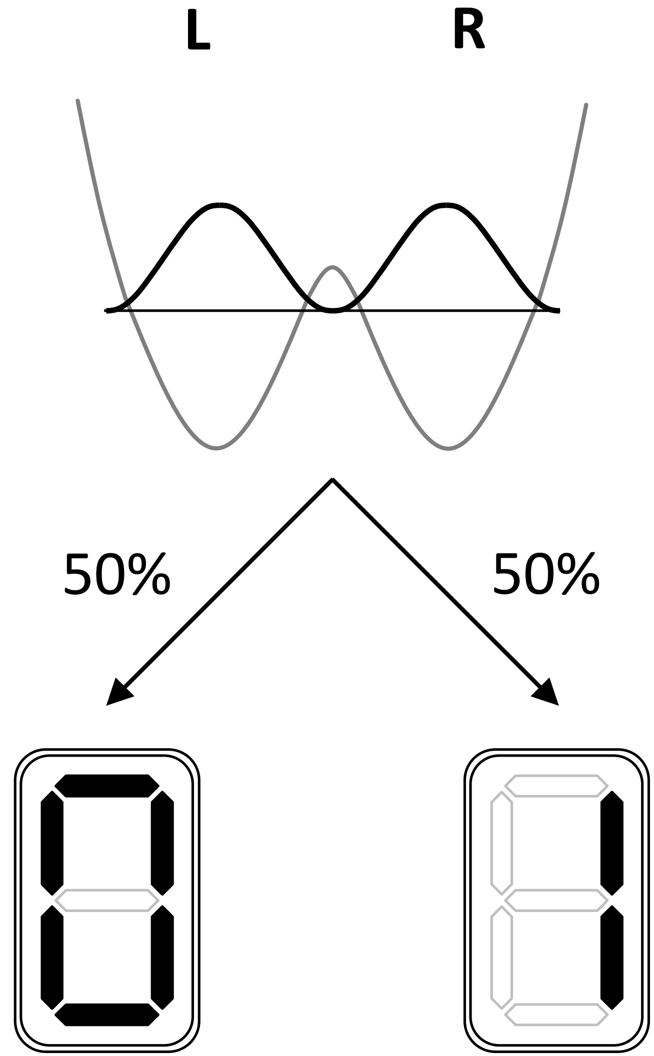

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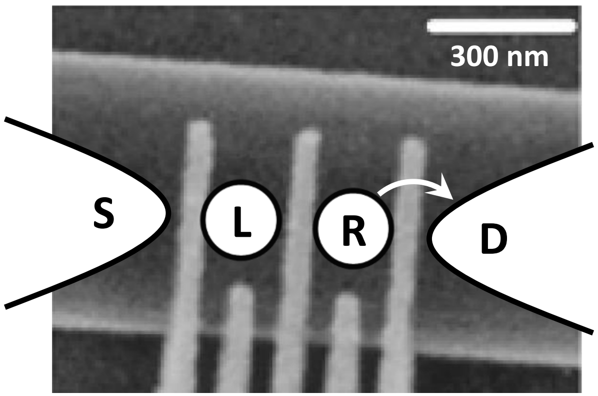
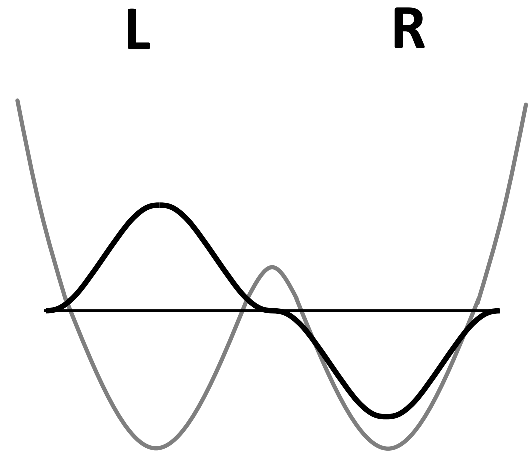




$$\psi_A(\boldsymbol{r}) = \frac{1}{\sqrt{2}}[\psi_L(\boldsymbol{r}) - \psi_R(\boldsymbol{r})]$$

Antibonding state

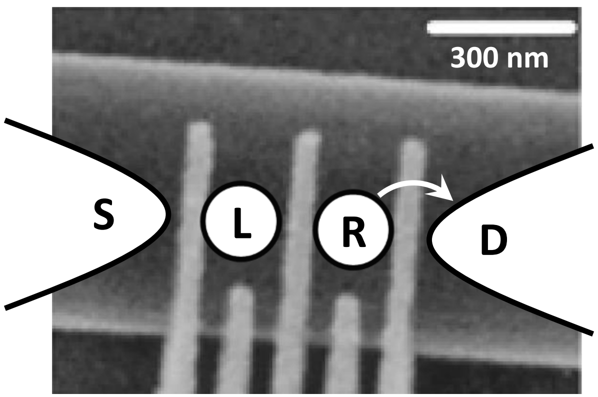
Born rule

$$\left|\int \psi_A^*(\boldsymbol{r})\psi_L(\boldsymbol{r})d\boldsymbol{r}\right|^2 = 0.5$$

$$\left|\int \psi_A^*(\boldsymbol{r})\psi_R(\boldsymbol{r})d\boldsymbol{r}\right|^2 = 0.5$$

Rep. Prog. Phys. **69**, 759 (2006) Fujisawa *et al.*

Measuring the location of an electron



Born rule

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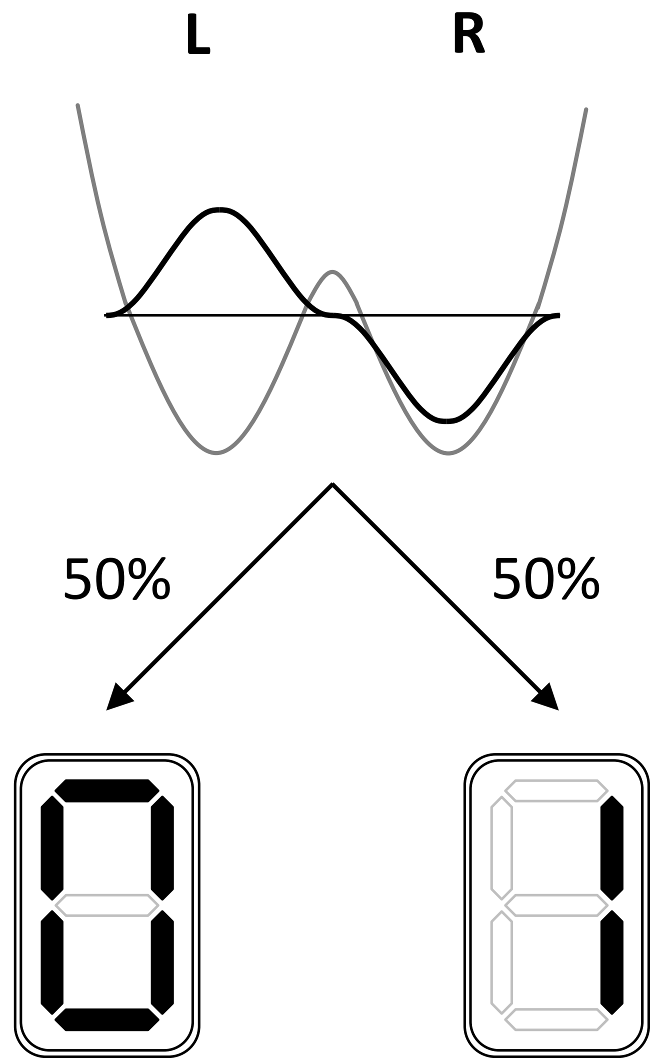

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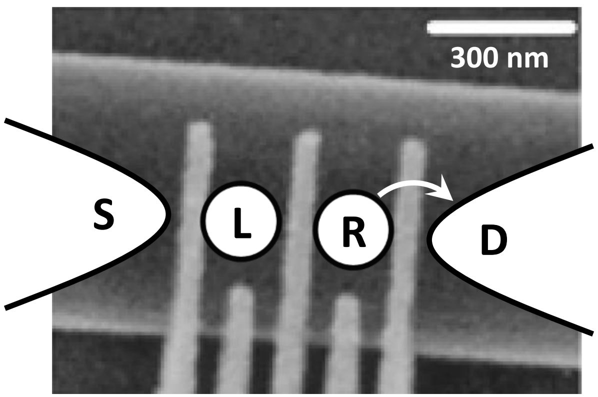

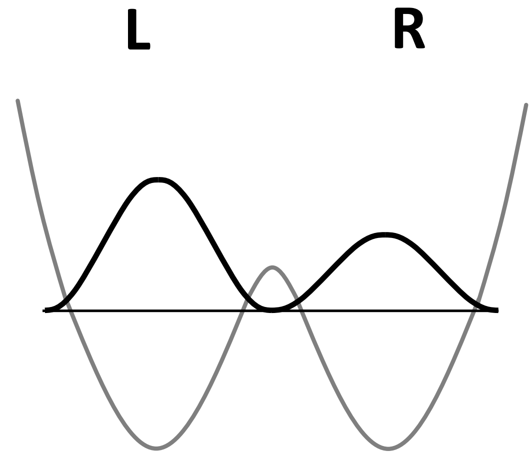

$$\psi(\boldsymbol{r}) = \frac{\sqrt{3}}{2}\psi_L(\boldsymbol{r}) + \frac{1}{2}\psi_R(\boldsymbol{r})$$

Superposition state

Born rule

$$\left| \int \psi^*(\boldsymbol{r})\psi_L(\boldsymbol{r})d\boldsymbol{r} \right|^2 = 0.75$$

$$\left| \int \psi^*(\boldsymbol{r})\psi_R(\boldsymbol{r})d\boldsymbol{r} \right|^2 = 0.25$$

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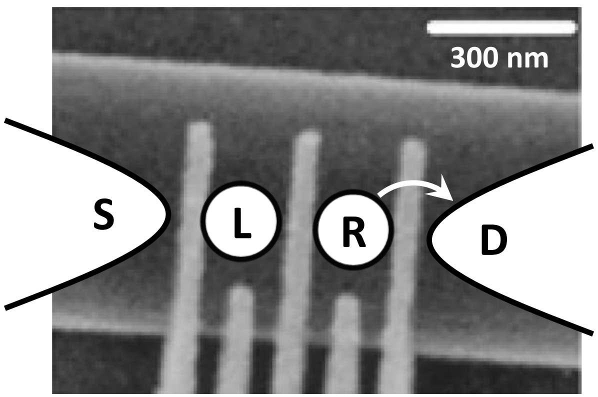
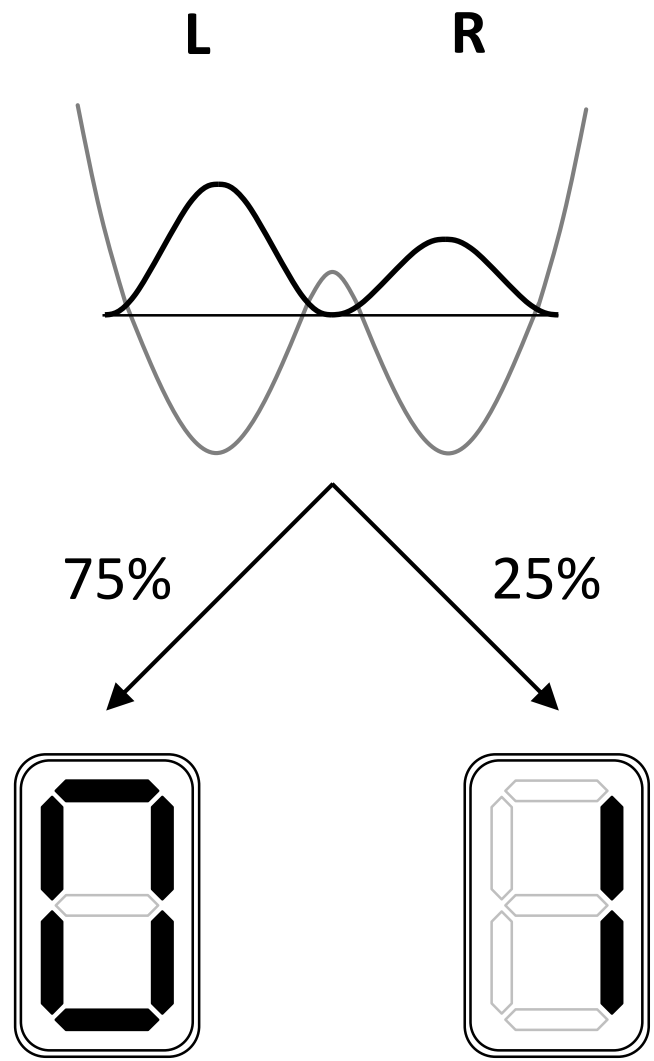
Measuring the location of an electron





Born rule

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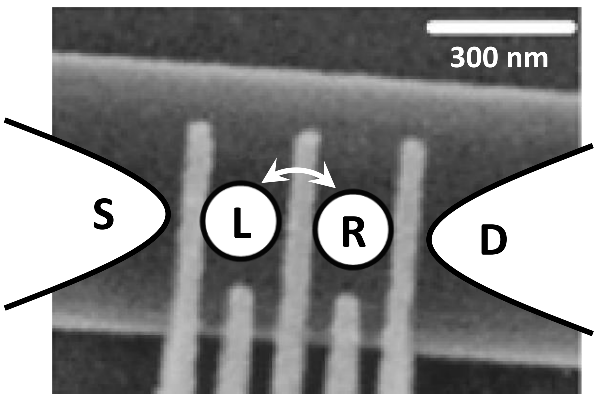

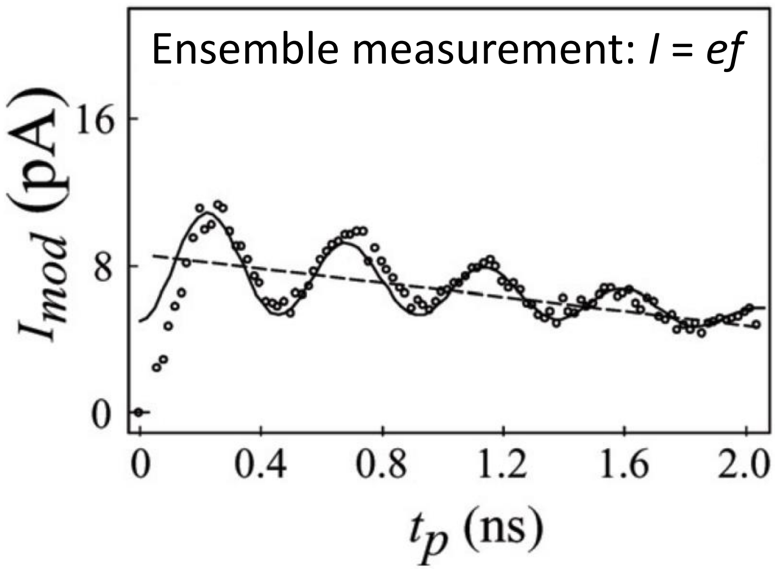

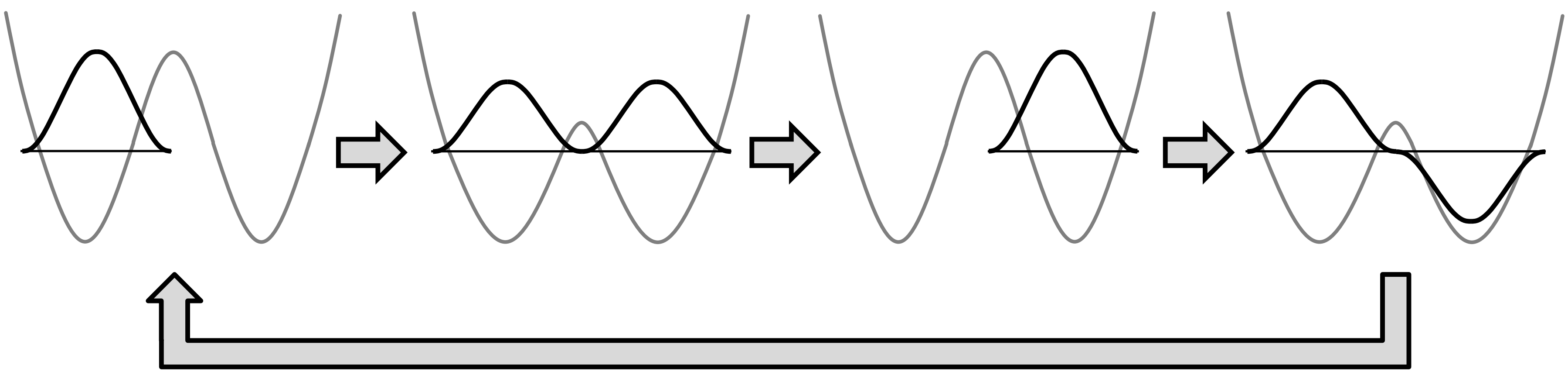

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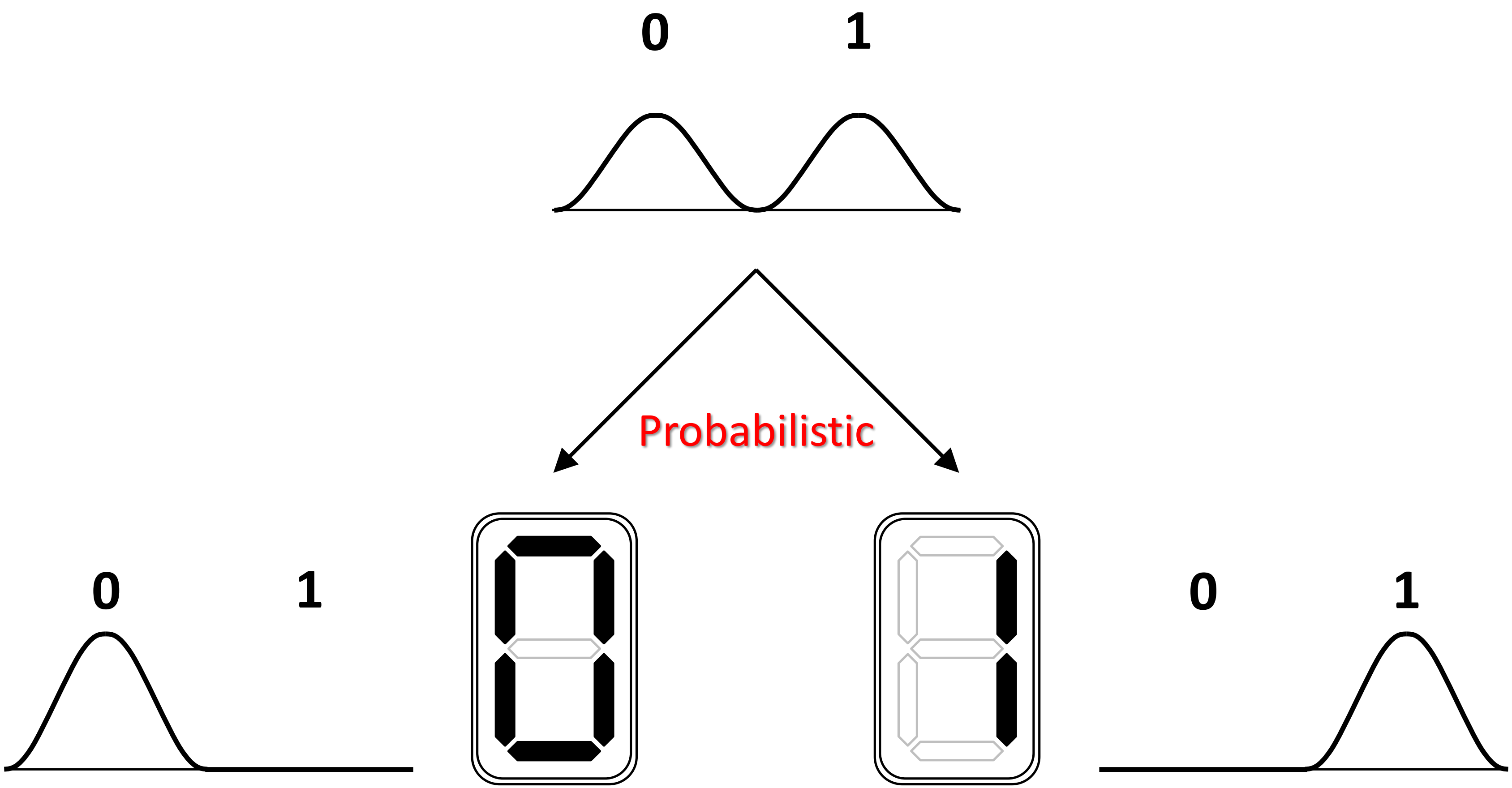
Quantum bit (Qubit)

Can we use the **WAVE** nature of an electron for computation?

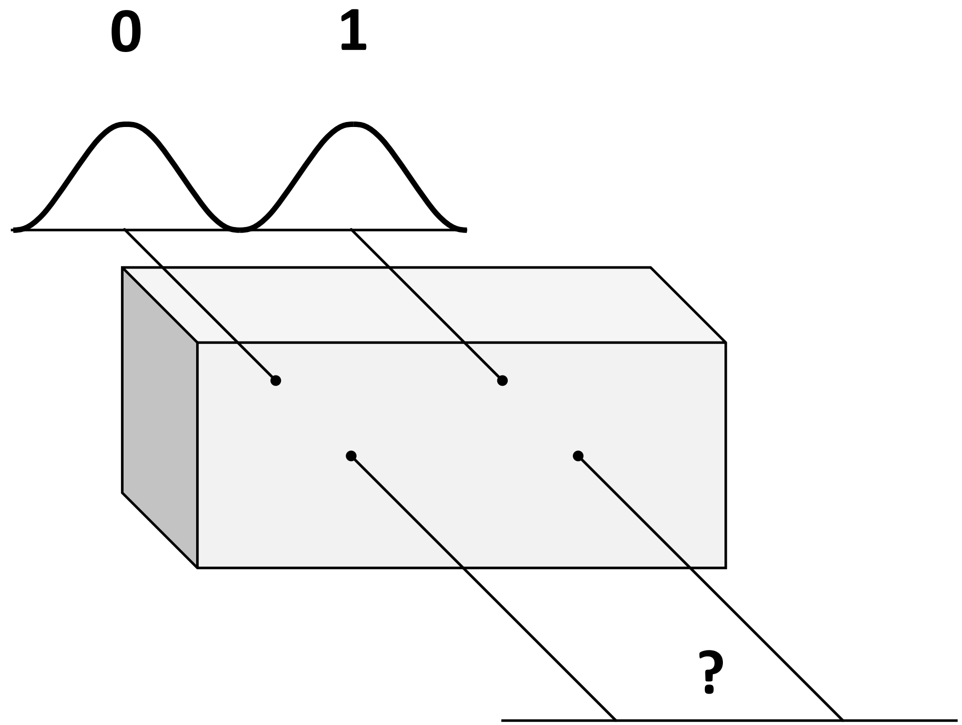


Measurement determines the electron’s location (Localization = **PARTICLE**)

Contents

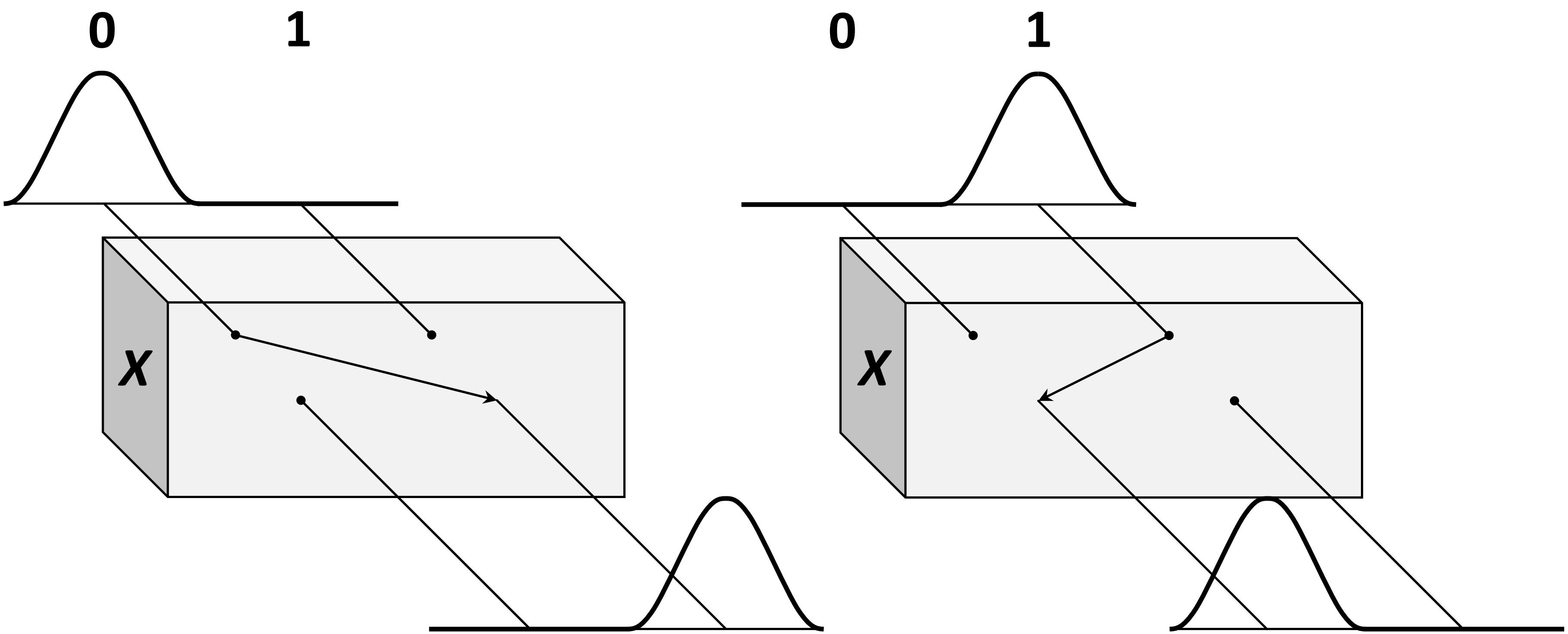
Quantum gate

Having bits is not enough to do computation



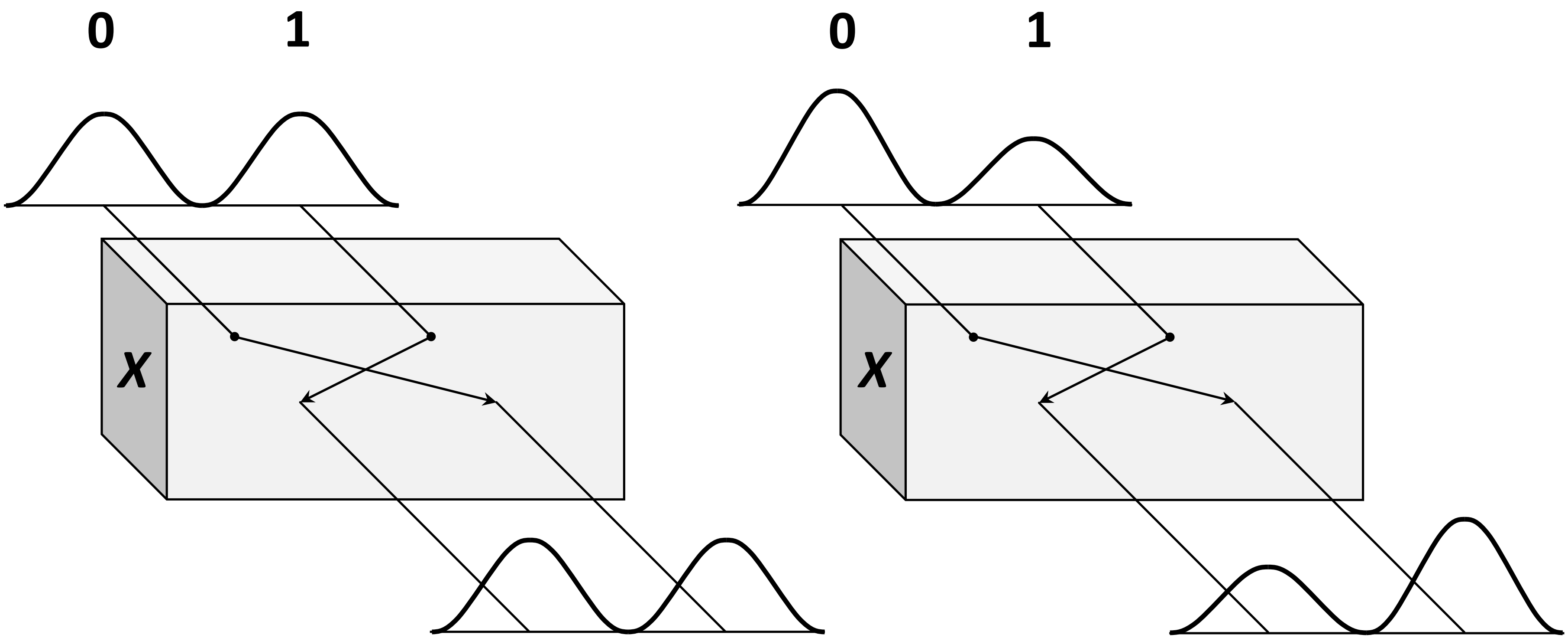
Operations (ops, gates) relevant for qubits?

X (NOT) gate



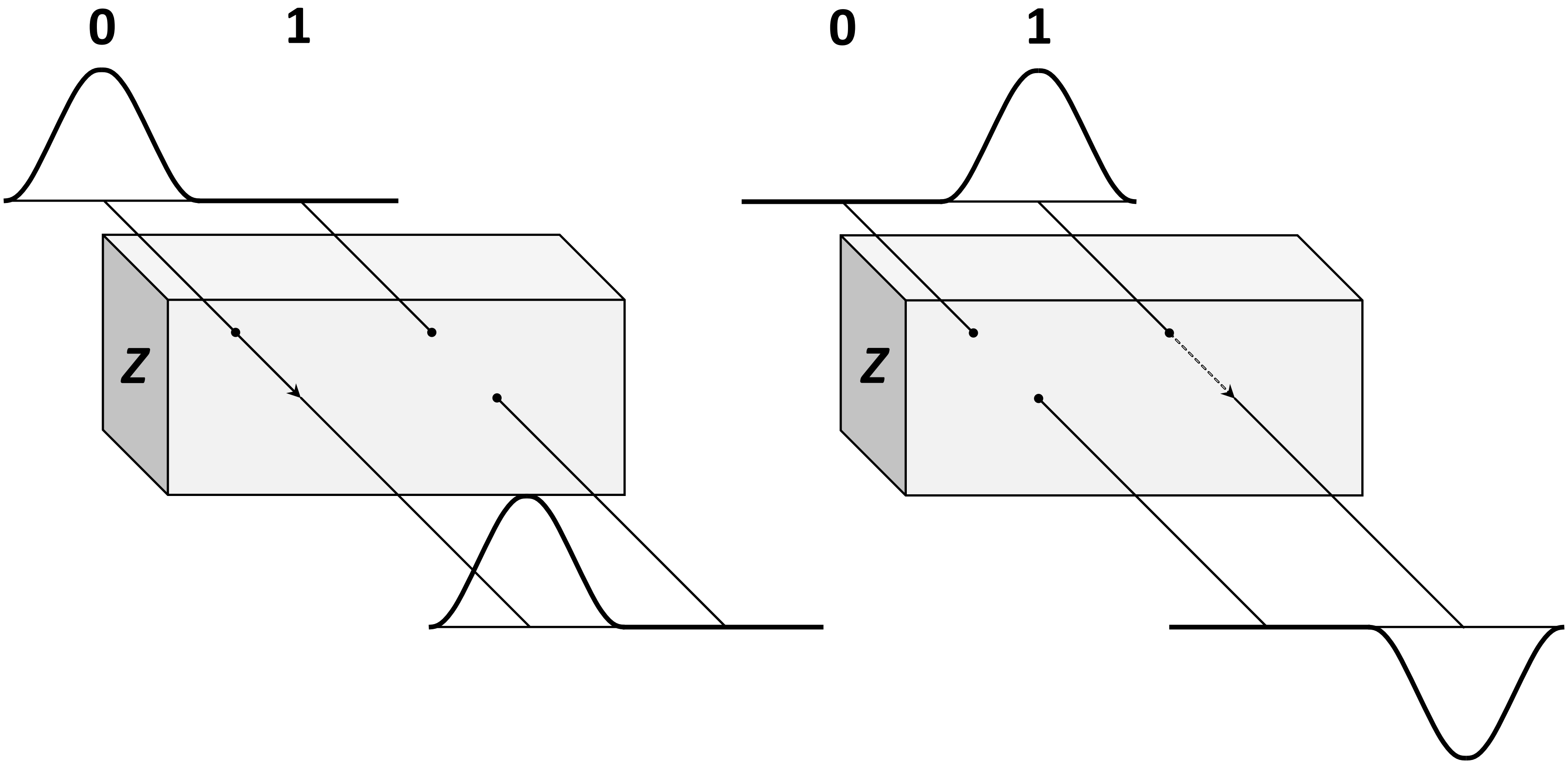
NOT is the only nontrivial 1-bit op for classical computation

X (NOT) gate

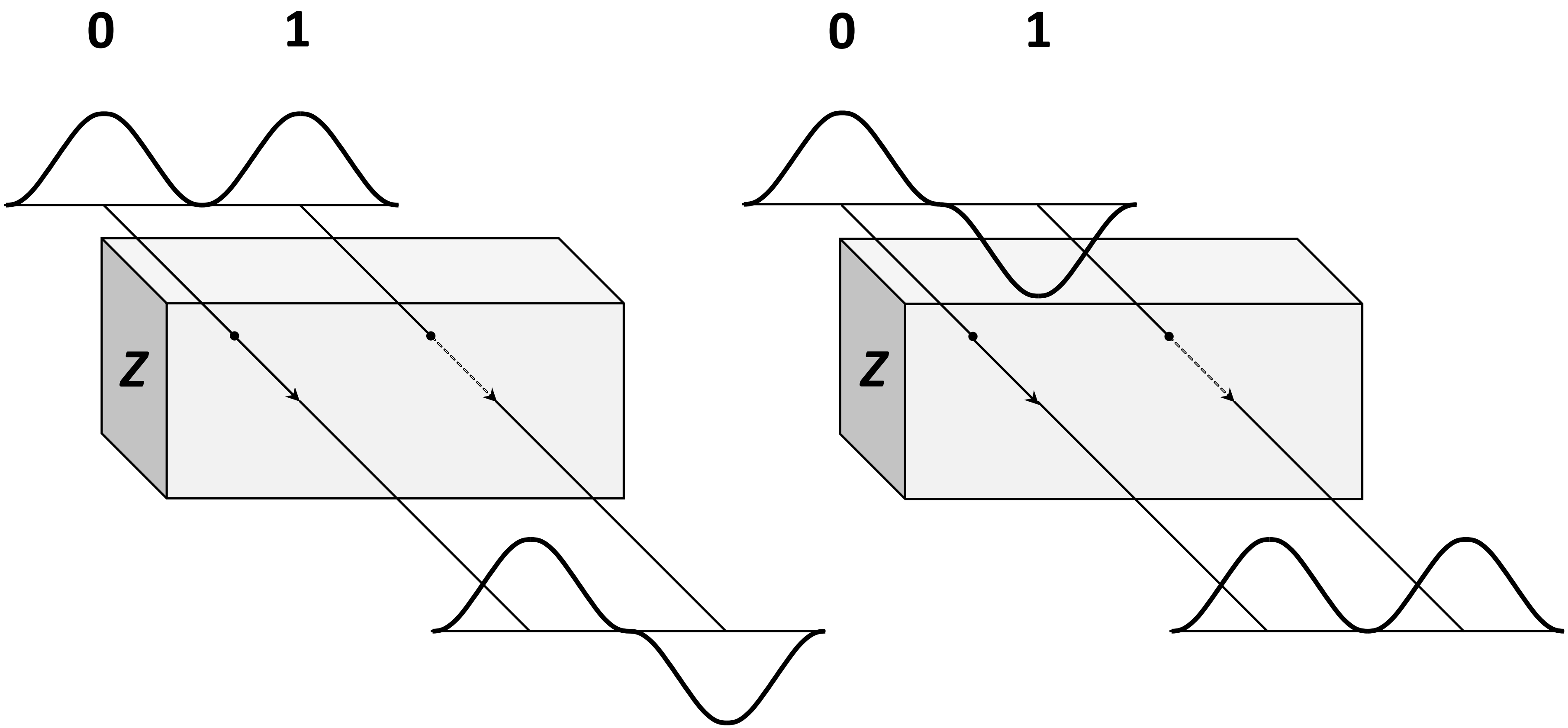


The same op but we can use a superposition state as input

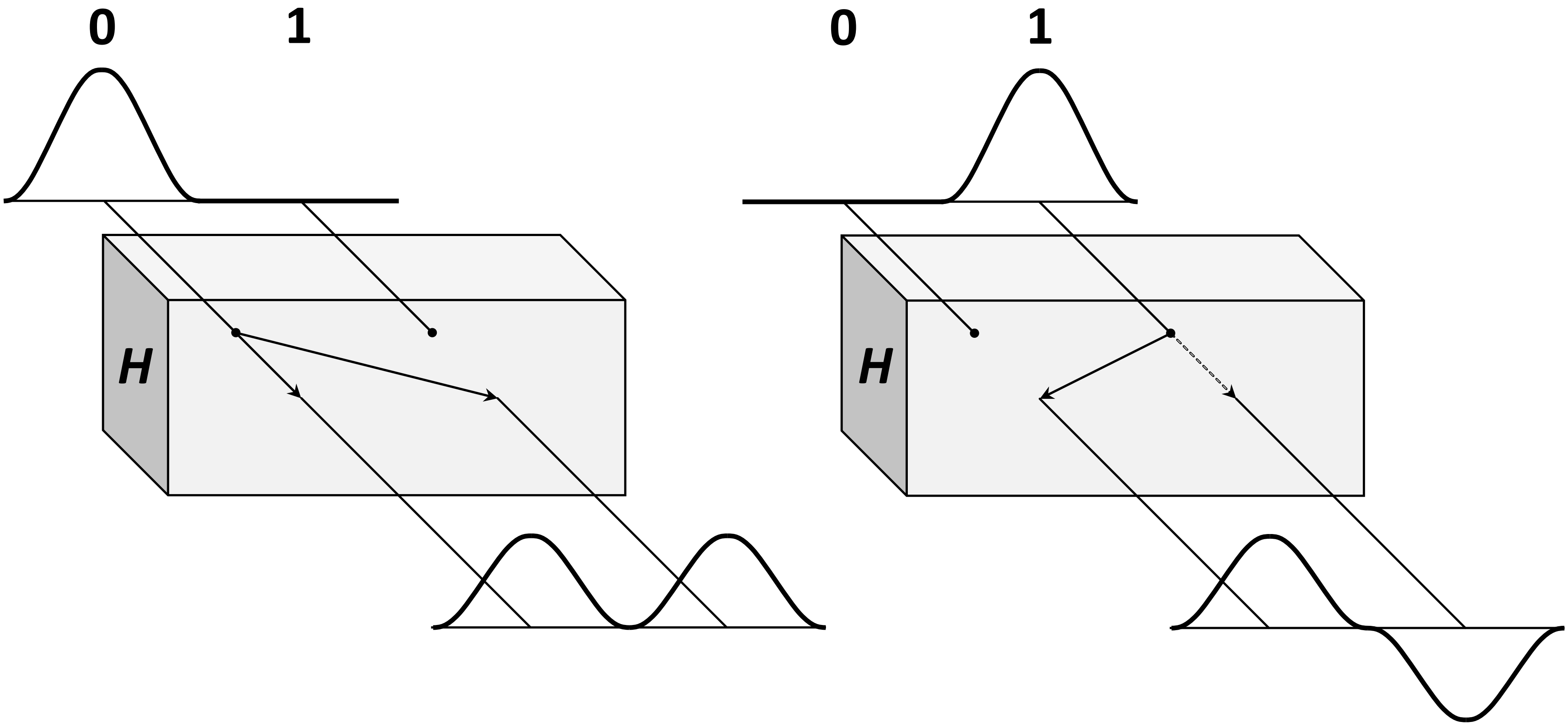
Z gate



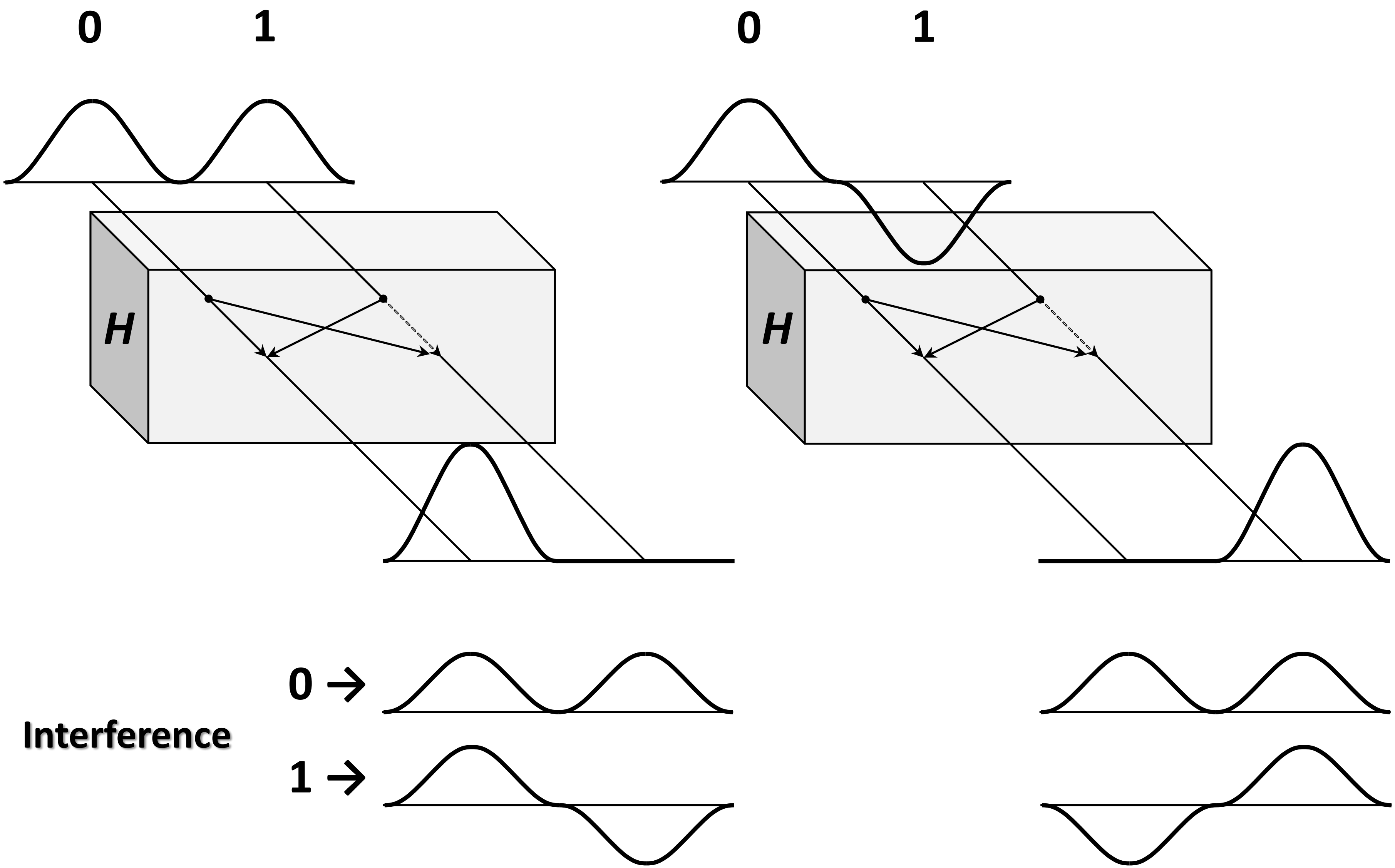
Z gate



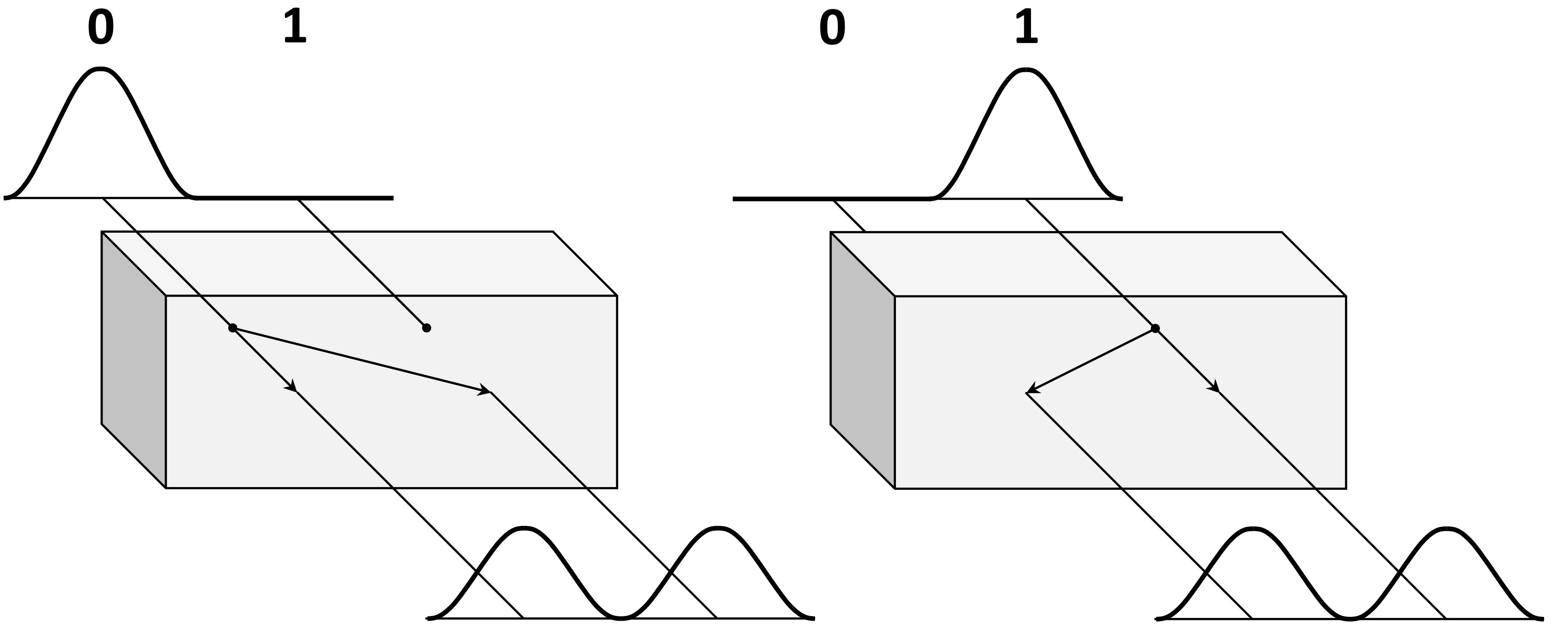
H gate



H gate



Impossible gate



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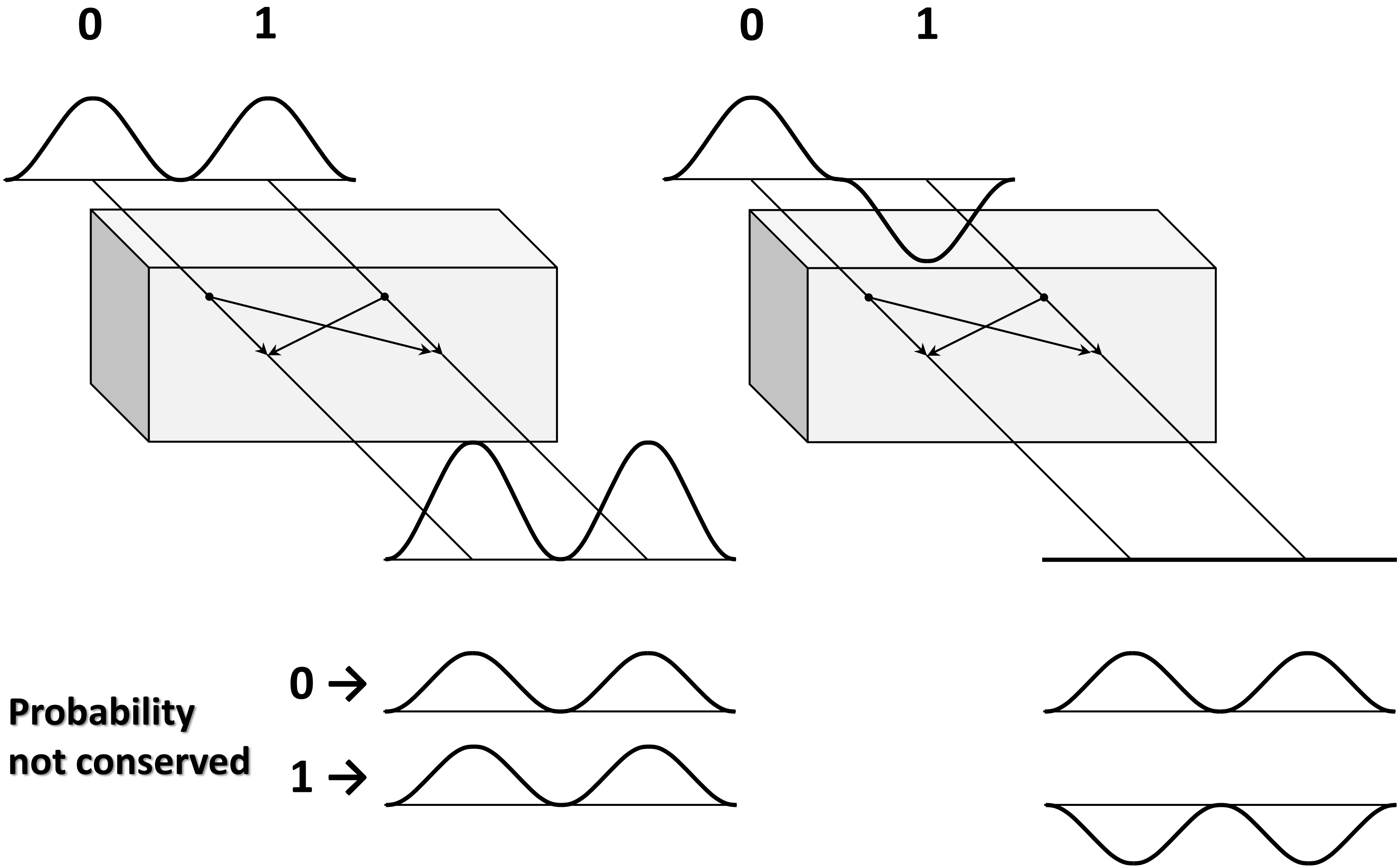
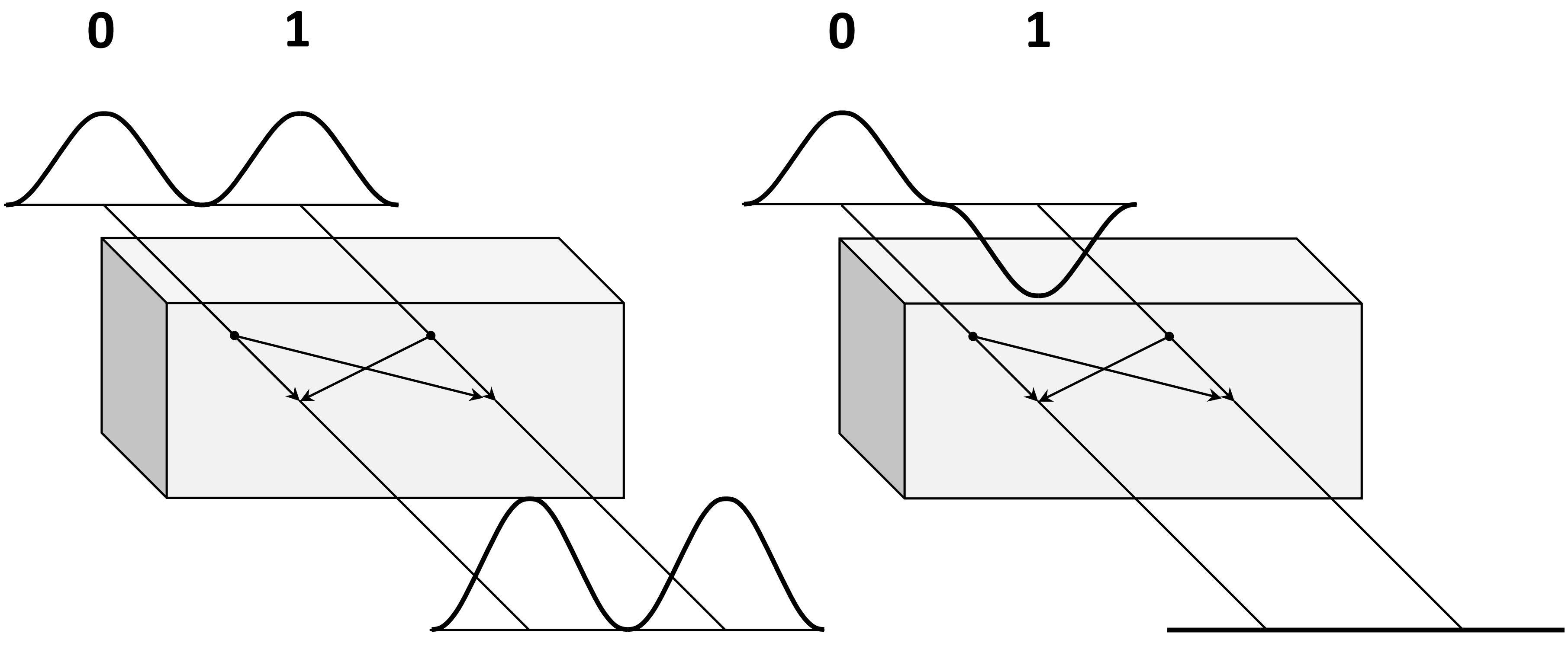


Impossible gate



Possible gates obeying the rule of quantum mechanics
are known as **unitary gates** (*X*, *H*…)

Qubit representation

Vector representation

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

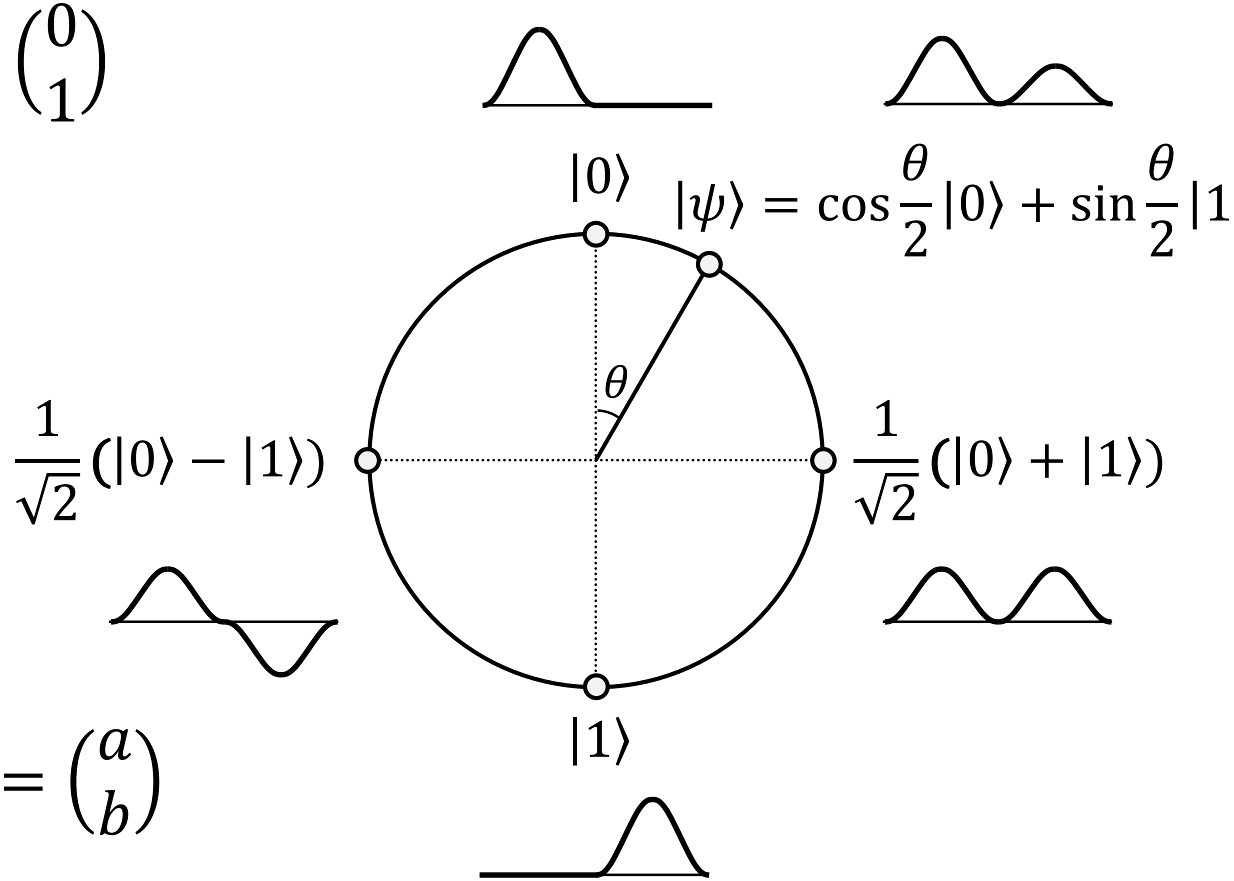
"Ket" in the Dirac notation

For now, you may just think of it as just a column vector

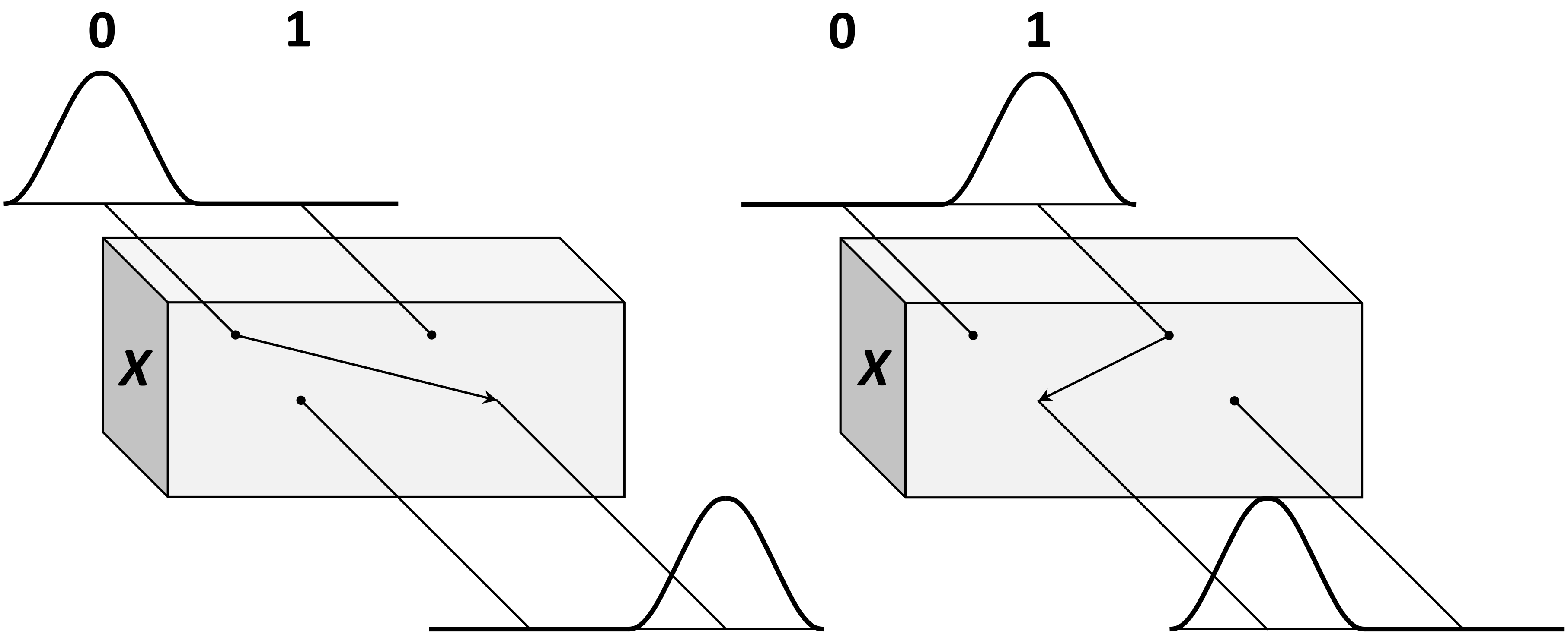
Superposition state

$$|\psi\rangle = a|0\rangle + b|1\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$|a|^2 + |b|^2 = 1$$



X gate

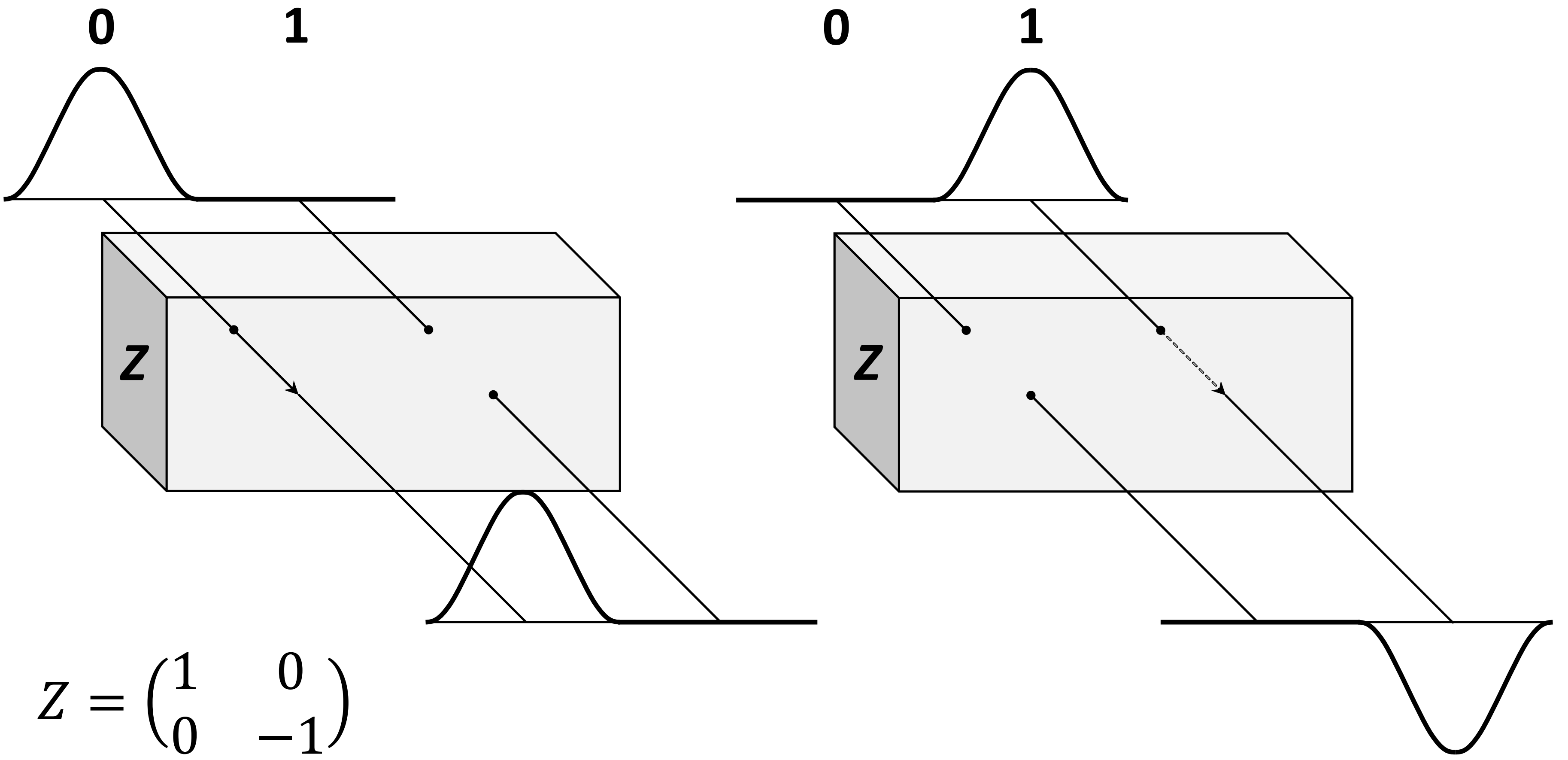


$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$X|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$X|1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

Z gate

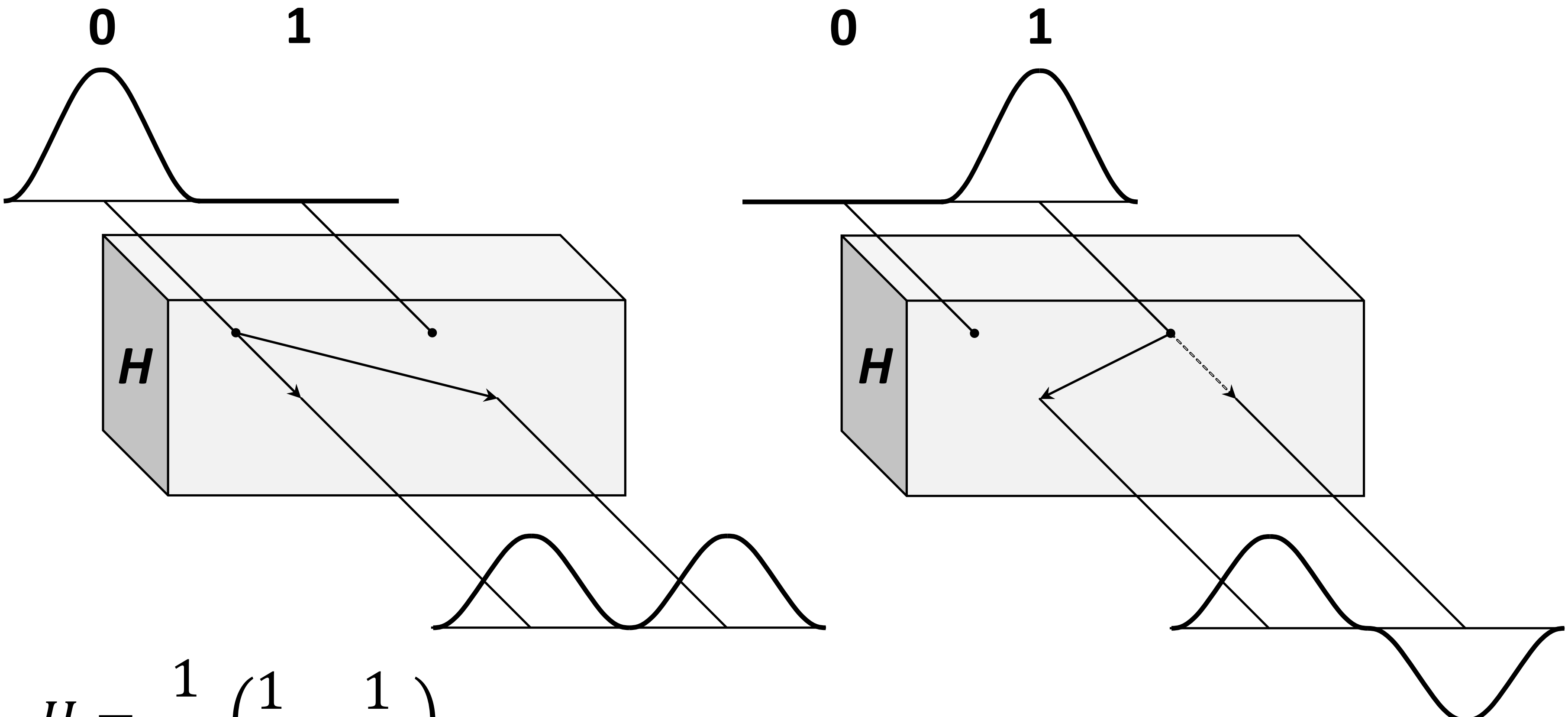


$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$Z|0\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

$$Z|1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -|1\rangle$$

H gate

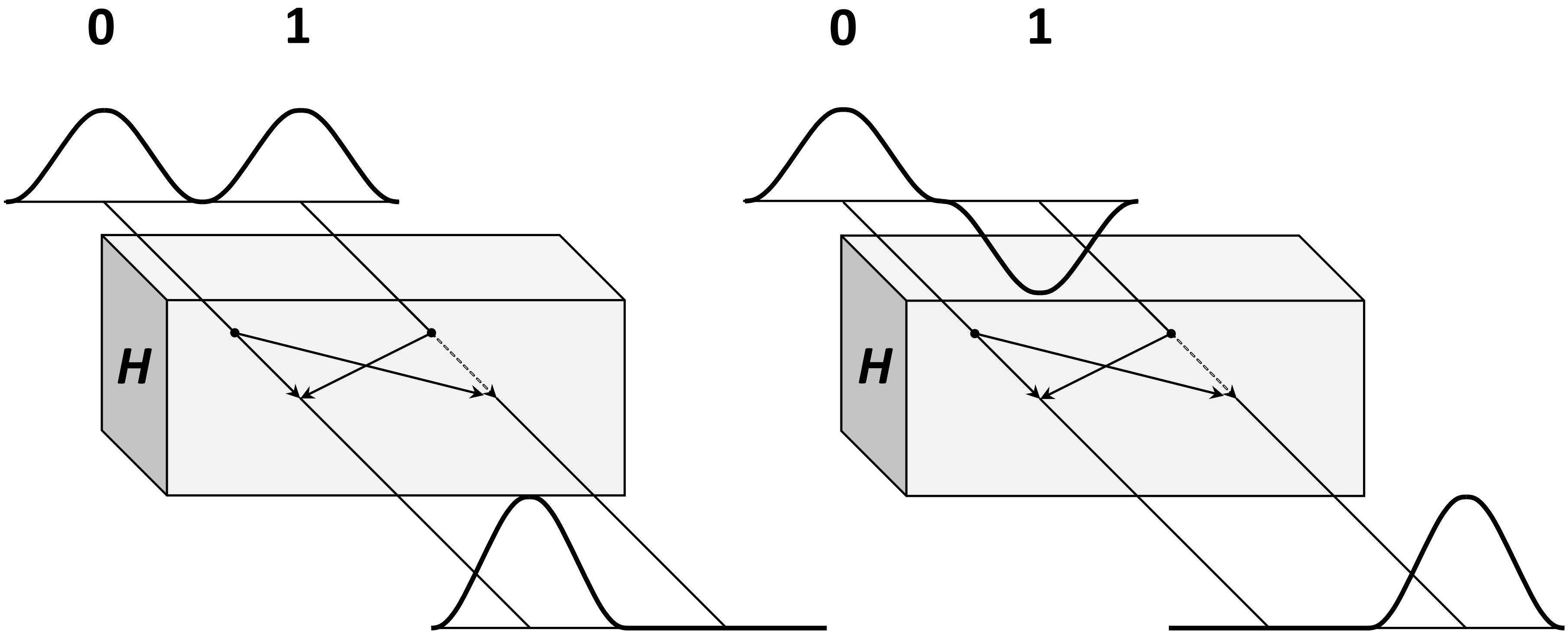


$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

H gate

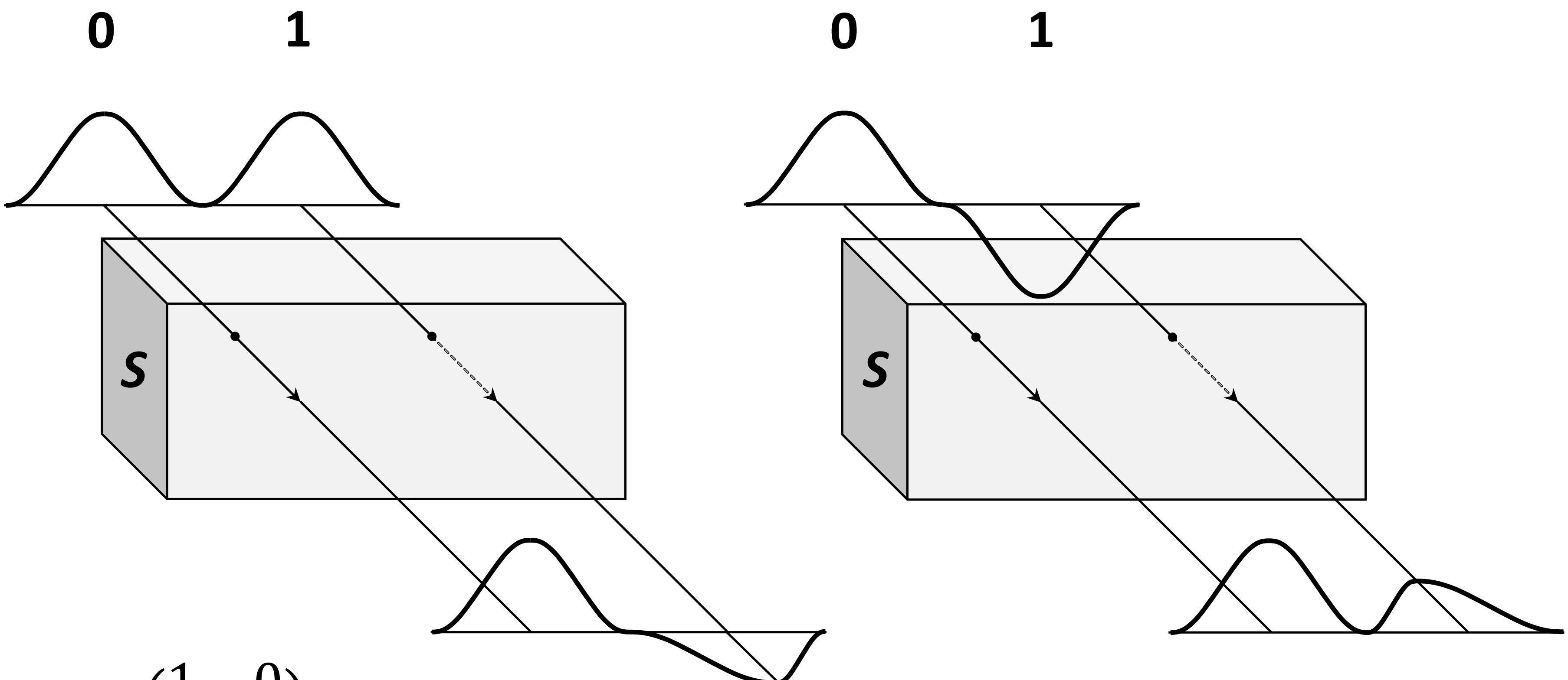


$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Interference

$$H\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle) = \frac{1}{2}(|0\rangle + |1\rangle \pm |0\rangle \mp |1\rangle) = \begin{cases} |0\rangle \\ |1\rangle \end{cases}$$

S gate



$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$i = \sqrt{-1}$ = quarter rotation, but of course, this is just a cartoon

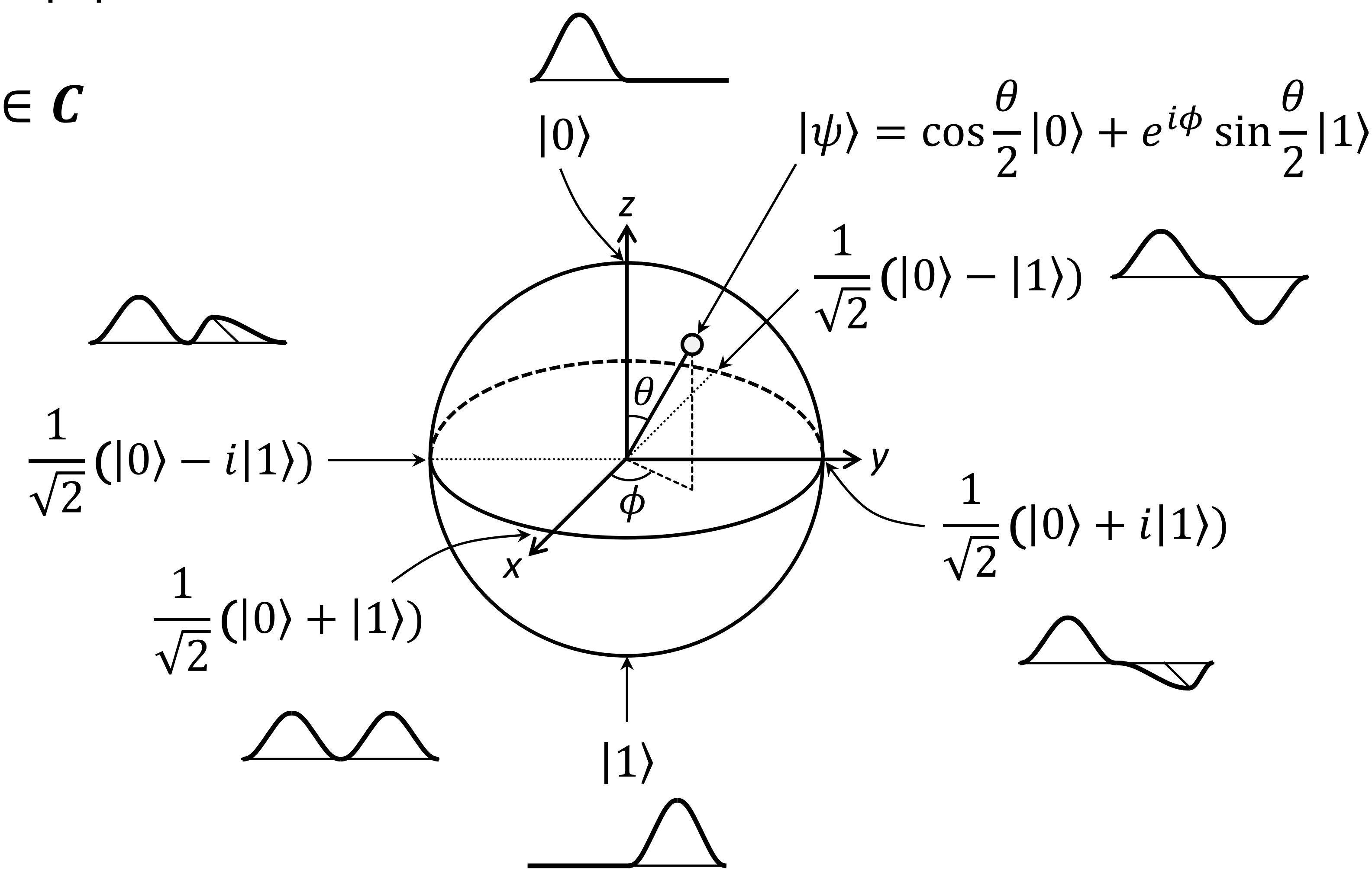
$$S\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

$$S\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$

Qubit representation: Bloch sphere

$$|a|^2 + |b|^2 = 1$$

$$a, b \in \boldsymbol{C}$$



2-qubit system

Vector representation of 2-qubit system

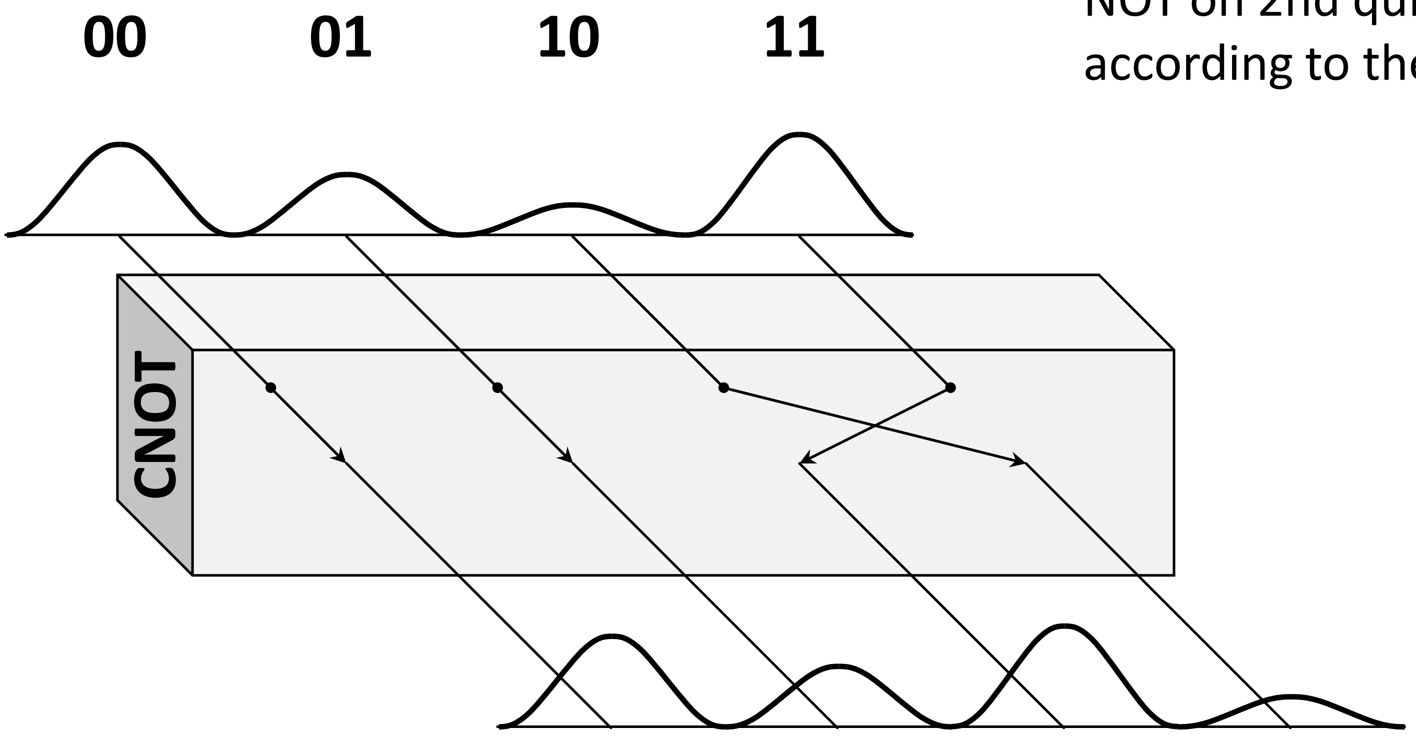
$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$$

CNOT gate

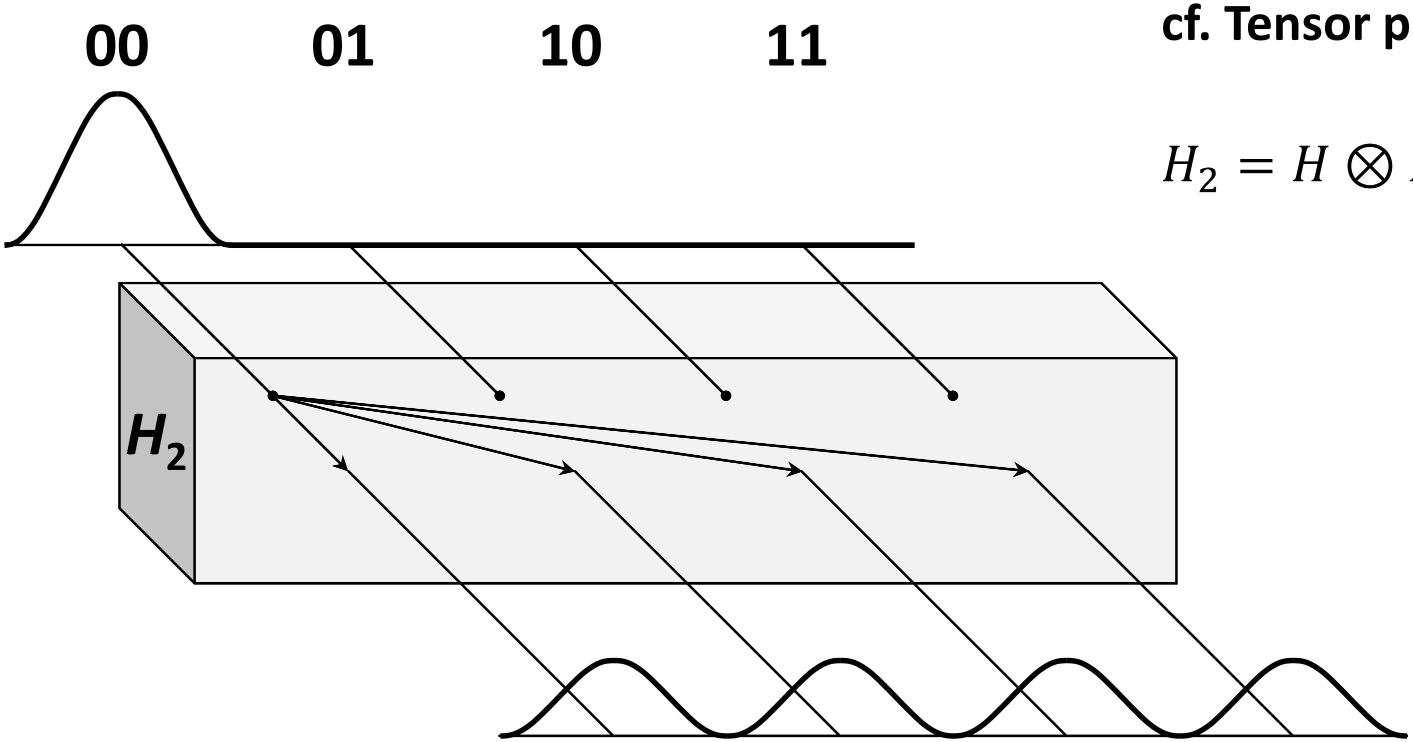
NOT on 2nd qubit
according to the state of 1st qubit



$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\mathrm{CNOT}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a \\ b \\ d \\ c \end{pmatrix}$$

H_2 gate

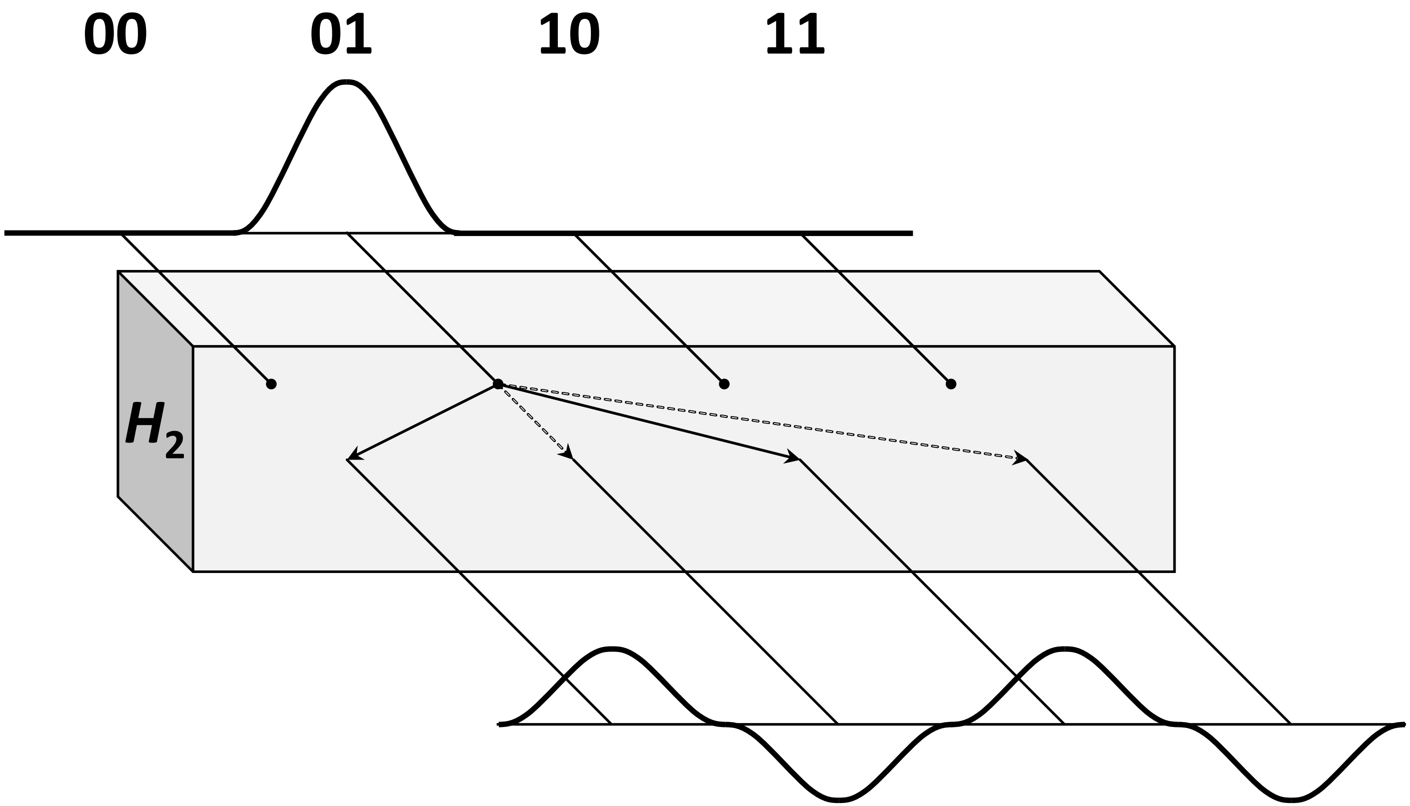


cf. Tensor product

$$H_2 = H \otimes H = \frac{1}{\sqrt{2}}\begin{pmatrix} H & H \\ H & -H \end{pmatrix}$$

$$H_2 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \qquad H_2\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

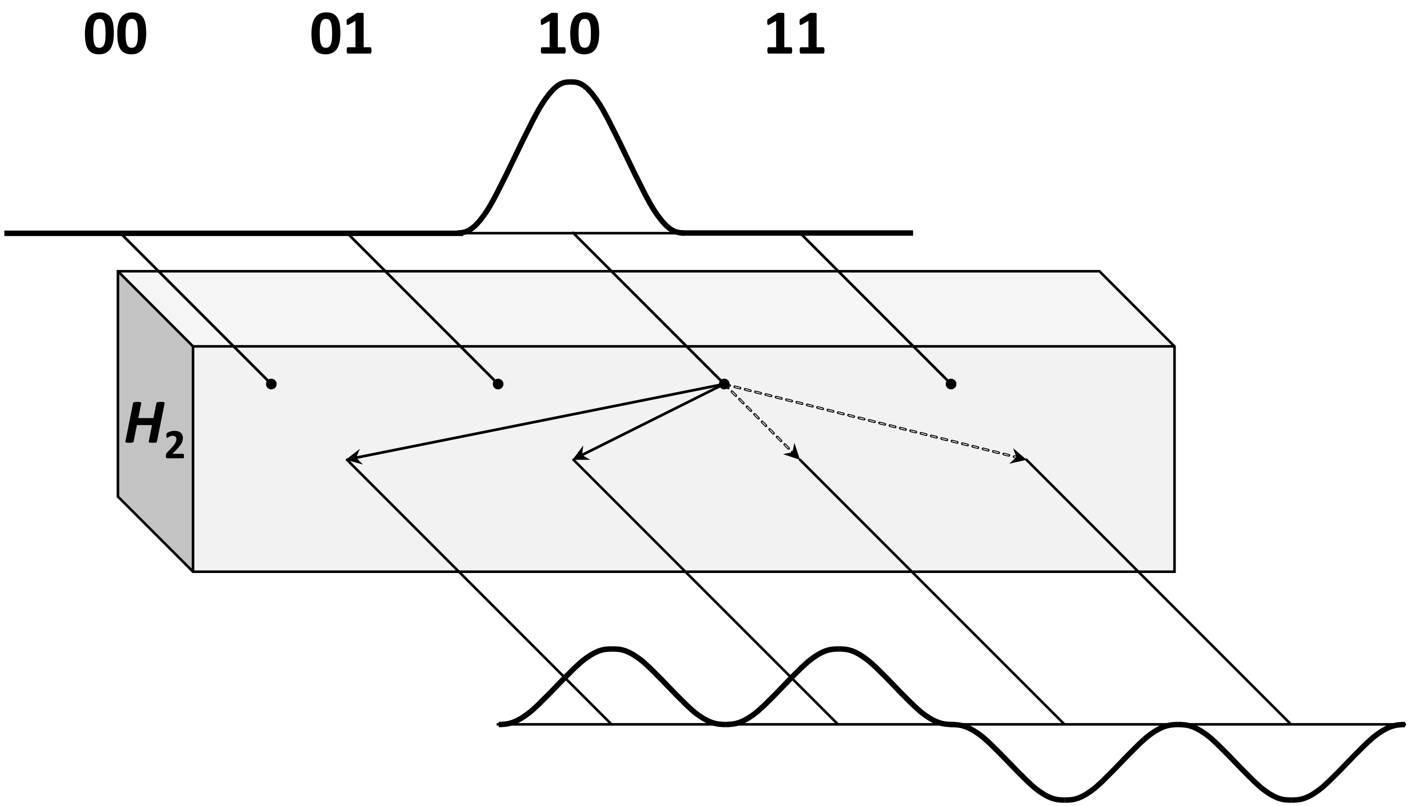
H_2 gate



$$H_2 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$$H_2 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

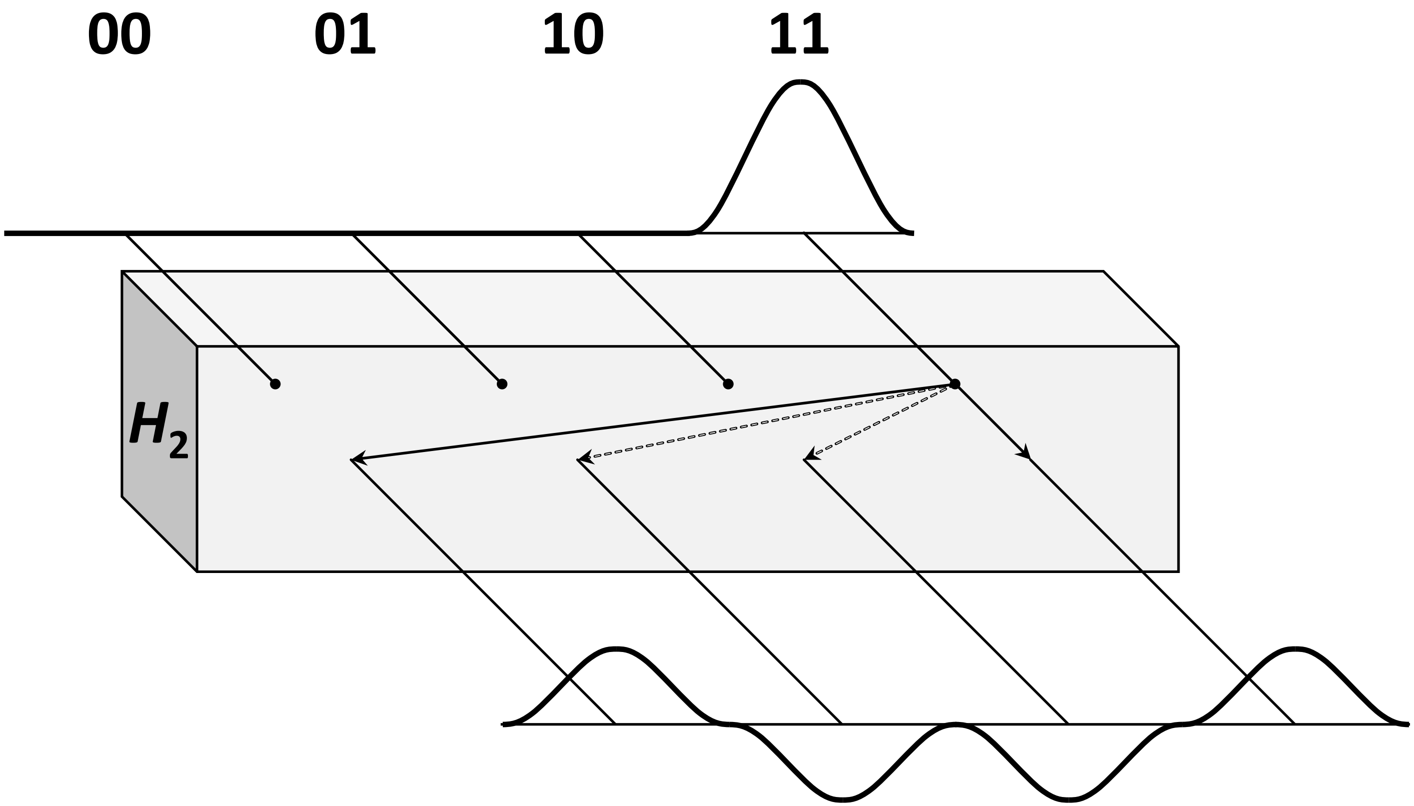
H_2 gate



$$H_2 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

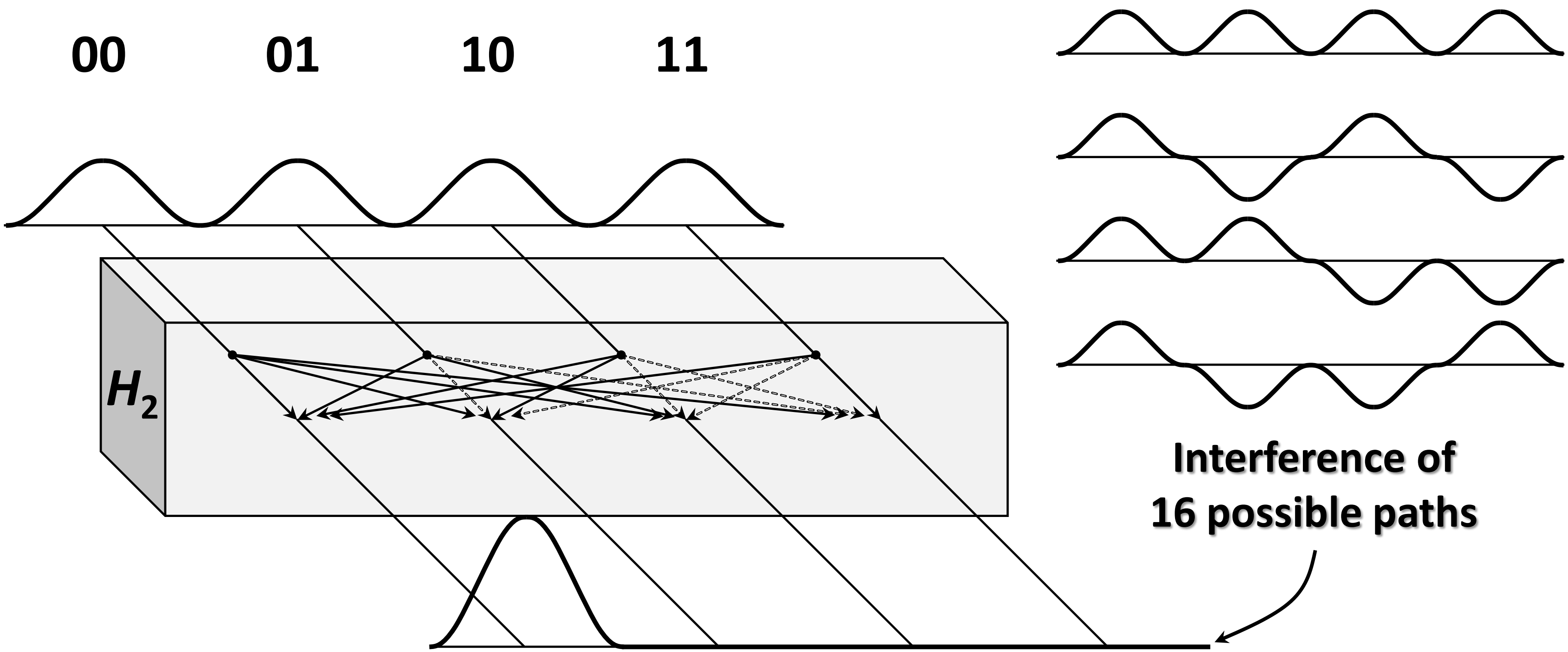
$$H_2 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$$

H_2 gate



$$H_2 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \qquad H_2\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

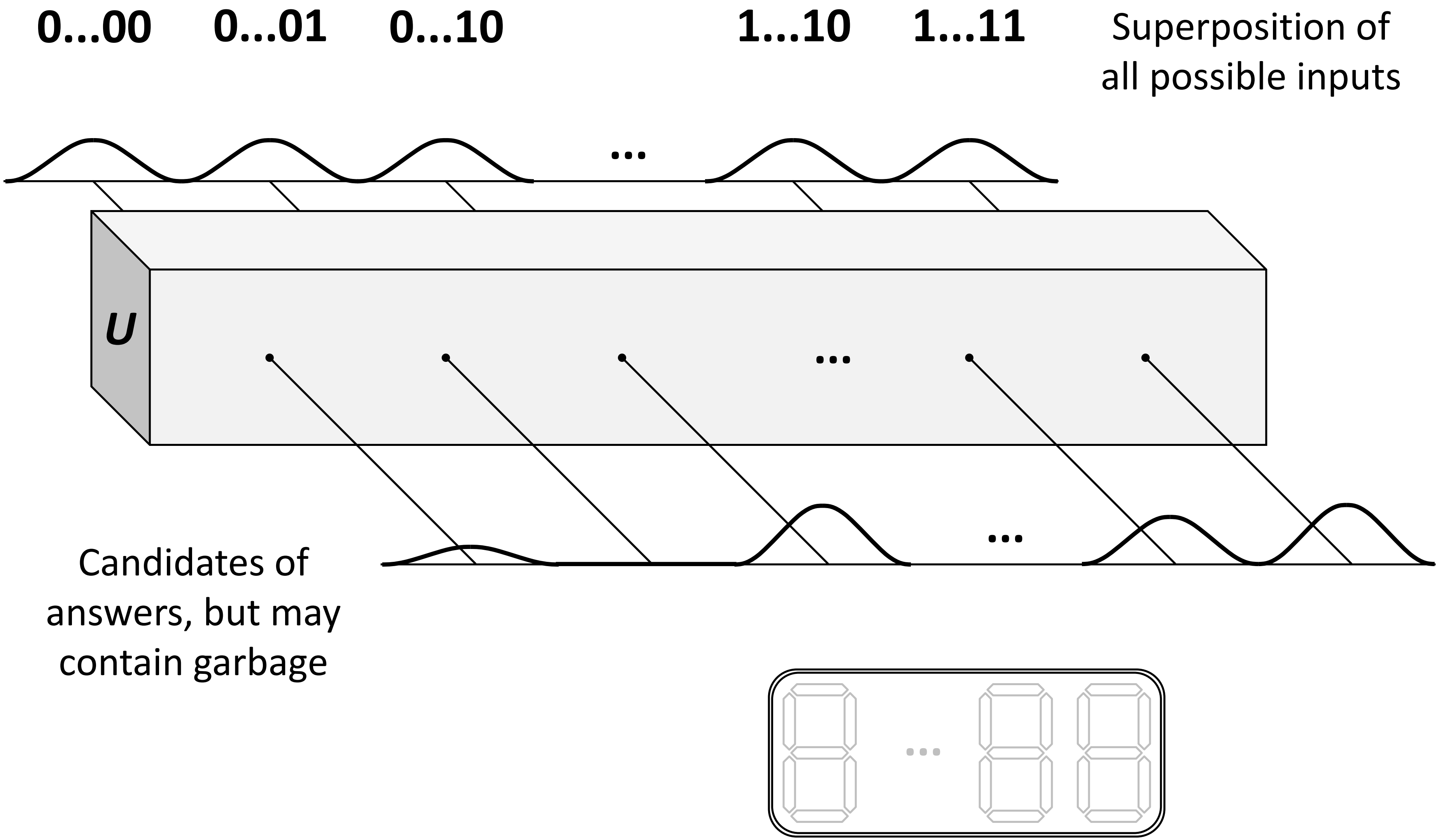
H_2 gate



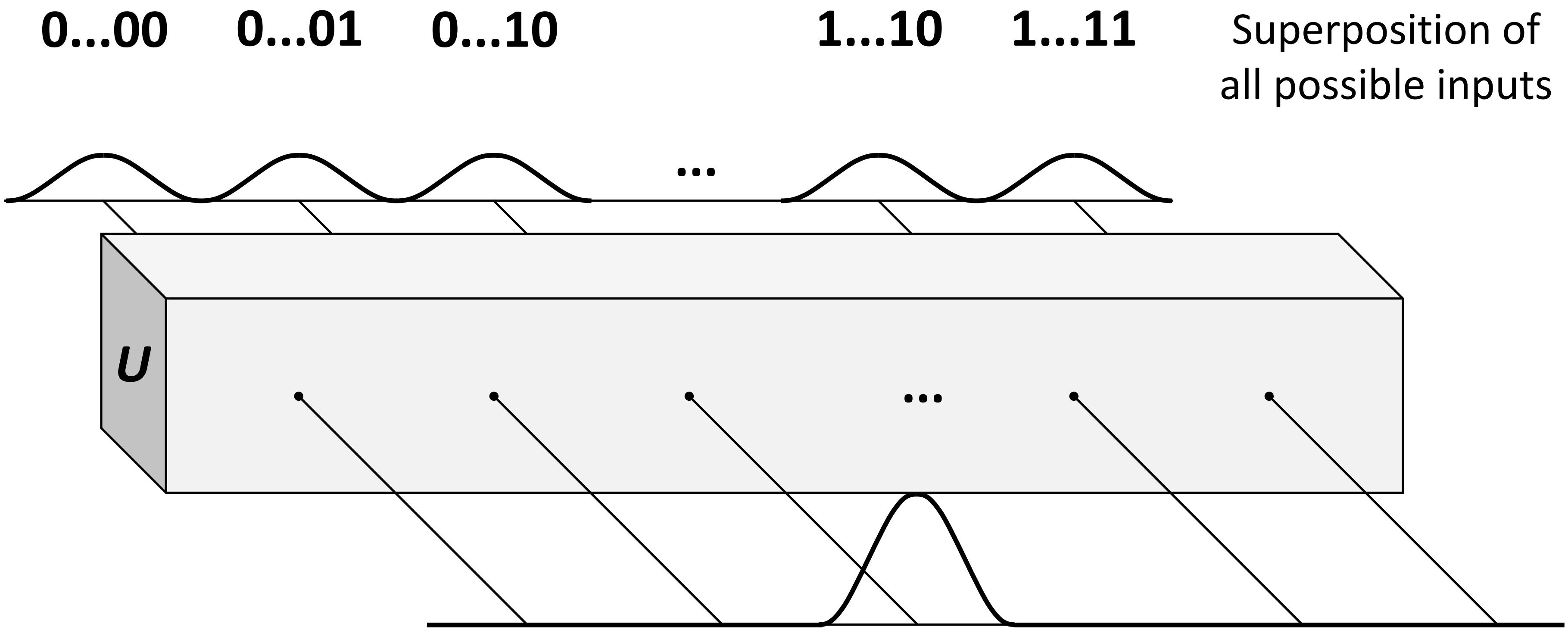
$$H_2 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$$H_2 \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

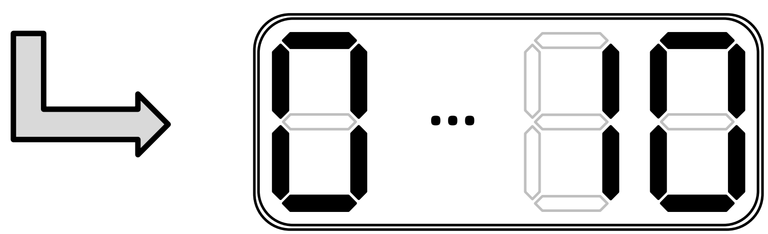
Quantum computation, conceptually



Quantum computation, conceptually

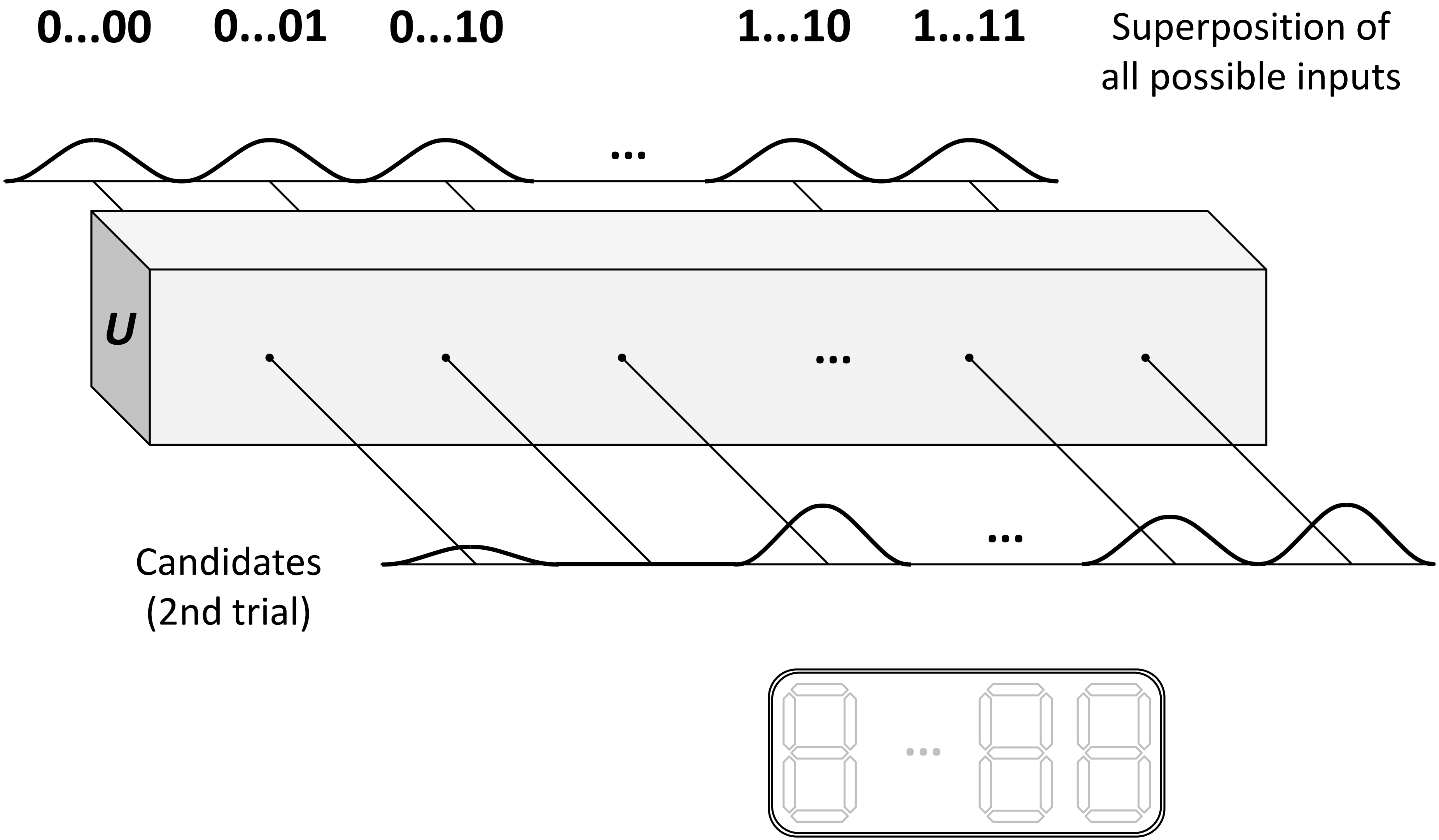


You may or may not get an answer. **Not so happy….**

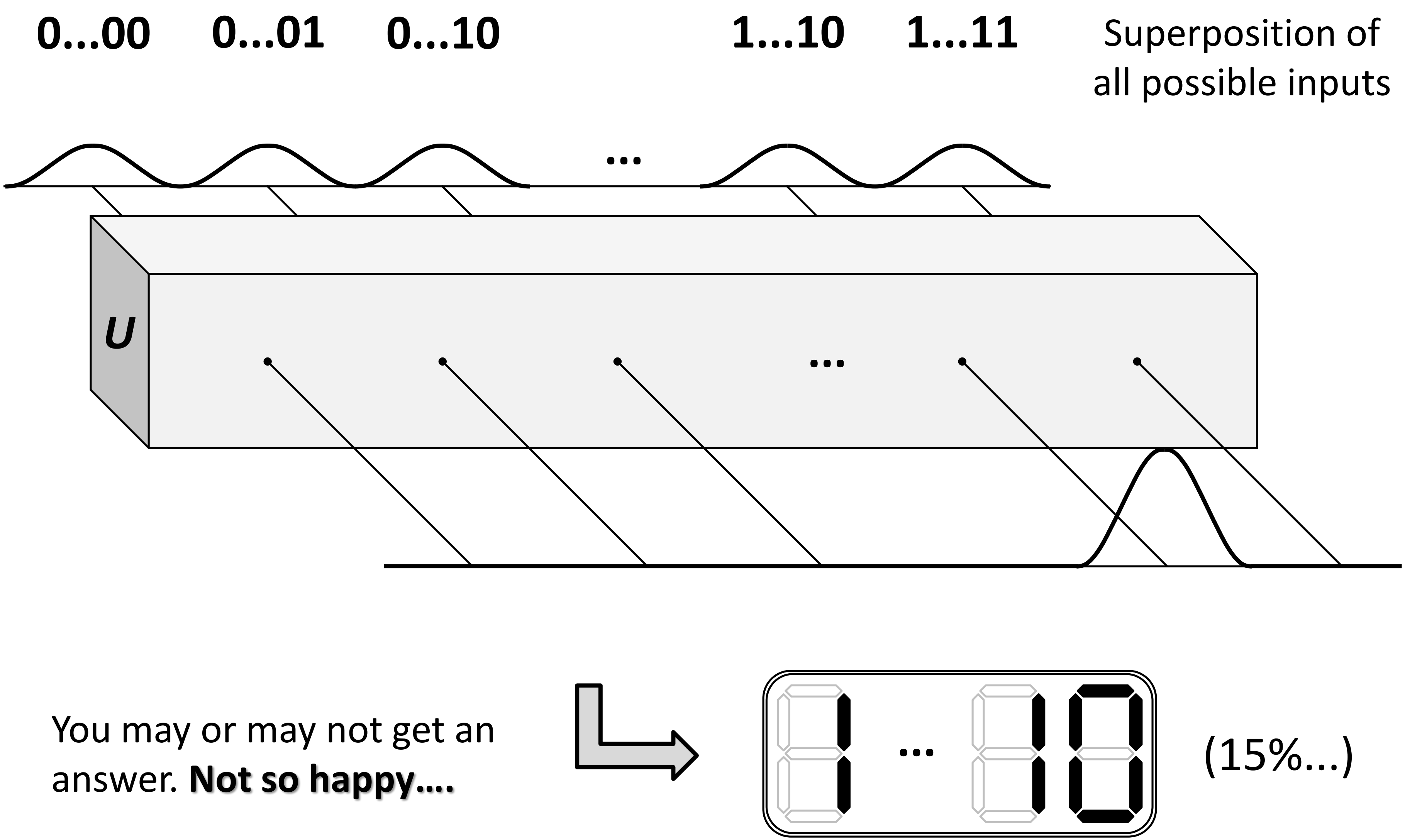


(20%...)

Quantum computation, conceptually

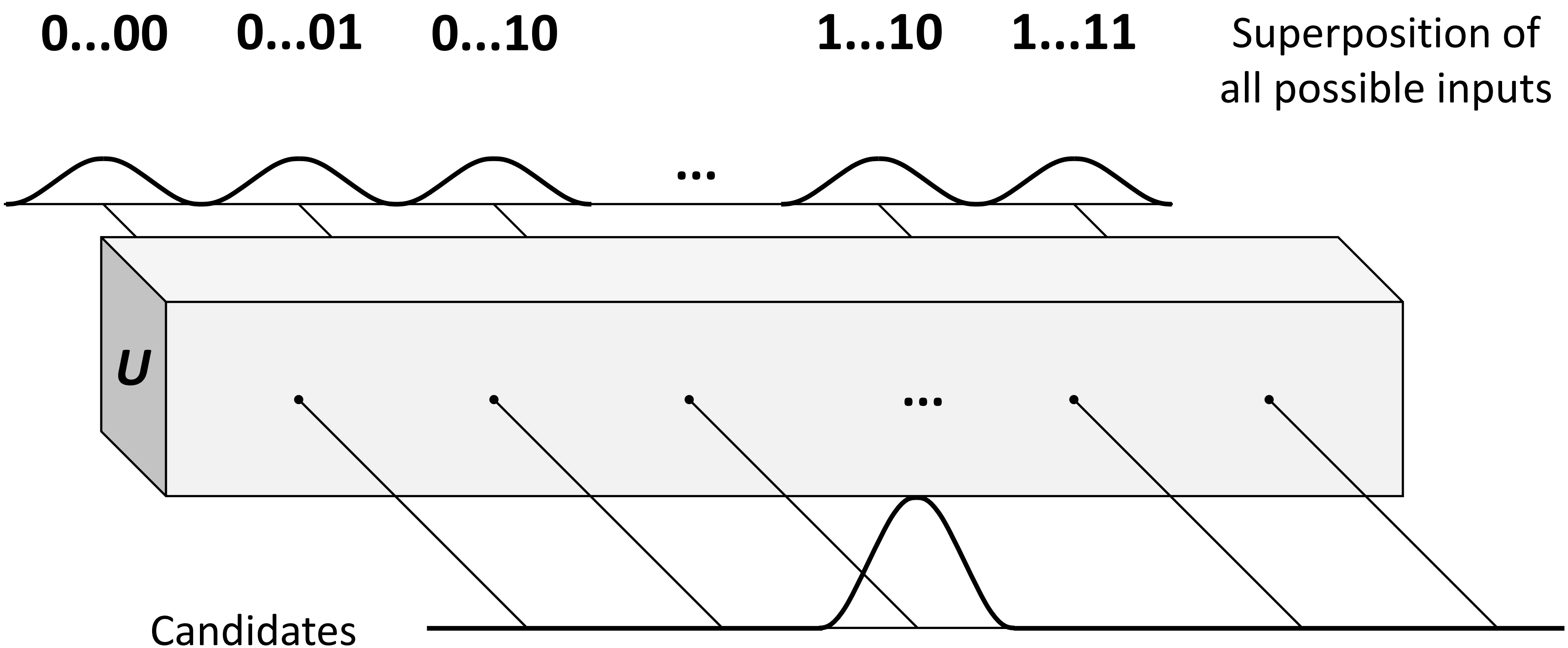


Quantum computation, conceptually



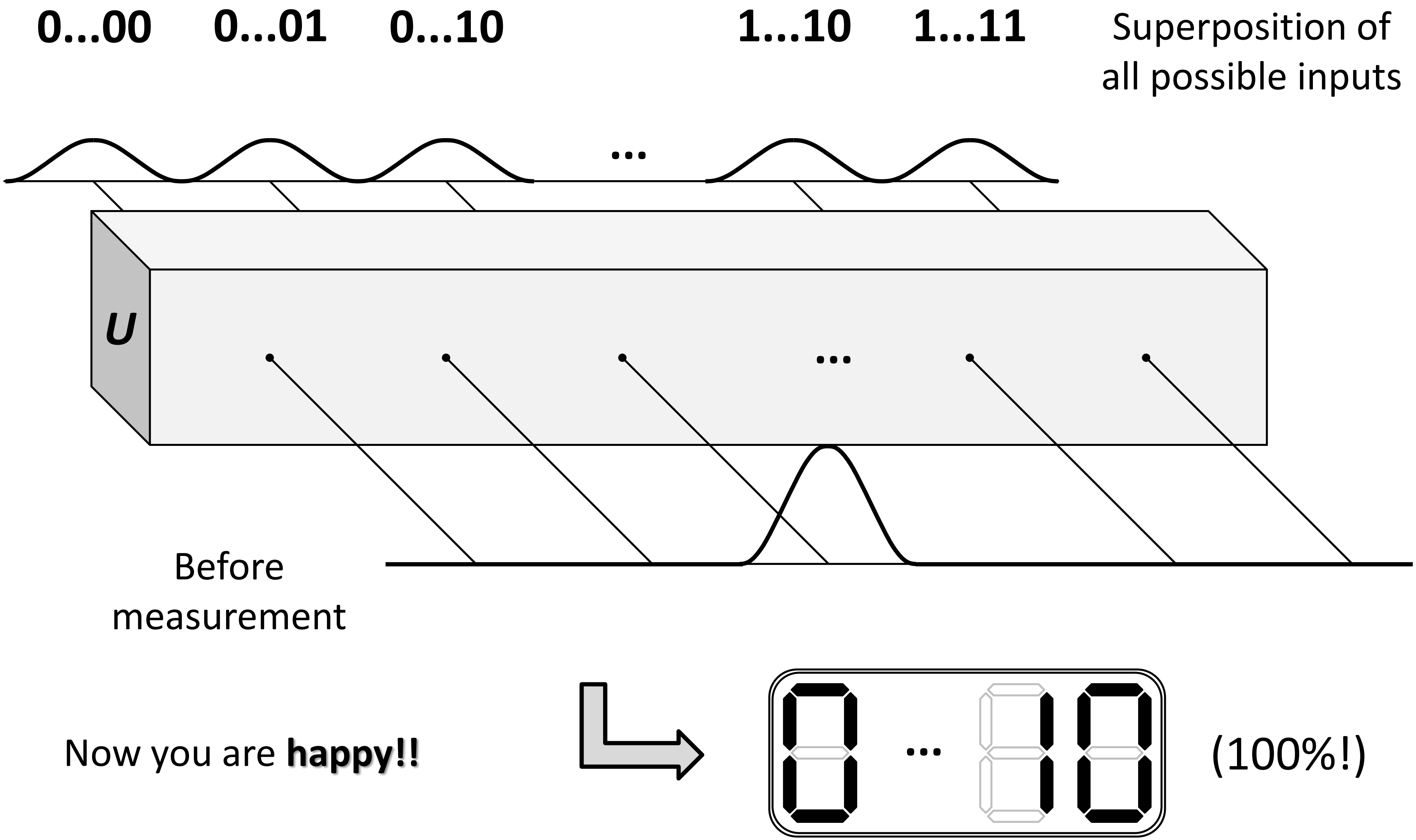
You may or may not get an answer. **Not so happy….**

Quantum computation, conceptually



After computation/before measurement,
we want **candidates** to be actually an **answer** (or at least very close to it)

Quantum computation, conceptually



Quantum computation

- Start from a superposition state (**quantum parallelism**), unitary-transform it into a state where the probability amplitude of the answer state is large enough (**quantum interference**), and **measure**
  → Deutsch–Jozsa algorithm (next topic)

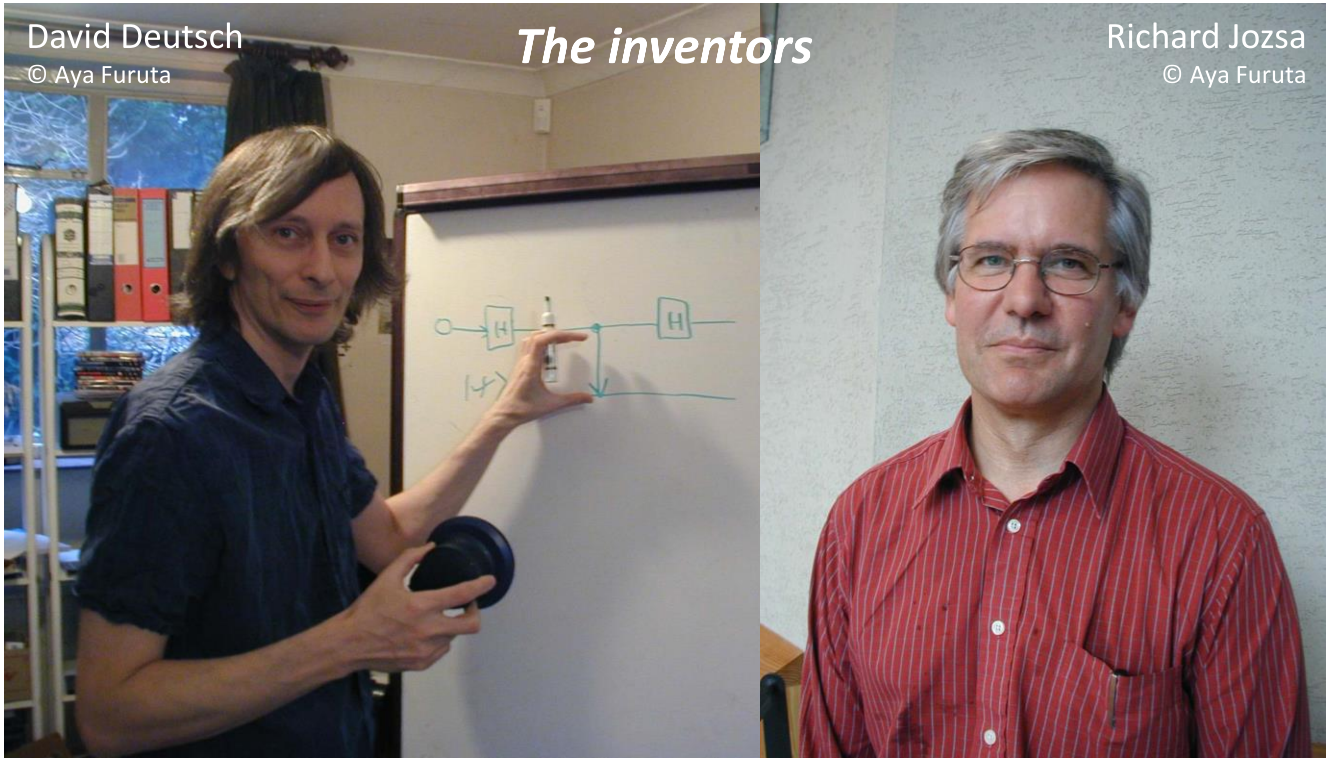
- For specific tasks, quantum computers can outperform classical computers, but **not almighty**
  → Scientific American **298**, (3) 62 (2008) Aaronson, "The limits of quantum computers"

- **Algorithms:** Data search (Grover), phase estimation (Kitaev), factoring (Shor), solving linear equations (Harrow–Hassidim–Lloyd), quantum simulation (Feynman) …
  → PRX Quantum **2**, 040203 (2021) Martyn *et al.*, "Grand Unification of Quantum Algorithms"

Contents

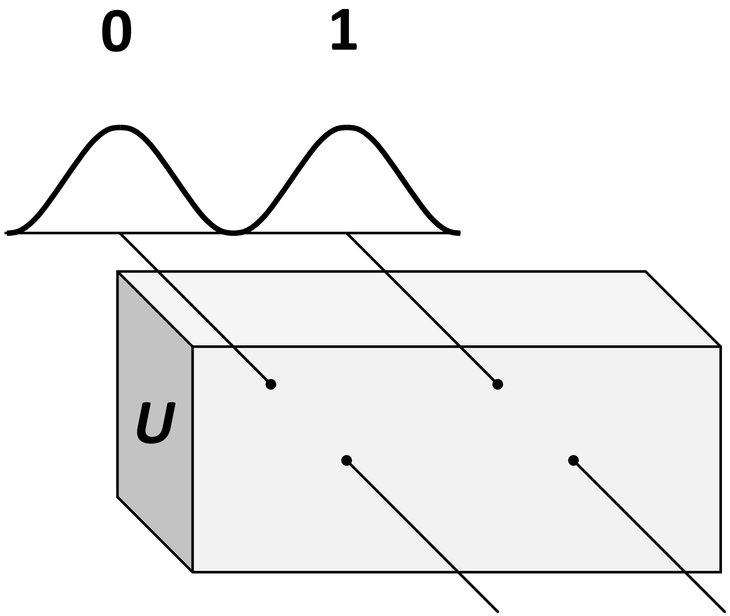
- **Quantum computation**
  - From an electron in a double-well potential to qubit
  - Quantum gates
  - Deutsch–Jozsa algorithm
- **Quantum error correction**
  - DiVincenzo’s criteria and the need of QEC
  - Spin, spin resonance, and spin relaxation
  - Basics of quantum error correction
- **Superconducting quantum circuits**
  - Circuit QED and transmon
  - Quantum control
  - Recent experiments by Google and ETH

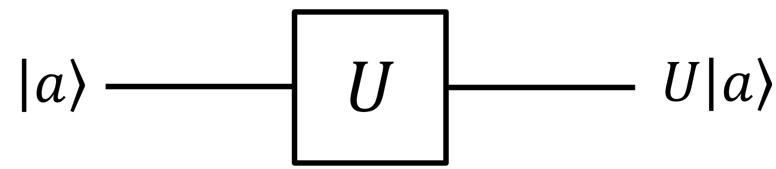
Deutsch–Jozsa algorithm



- The first quantum algorithm that showed the potential of quantum computers
- Deterministic (give a 100% answer)
- Of no practical use
- **Easy to see the roles of quantum parallelism and quantum interference**

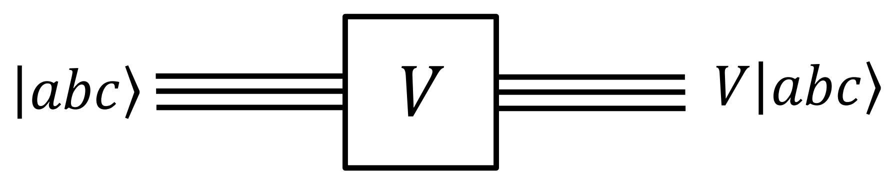
Quantum circuit





$a = 0{,}1$

Only *n* “wires” are required to represent *n*-qubit gates
(2^n wires in the left figure)

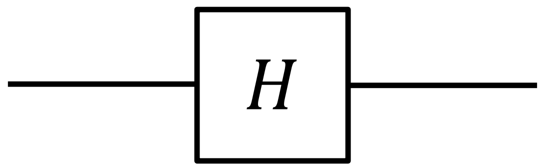


$a, b, c = 0{,}1$

Not to be confused with “superconducting quantum circuit,” which refers to a physical device based on circuit QED

H gate

$|a\rangle$

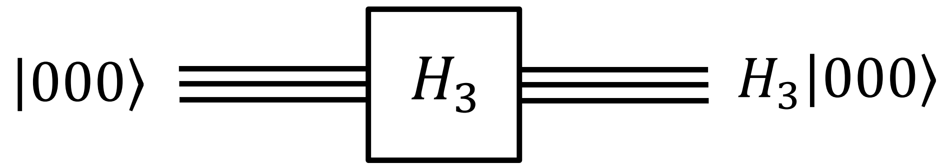


$$\frac{1}{\sqrt{2}}\sum_{b=0,1}(-1)^{a\cdot b}|b\rangle = \frac{|0\rangle + (-1)^{a}|1\rangle}{\sqrt{2}}$$

$a = 0{,}1$

$$\begin{cases} H|0\rangle = \dfrac{|0\rangle + |1\rangle}{\sqrt{2}} \\ H|1\rangle = \dfrac{|0\rangle - |1\rangle}{\sqrt{2}} \end{cases} \Longleftrightarrow \begin{cases} H\begin{pmatrix}1\\0\end{pmatrix} = \dfrac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \\ H\begin{pmatrix}0\\1\end{pmatrix} = \dfrac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \end{cases} \Longleftrightarrow H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$$

H_3 gate



$$H_3|000\rangle$$

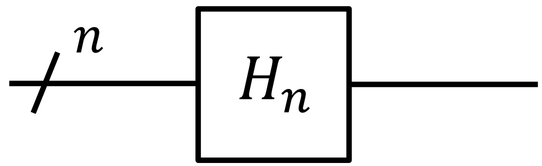
$$= \frac{1}{\sqrt{2^3}}(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$= \frac{1}{\sqrt{2^3}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle)$$

$$= \frac{1}{\sqrt{2^3}} \sum_{a,b,c=0,1} |abc\rangle = \frac{1}{\sqrt{2^3}} \sum_{x=0}^{2^3-1} |x\rangle$$

H_n gate

$|x\rangle = |a_1\rangle|a_2\rangle\cdots|a_n\rangle$



$\frac{1}{\sqrt{2^n}}\sum_y (-1)^{x\cdot y}|y\rangle$

$x\cdot y \equiv a_1\cdot b_1 + a_2\cdot b_2 + \cdots + a_n\cdot b_n$

$$H_n|x\rangle = \frac{1}{\sqrt{2^n}}\left(\sum_{b_1=0,1}(-1)^{a_1\cdot b_1}|b_1\rangle\right)\cdots\left(\sum_{b_n=0,1}(-1)^{a_n\cdot b_n}|b_n\rangle\right)$$

$$= \frac{1}{\sqrt{2^n}}\sum_{b_1,b_2\cdots b_n}(-1)^{a_1\cdot b_1+a_2\cdot b_2+\cdots+a_n\cdot b_n}|b_1 b_2\cdots b_n\rangle$$

$$= \frac{1}{\sqrt{2^n}}\sum_y (-1)^{x\cdot y}|y\rangle$$

Deutsch's problem

Definition: Binary function $f(x)$ is called "**constant**" if it returns the same output (all 0s or all 1s) for all the inputs x, and is called "**balanced**" if it returns half 0s and half 1s

Examples:

Constant

x	$f(x)$
0	0
1	0
2	0
3	0

Balanced

x	$f(x)$
0	0
1	1
2	1
3	0

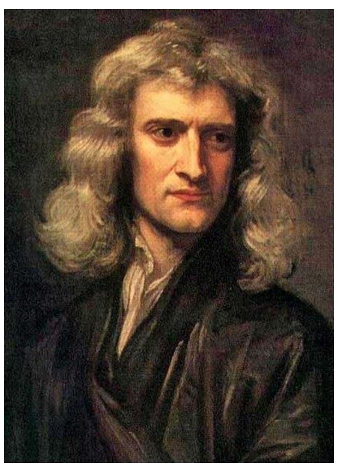
Non-of-the-above

x	$f(x)$
0	0
1	1
2	1
3	1

Deutsch's problem

Deutsch has a bit-string $f(x)$ that is known to be either **constant** or **balanced.** **How many queries** will **Newton** and **Schrödinger** have to make in order to judge the type (constant or balanced) of $f(x)$?

"Classical" query



Isaac Newton
By Godfrey Kneller

$x = 0$

$f(0) = 0$

$x = 1$

$f(1) = 0$

⋮

In the worst case (# elements/2+1) times

x	$f(x)$
0	0
1	0
2	0
3	0

$\sum_x |x\rangle|0\rangle$

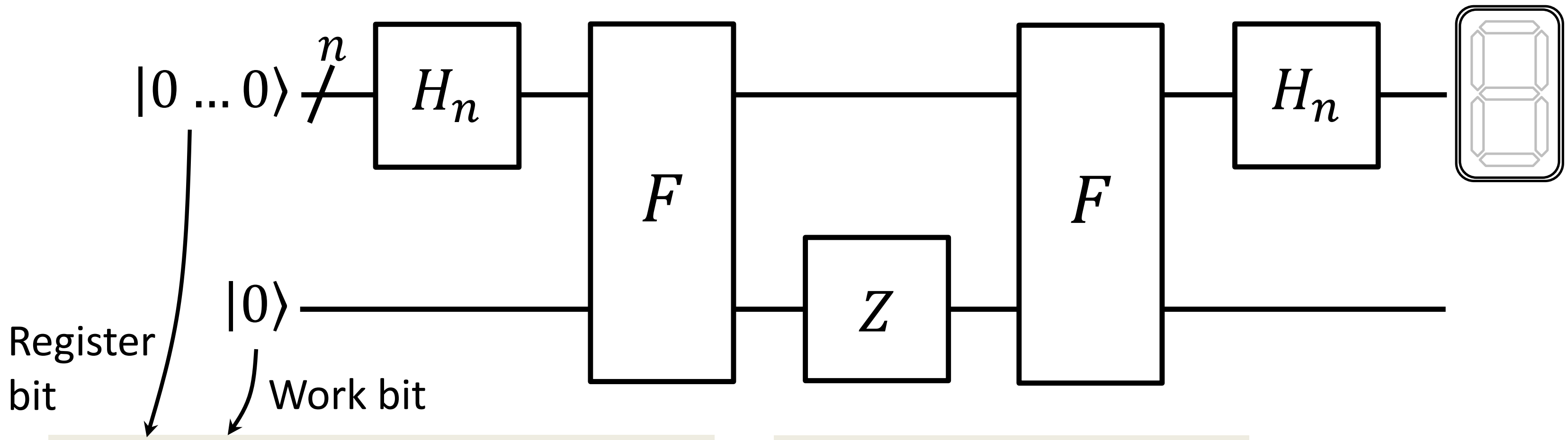
$|\Psi\rangle$

Always only once

"Quantum" query

Erwin Schrödinger
©Nobel Foundation

Deutsch–Jozsa algorithm



$F|x\rangle|a\rangle = |x\rangle|a \oplus f(x)\rangle$

$Z|a\rangle = (-1)^a|a\rangle$

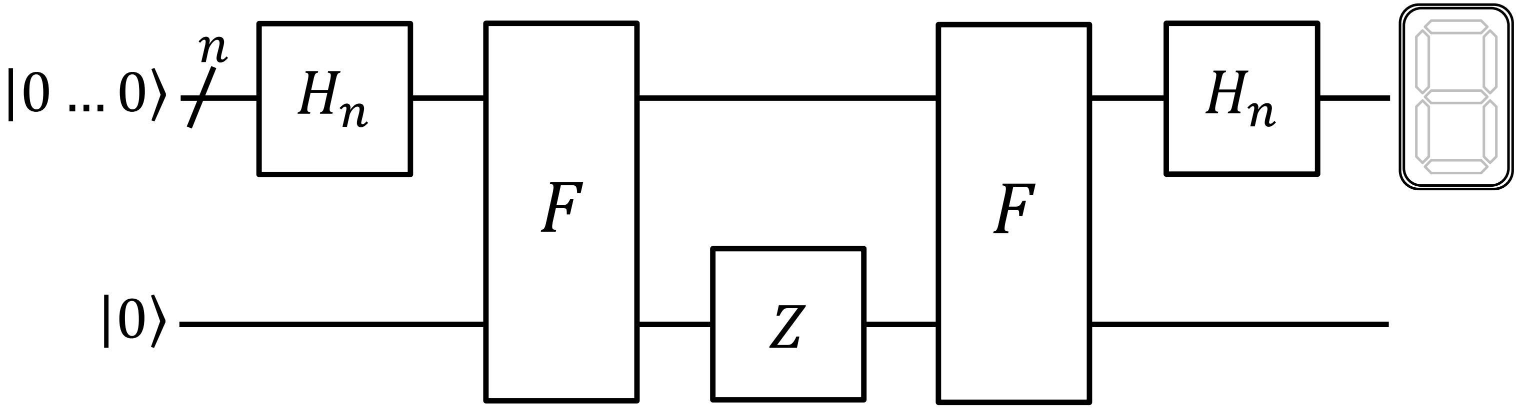
Entangled state carrying all the info on $f(x)$

$$|0 \ldots 0\rangle|0\rangle \xrightarrow{H_n} \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle|0\rangle \xrightarrow{F} \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle|f(x)\rangle$$

$$\xrightarrow{Z} \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} (-1)^{f(x)}|x\rangle|f(x)\rangle$$

Encode the info on $f(x)$ into the **phase**

Deutsch–Jozsa algorithm



$$F|x\rangle|a\rangle = |x\rangle|a \oplus f(x)\rangle$$

Erase the info on $f(x)$ from the work bit

$$\frac{1}{\sqrt{2^n}}\sum_{x=0}^{2^n-1}(-1)^{f(x)}|x\rangle|f(x)\rangle \xrightarrow{F} \frac{1}{\sqrt{2^n}}\sum_{x=0}^{2^n-1}(-1)^{f(x)}|x\rangle|0\rangle$$

$$\xrightarrow{H_n} \sum_y\left(\sum_x \frac{(-1)^{f(x)+x\cdot y}}{2^n}\right)|y\rangle|0\rangle$$

$$H_n|x\rangle = \frac{1}{\sqrt{2^n}}\sum_y(-1)^{x\cdot y}|y\rangle$$

Deutsch–Jozsa algorithm

Probablity amplitude of returning to |000...⟩

$$\sum_{x=0}^{2^n-1} \frac{(-1)^{f(x)+x\cdot 0}}{2^n} = \begin{cases} \pm 1 & \text{(Constant)} \\ 0 & \text{(Balanced)} \end{cases}$$

***n* = 2, constant** Constructive interference

$$\sum_{x=0}^{3} \frac{(-1)^{f(x)}}{2^n} = \frac{(-1)^0 + (-1)^0 + (-1)^0 + (-1)^0}{4} = 1$$

***n* = 2, balanced** Destructive interference

$$\sum_{x=0}^{3} \frac{(-1)^{f(x)}}{2^n} = \frac{(-1)^0 + (-1)^1 + (-1)^1 + (-1)^0}{4} = 0$$

2-bit $f(x)$

x	ab	Constant		Balanced ($_4C_2$ = 6)					
		f_{c0}	f_{c1}	f_{b0}	f_{b1}	f_{b2}	f_{b3}	f_{b4}	f_{b5}
0	00	0	1	0	0	0	1	1	1
1	01	0	1	0	1	1	1	0	0
2	10	0	1	1	0	1	0	1	0
3	11	0	1	1	1	0	0	0	1

$$f_{\mathrm{c0}}(x) = 0$$

$$f_{\mathrm{c1}}(x) = 1$$

$$f_{\mathrm{b0}}(x) = a$$

$$f_{\mathrm{b1}}(x) = b$$

$$f_{\mathrm{b2}}(x) = a \oplus b$$

$$f_{\mathrm{b3}}(x) = \bar{a}$$

$$f_{\mathrm{b4}}(x) = \bar{b}$$

$$f_{\mathrm{b5}}(x) = \overline{a \oplus b}$$

Homework 1

Constrict all the 2-bit F gates using only *X*s (NOTs) and CNOTs
Note: 2-bit F gates are 3Q gates

Contents

DiVincenzo's criteria

Fortschr. Phys. **48**, 771 (2000) DiVincenzo

1. A scalable physical system with well characterized qubits
2. The ability to initialize the state of the qubits to a simple fiducial state, such as $|000...\rangle$
3. Long relevant decoherence times, much longer than the gate operation time
4. A "universal" set of quantum gates
5. A qubit-specific measurement capability

David DiVincenzo

Universal

- 1Q gates + CNOT (can construct arbitrary *n*-qubit gates)
- *T, H*, *S* + CNOT (can approximate arbitrary *n*-qubit gates with arbitrary accuracy)

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \quad \underbrace{H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad S = T^2 = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad \mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{Clifford gates}}$$

Difficulty of quantum computation

experimental

- Encode quantum information into the **phase** and extract the answer by using **quantum interference**
  → **Quantum coherence** must be preserved during computation

Rolf Landauer
(1927–1999)

The physical nature of information

Rolf Landauer [1]
IBM T.J. Watson Research Center, P.O. Box 218, Yorktown Heights, NY 10598, USA

Received 9 May 1996
Communicated by V.M. Agranovich

Abstract

Information is inevitably tied to a physical representation and therefore to restrictions and possibilities related to the laws of physics and the parts available in the universe. Quantum mechanical superpositions of information bearing states can be used, and the real utility of that needs to be understood. Quantum parallelism in computation is one possibility and will be assessed pessimistically. The energy dissipation requirements of computation, of measurement and of the communications link are discussed. The insights gained from the analysis of computation has caused a reappraisal of the perceived wisdom in the other two fields. A concluding section speculates about the nature of the laws of physics, which are algorithms for the handling of information, and must be executable in our real physical universe.

Phys. Lett. A **217**, 188 (1996) Landauer

Difficulty of quantum computation

experimental

- Encode quantum information into the **phase** and extract the answer by using **quantum interference**
  → **Quantum coherence** must be preserved during computation

Rolf Landauer
(1927–1999)

Landauer's footnote
(…) all papers on quantum computing should carry a footnote: "This proposal, like all proposals for quantum computation, relies on speculative technology, does not in its current form take into account all possible sources of noise, unreliability and manufacturing error, and probably will not work."

Nature **400**, 720 (1999) Lloyd

Difficulty of quantum computation

experimental

- Encode quantum information into the **phase** and extract the answer by using **quantum interference**
  → **Quantum coherence** must be preserved during computation

- Quantum states cannot be copied (**no-cloning theorem**)
  → **Quantum error correction** & fault-tolerant quantum computation

Landauer's view on QEC

(...) progress has been made toward error reduction, and we can cite only a sample of the material on its way [21]. This is far more progress in fact than this author thought possible, but not enough to permit computation. (...) Undoubtedly, further progress will be made, but victory is not yet in sight.

[21] C.H. Bennett, D.P. DiVincenzo, J.A. Smolin and W.K. Wootters, Mixed state entanglement and quantum error correction, Phys. Rev. A., to be published;
R. Laflamme, C. Miquel, J.-P. Paz and W.H. Zurek, Perfect quantum error correction code, to be published;
P. Shor, Phys. Rev. A. 52 (1995) 2493.

Phys. Lett. A **217**, 188 (1996) Landauer

No-cloning theorem

There exists no unitary gate that realizes $U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$ for an arbitrary state $|\psi\rangle$

LETTERS TO NATURE

A single quantum cannot be cloned

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If a photon of definite polarization encounters an excited atom, there is typically some nonvanishing probability that the atom will emit a second photon by stimulated emission. Such a photon is guaranteed to have the same polarization as the original photon. But is it possible by this or any other process to amplify a quantum state, that is, to produce several copies of a quantum system (the polarized photon in the present case) each having the same state as the original? If it were, the amplifying process could be used to ascertain the exact state of a quantum system: in the case of a photon, one could determine its polarization by first producing a beam of identically polarized copies and then measuring the Stokes parameters[1]. We show here that the linearity of quantum mechanics forbids such replication and that this conclusion holds for all quantum systems.

Note that if photons could be cloned, a plausible argument could be made for the possibility of faster-than-light communication[2]. It is well known that for certain non-separably correlated Einstein–Podolsky–Rosen pairs of photons, once an observer has made a polarization measurement (say, vertical versus horizontal) on one member of the pair, the other one, which may be far away, can be for all purposes of prediction regarded as having the same polarization[3]. If this second photon could be replicated and its precise polarization measured as above, it would be possible to ascertain whether, for example, the first photon had been subjected to a measurement of linear or circular polarization. In this way the first observer would be able to transmit information faster than light by encoding his message into his choice of measurement. The actual impossibility of cloning photons, shown below, thus prohibits superluminal communication by this scheme. That such a scheme must fail for some reason despite the well-established existence of long-range quantum correlations[6–8], is a general consequence of quantum mechanics[9].

A perfect amplifying device would have the following effect on an incoming photon with polarization state $|s\rangle$:

$$|A_0\rangle|s\rangle \rightarrow |A_s\rangle|ss\rangle \qquad (1)$$

Here $|A_0\rangle$ is the 'ready' state of the apparatus, and $|A_s\rangle$ is its final state, which may or may not depend on the polarization of the original photon. The symbol $|ss\rangle$ refers to the state of the radiation field in which there are two photons each having the polarization $|s\rangle$. Let us suppose that such an amplification can in fact be accomplished for the vertical polarization $|\updownarrow\rangle$ and for the horizontal polarization $|\leftrightarrow\rangle$. That is,

$$|A_0\rangle|\updownarrow\rangle \rightarrow |A_{\rm vert}\rangle|\updownarrow\updownarrow\rangle \qquad (2)$$

and

$$|A_0\rangle|\leftrightarrow\rangle \rightarrow |A_{\rm hor}\rangle|\leftrightarrow\leftrightarrow\rangle \qquad (3)$$

According to quantum mechanics this transformation should be representable by a linear (in fact unitary) operator. It therefore follows that if the incoming photon has the polarization given by the linear combination $\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle$—for example, it could be linearly polarized in a direction 45° from the vertical, so that $\alpha=\beta=2^{-1/2}$—the result of its interaction with the apparatus will be the superposition of equations (2) and (3):

$$|A_0\rangle(\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle) \rightarrow \alpha|A_{\rm vert}\rangle|\updownarrow\updownarrow\rangle+\beta|A_{\rm hor}\rangle|\leftrightarrow\leftrightarrow\rangle \qquad (4)$$

If the apparatus states $|A_{\rm vert}\rangle$ and $|A_{\rm hor}\rangle$ are not identical, then the two photons emerging from the apparatus are in a mixed state of polarization. If these apparatus states are identical, then the two photons are in the pure state

$$\alpha|\updownarrow\updownarrow\rangle+\beta|\leftrightarrow\leftrightarrow\rangle \qquad (5)$$

In neither of these cases is the final state the same as the state with two photons both having the polarization $\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle$. That state, the one which would be required if the apparatus were to be a perfect amplifier, can be written as

$$2^{-1/2}(\alpha a^{+}_{\rm vert}+\beta a^{+}_{\rm hor})^2|0\rangle = \alpha^2|\updownarrow\updownarrow\rangle+2^{1/2}\alpha\beta|\updownarrow\leftrightarrow\rangle+\beta^2|\leftrightarrow\leftrightarrow\rangle$$

which is a pure state different from the one obtained above by superposition [equation (5)].

Thus no apparatus exists which will amplify an arbitrary polarization. The above argument does not rule out the possibility of a device which can amplify two special polarizations, such as vertical and horizontal. Indeed, any measuring device which distinguishes between these two polarizations, a Nicol prism for example, could be used to trigger such an amplification.

The same argument can be applied to any other kind of quantum system. As in the case of photons, linearity does not forbid the amplification of any given state by a device designed especially for that state, but it does rule out the existence of a device capable of amplifying an arbitrary state.

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Milonni (unpublished work) has shown that the process of stimulated emission does not lead to quantum amplification, because if there is stimulated emission there must also be—with equal probability in the case of one incoming photon—spontaneous emission, and the polarization of a spontaneously emitted photon is entirely independent of the polarization of the original.

It is conceivable that a more sophisticated amplifying apparatus could get around Milonni's argument. We have therefore presented the above simple argument, based on the linearity of quantum mechanics, to show that no apparatus, however complicated, can amplify an arbitrary polarization.

We stress that the question of replicating individual photons is of practical interest. It is obviously closely related to the quantum limits on the noise in amplifiers[10,11]. Moreover, an experiment devised to establish the extent to which polarization of single photons can be replicated through the process of stimulated emission is under way (A. Gozzini, personal communication; and see ref. 12). The quantum mechanical prediction is quite definite; for each perfect clone there is also one randomly polarized, spontaneously emitted, photon.

We thank Alain Aspect, Carl Caves, Ron Dickman, Ted Jacobson, Peter Milonni, Marlan Scully, Pierre Meystre, Don Page and John Archibald Wheeler for enjoyable and stimulating discussions.

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Nature **299**, 802 (1982) Wootters & Zurek

No-cloning theorem

There exists no unitary gate that realizes $U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$ for an arbitrary state $|\psi\rangle$

Proof: If such U exits…

$$U|0\rangle|0\rangle = |0\rangle|0\rangle$$

$$U|1\rangle|0\rangle = |1\rangle|1\rangle$$

$$\Downarrow$$

$$U(\underbrace{a|0\rangle + b|1\rangle}_{|\psi\rangle})|0\rangle = aU|0\rangle|0\rangle + bU|1\rangle|0\rangle$$

$$= a|0\rangle|0\rangle + b|1\rangle|1\rangle$$

$$\neq (a|0\rangle + b|1\rangle)(a|0\rangle + b|1\rangle)$$

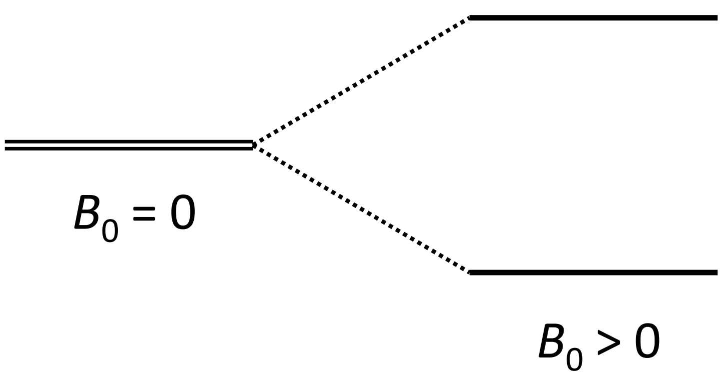
Contents

Electron spin

Intrinsic, quantum mechanical angular momentum of an electron S = ½ ($m_s = \pm$ ½)

Hamiltonian & energy levels

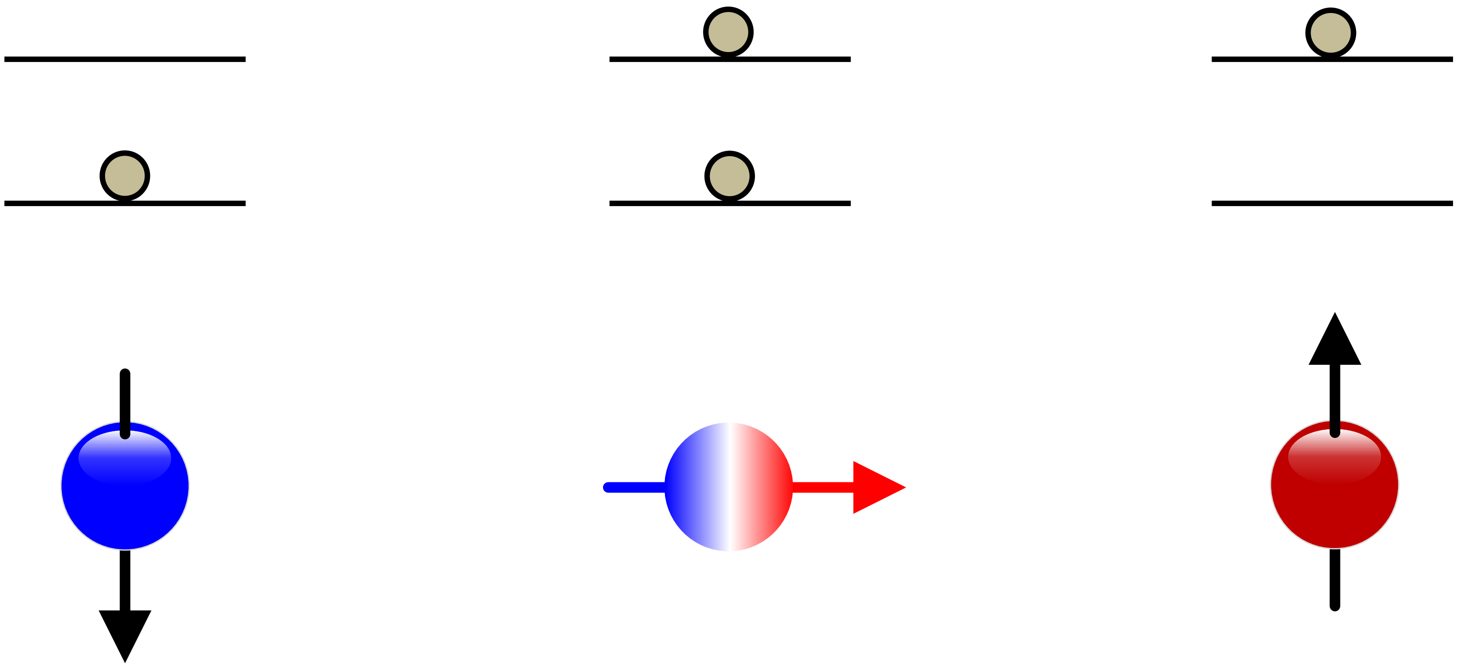
$$H_Z = \frac{g\mu_B B_0}{\hbar} S_z$$



$$H_z|\uparrow\rangle = \frac{g\mu_B B_0}{2}|\uparrow\rangle \qquad S_z|\uparrow\rangle = \frac{\hbar}{2}|\uparrow\rangle$$

$$H_z|\downarrow\rangle = -\frac{g\mu_B B_0}{2}|\downarrow\rangle \qquad S_z|\downarrow\rangle = -\frac{\hbar}{2}|\downarrow\rangle$$

Quantum coherence



$|0\rangle \equiv |\downarrow\rangle$ $\quad$ $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ $\quad$ $|1\rangle \equiv |\uparrow\rangle$

In many cases, the spin dynamics can be described phenomenologically (Bloch equation)

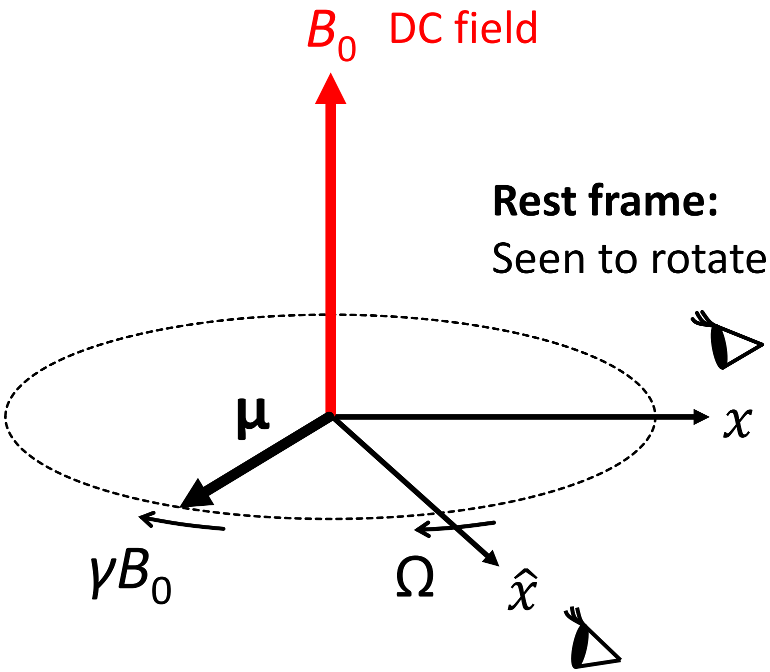
Larmor precession

Torque equation

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\mu} \times \gamma \boldsymbol{B}_0$$

Gyromagnetic ratio ($g\mu_B$)

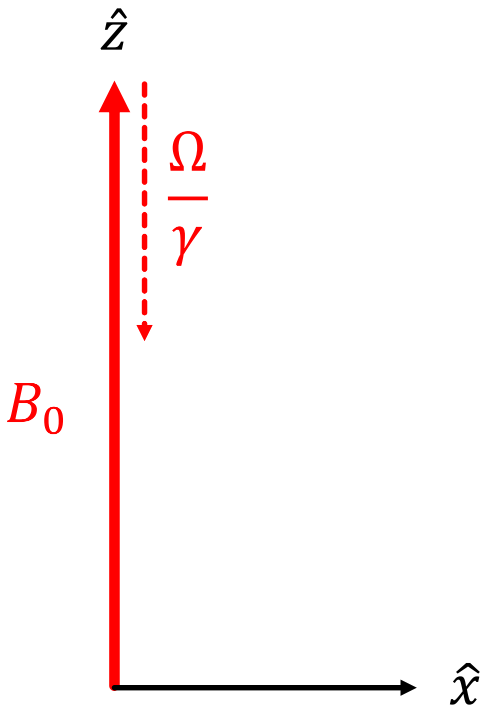
Magnetic moment: $\boldsymbol{\mu} = \gamma \boldsymbol{J}$

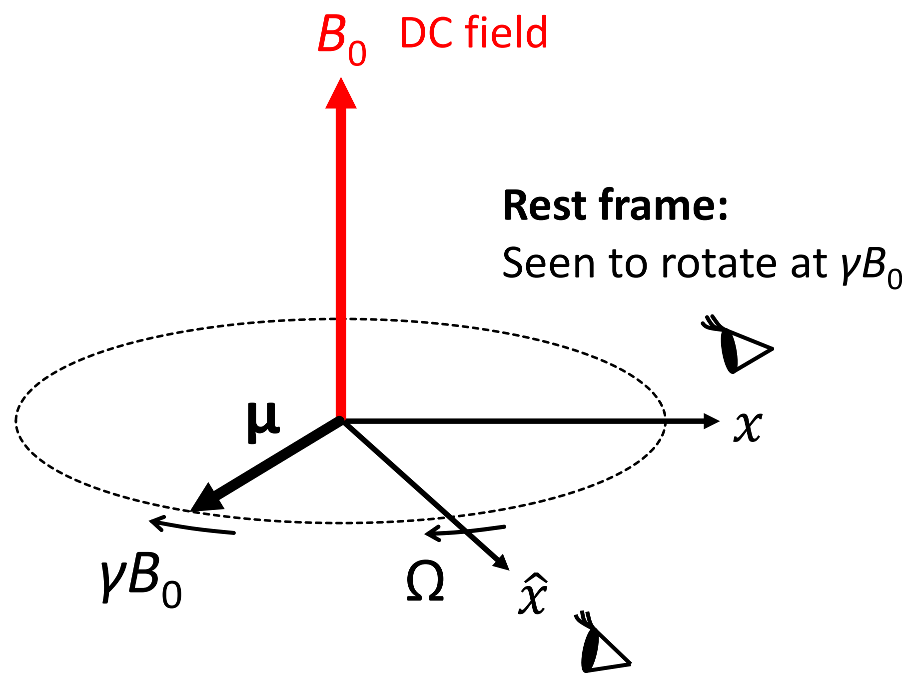


Rest frame:
Seen to rotate at γB_0

Frame rotating at angular velocity Ω:
Rotate slower…why?

Larmor precession





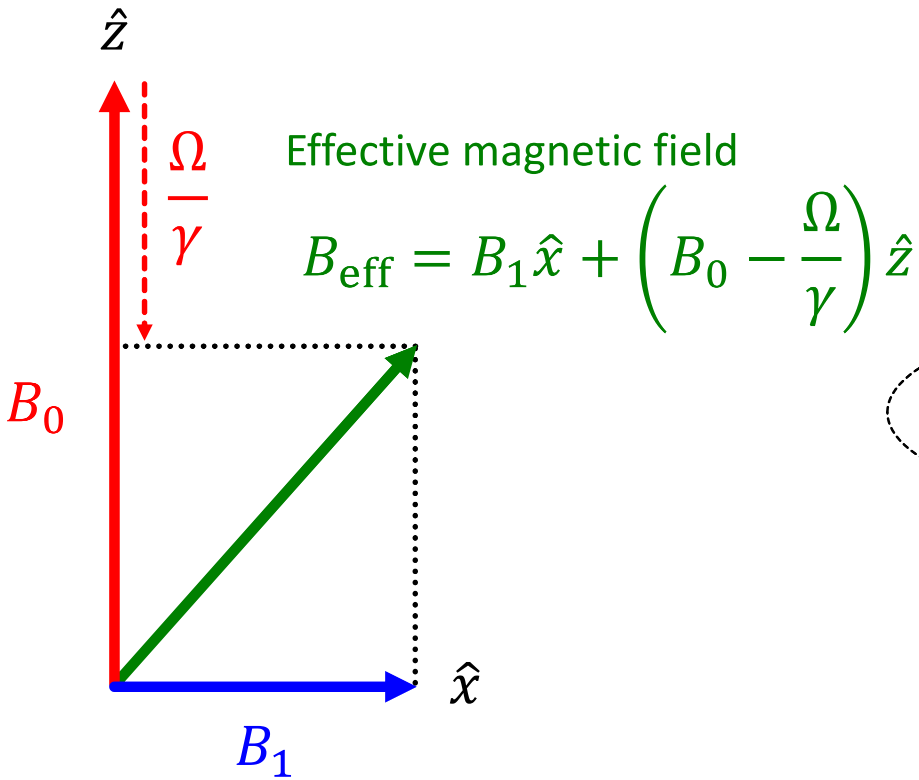
Frame rotating at angular velocity Ω:

Rotate slower…why?

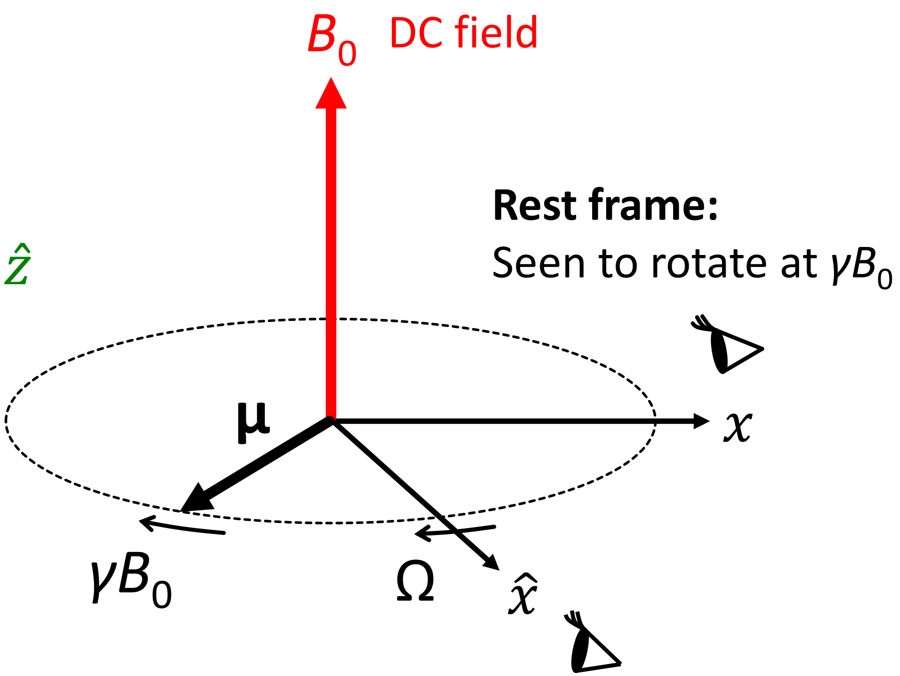
⇩

DC field along the *z* direction becomes weaker

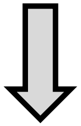
Spin resonance



AC field rotating in the *xy* plane at Ω

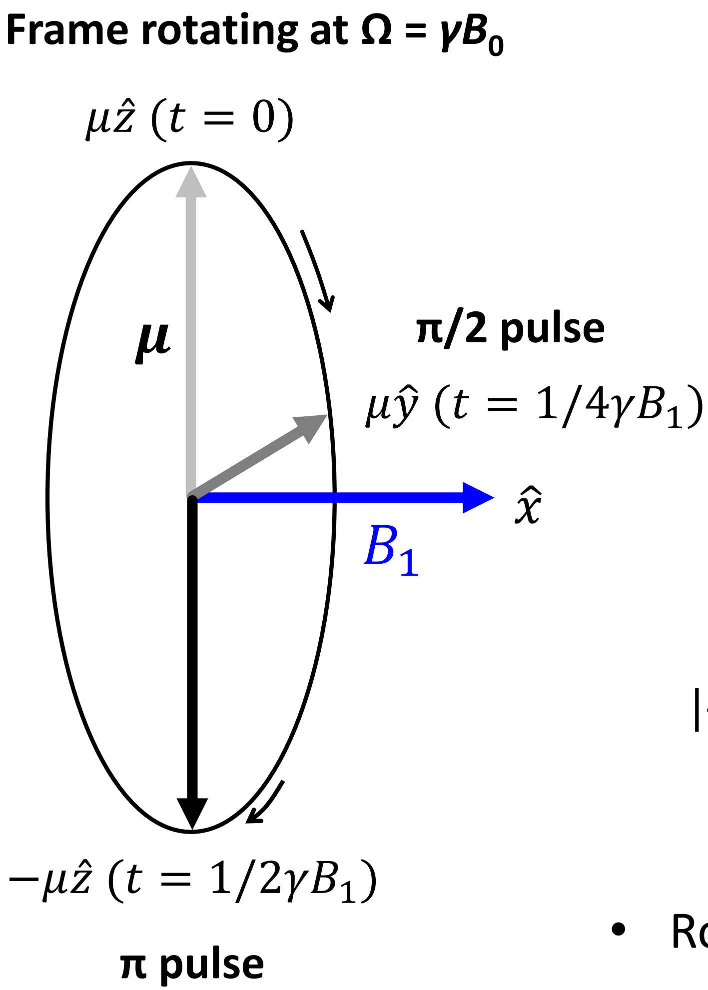


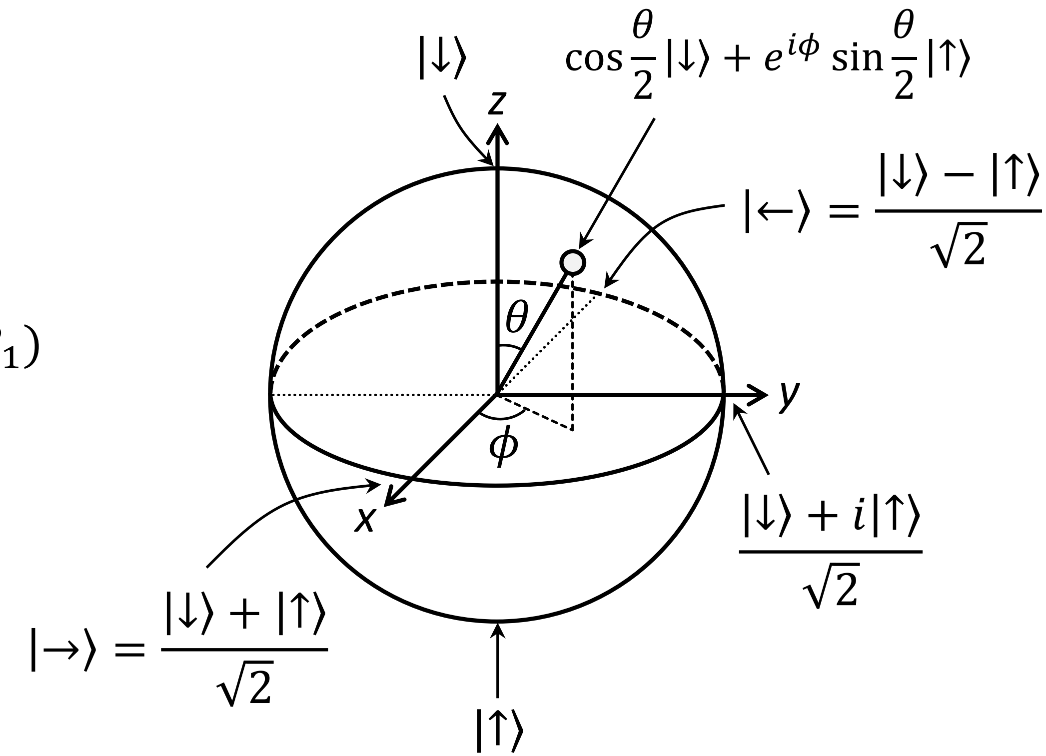
Frame rotating at angular velocity Ω:
Rotate slower…why?



DC field along the *z* direction becomes weaker

Spin resonance



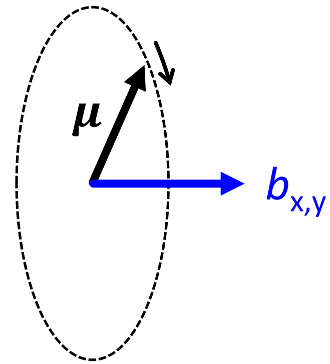


- Rotation on the Bloch sphere
- Rotations about the $\pm\hat{x}, \pm\hat{y}$ axes are realized by adjusting the microwave phases

Spin relaxation: T_1 & T_2

Bloch equation

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\mu} \times \gamma \boldsymbol{B}_0 - \frac{\boldsymbol{\mu}_{\parallel} - \boldsymbol{\mu}_0}{T_1} - \frac{\boldsymbol{\mu}_{\perp}}{T_2}$$



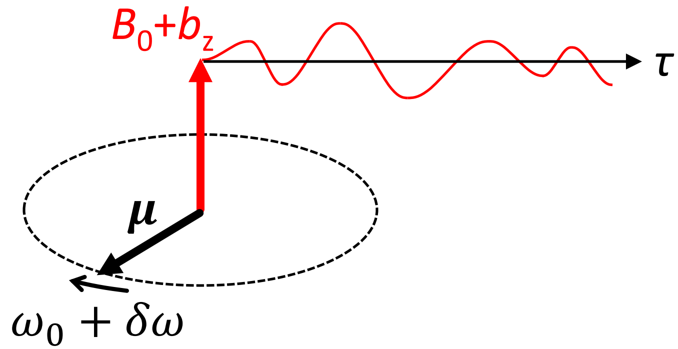
Felix Bloch
(1905–1983)

Energy relaxation (Change the direction of a spin)

$$\frac{1}{T_1} = \frac{\gamma^2}{2} \int_{-\infty}^{\infty} \left[\langle b_x(\tau) b_x(0)\rangle + \langle b_y(\tau) b_y(0)\rangle\right] \cos(\omega_0 \tau)\, d\tau$$

Phase relaxation (Change the precession frequency)

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{\gamma^2}{2} \int_{-\infty}^{\infty} \langle b_z(\tau) b_z(0)\rangle d\tau$$

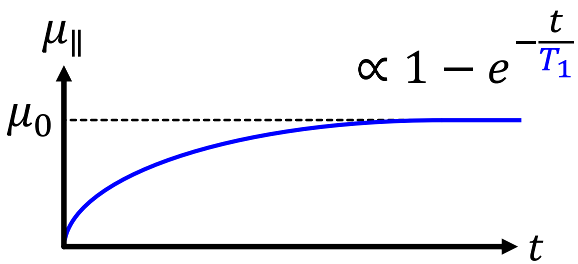


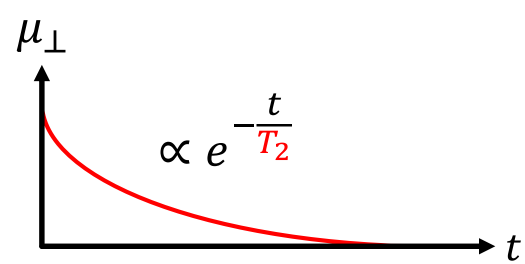
→ Incoherent process (Error!)

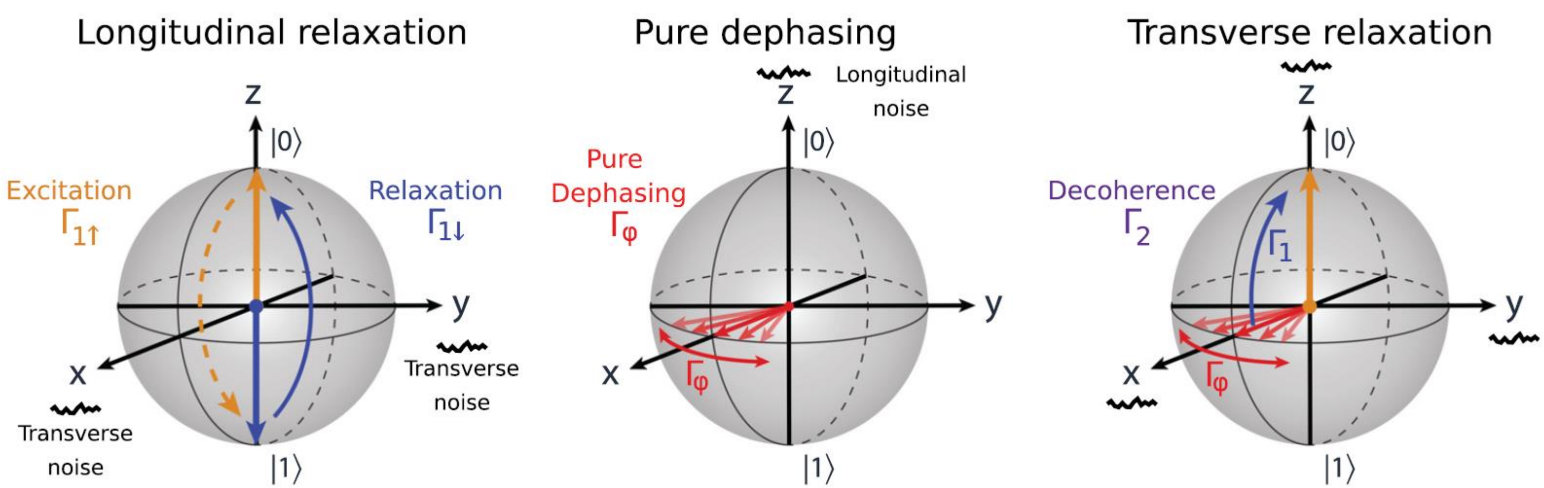
Spin relaxation: T_1 & T_2

Bloch equation

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\mu} \times \gamma \boldsymbol{B}_0 - \frac{\boldsymbol{\mu}_\parallel - \boldsymbol{\mu}_0}{T_1} - \frac{\boldsymbol{\mu}_\perp}{T_2}$$







Appl. Phys. Rev. **6**, 021318 (2019) Krantz *et al.*

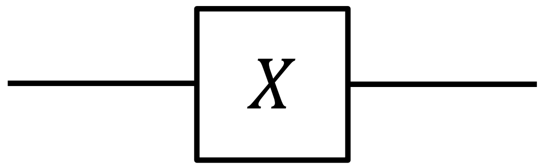
Contents

Errors in quantum circuits

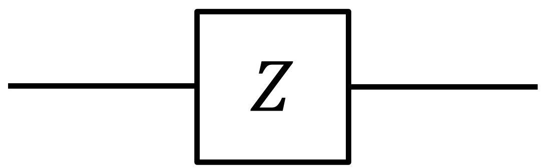
Coupling with the environment

$$|\psi\rangle|e_0\rangle \xrightarrow{U_{\mathrm{E}}} |\psi\rangle|e_I\rangle + X|\psi\rangle|e_X\rangle + Z|\psi\rangle|e_Z\rangle + XZ|\psi\rangle|e_Y\rangle$$

Bit-flip error



Phase-flip error



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$XZ = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -iY$$

Basic ideas of quantum error correction

- Continuous errors can be discretized by measurements
- Any 1Q errors are correctable as long as we can detect & correct bit-flip (*X*), phase-flip (*Z*), phase–bit-flip (*XZ*) errors

Check

$$U_{\mathrm{E}}|\psi\rangle|e_0\rangle = |\psi\rangle|e_I\rangle + X|\psi\rangle|e_X\rangle + Z|\psi\rangle|e_Z\rangle + XZ|\psi\rangle|e_Y\rangle$$

$$\begin{cases} U_{\mathrm{E}}|0\rangle|e_0\rangle = |0\rangle|e_{00}\rangle + |1\rangle|e_{10}\rangle \\ U_{\mathrm{E}}|1\rangle|e_0\rangle = |0\rangle|e_{01}\rangle + |1\rangle|e_{11}\rangle \end{cases}$$

L.H.S $U_{\mathrm{E}}(\alpha|0\rangle + \beta|1\rangle)|e_0\rangle = \alpha|0\rangle|e_{00}\rangle + \alpha|1\rangle|e_{10}\rangle + \beta|0\rangle|e_{01}\rangle + \beta|1\rangle|e_{11}\rangle$

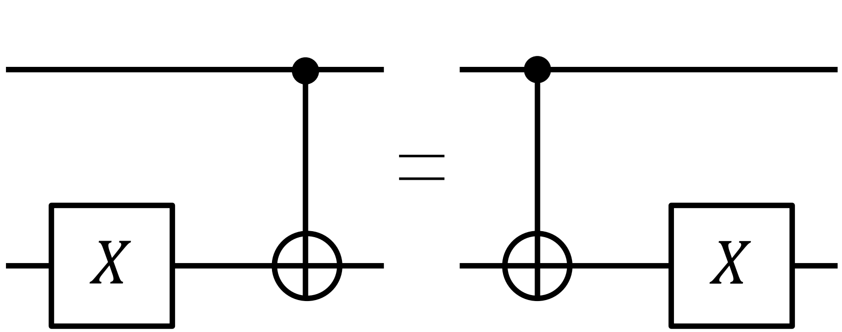
R.H.S.

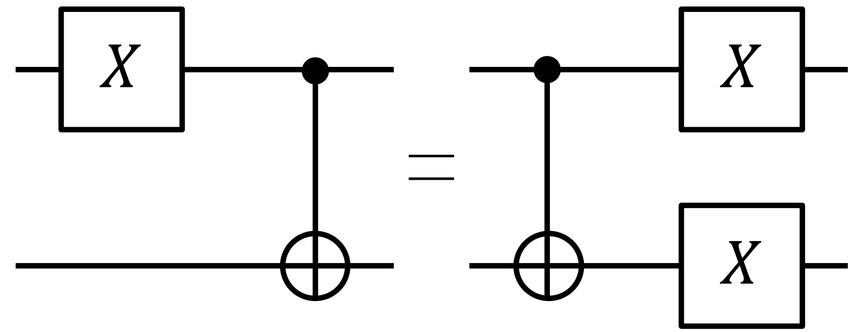
$$(\alpha|0\rangle + \beta|1\rangle)|e_I\rangle + (\alpha|1\rangle + \beta|0\rangle)|e_X\rangle + (\alpha|0\rangle - \beta|1\rangle)|e_Z\rangle + (\alpha|1\rangle - \beta|0\rangle)|e_Y\rangle$$

$$= \alpha|0\rangle(|e_I\rangle + |e_Z\rangle) + \alpha|1\rangle(|e_X\rangle + |e_Y\rangle) + \beta|0\rangle(|e_X\rangle - |e_Y\rangle) + \beta|1\rangle(|e_I\rangle - |e_Z\rangle)$$

$$\longrightarrow \quad |e_{I,Z}\rangle = \frac{|e_{00}\rangle \pm |e_{11}\rangle}{2} \qquad |e_{X,Y}\rangle = \frac{|e_{10}\rangle \pm |e_{01}\rangle}{2}$$

CNOT & X gate





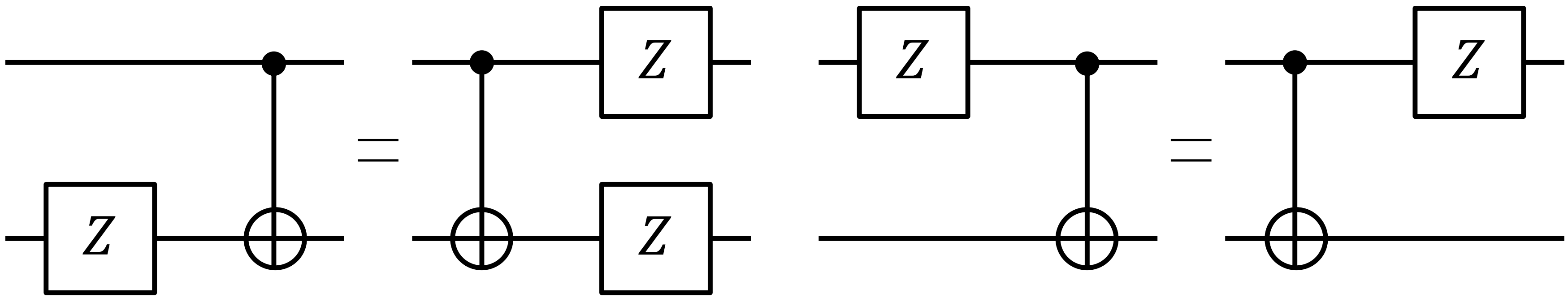
$$C_{12}X_2|a\rangle|b\rangle = C_{12}|a\rangle|b \oplus 1\rangle$$
$$= |a\rangle|a \oplus b \oplus 1\rangle$$

$$C_{12}X_1|a\rangle|b\rangle = C_{12}|a \oplus 1\rangle|b\rangle$$
$$= |a \oplus 1\rangle|a \oplus b \oplus 1\rangle$$

$$X_2C_{12}|a\rangle|b\rangle = X_2|a\rangle|a \oplus b\rangle$$
$$= |a\rangle|a \oplus b \oplus 1\rangle$$

$$X_1X_2C_{12}|a\rangle|b\rangle = X_1X_2|a\rangle|a \oplus b\rangle$$
$$= |a \oplus 1\rangle|a \oplus b \oplus 1\rangle$$

CNOT & Z gate



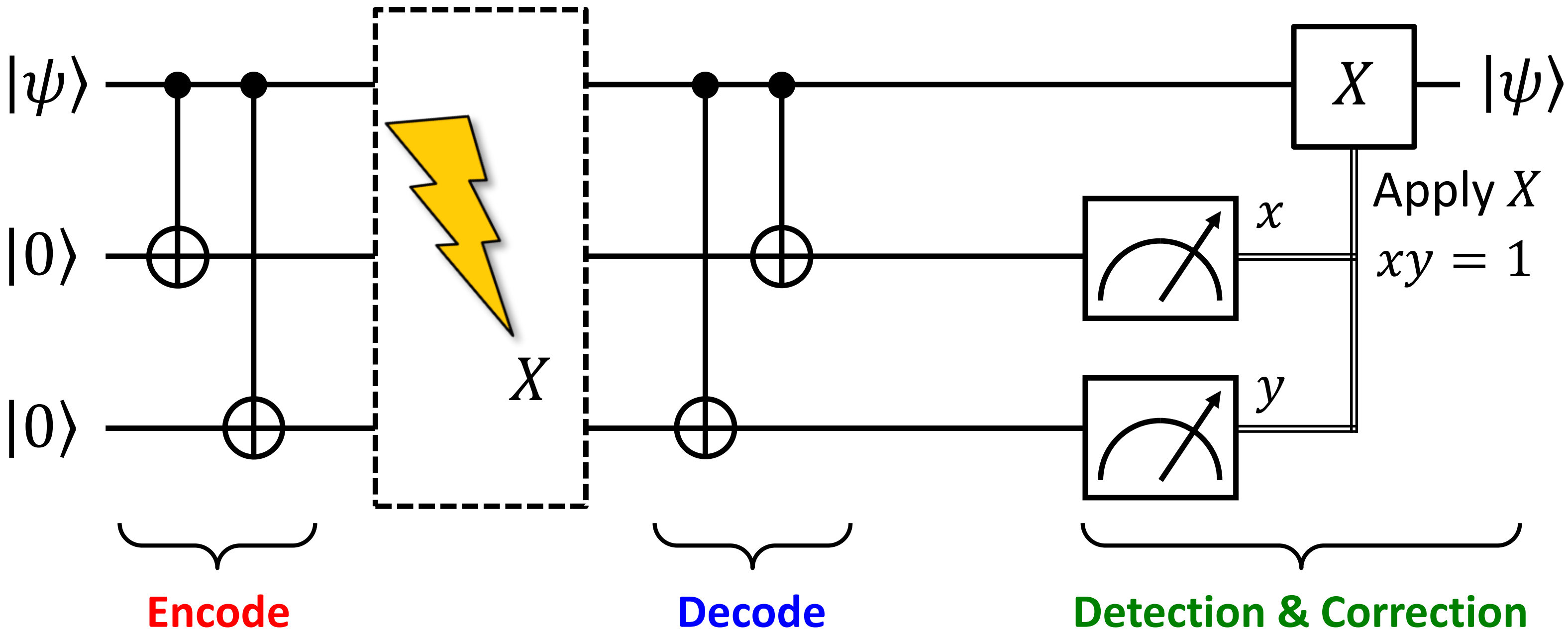
Homework 2

Verify the above circuit relations using

$$C_{12}|a\rangle|b\rangle = |a\rangle|a \oplus b\rangle$$

$$Z|a\rangle = (-1)^a|a\rangle$$

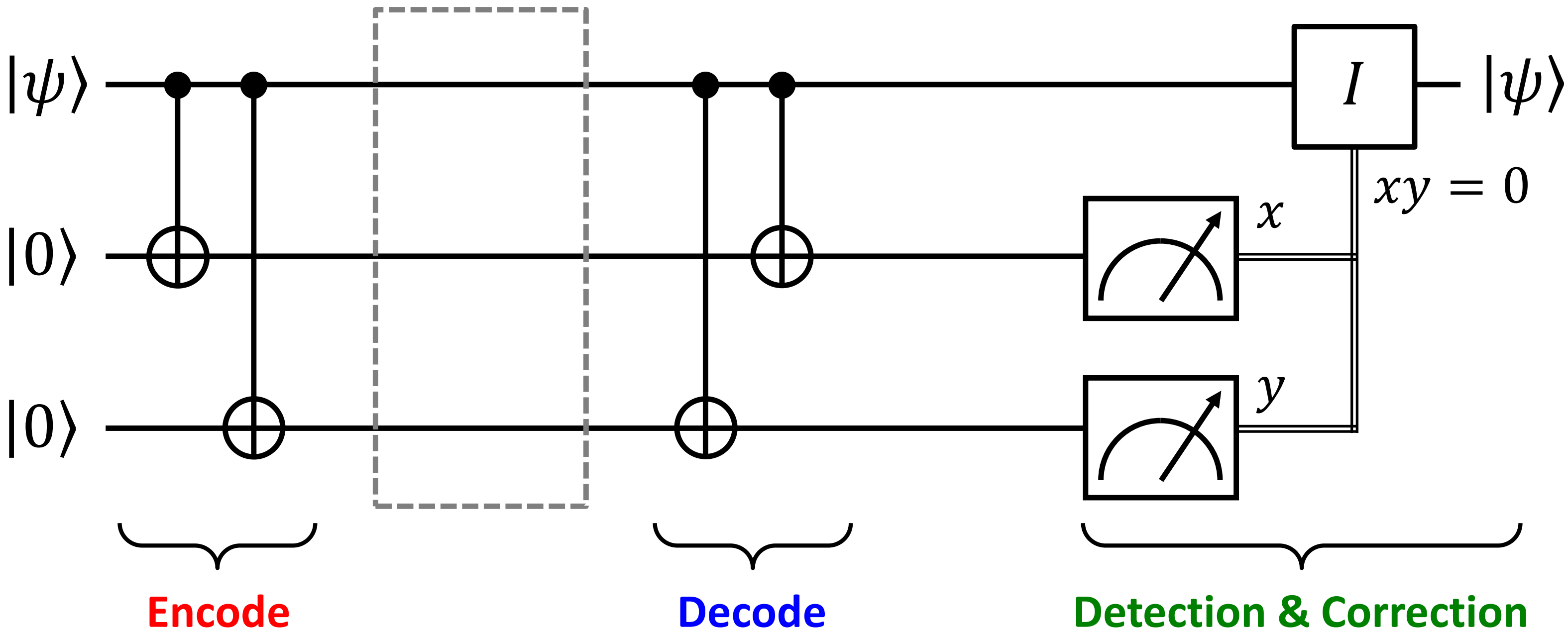
Detection & correction of bit-flip error



$$|\psi\rangle|00\rangle = (\alpha|0\rangle + \beta|1\rangle)|00\rangle$$

$$|\psi\rangle_L = \alpha|000\rangle + \beta|111\rangle$$

Detection & correction of bit-flip error



$$|\psi\rangle|00\rangle = (\alpha|0\rangle + \beta|1\rangle)|00\rangle$$

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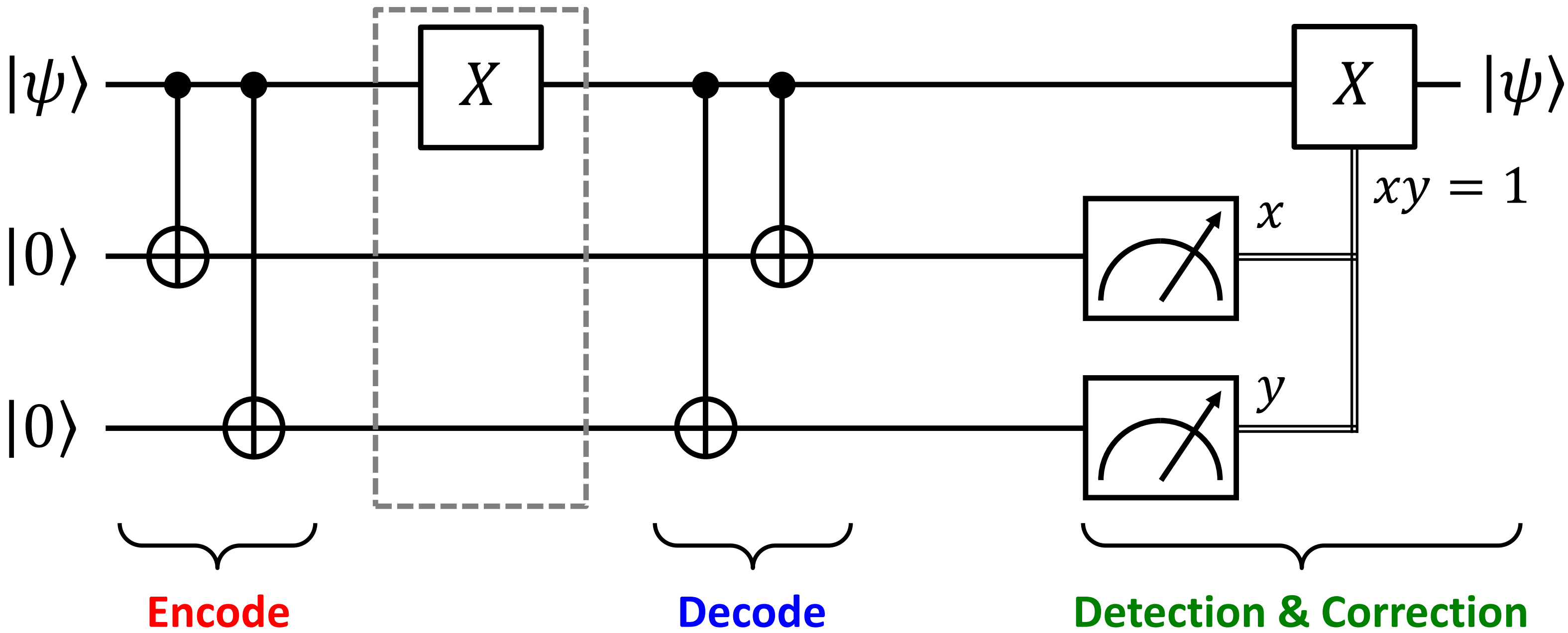
Detection & correction of bit-flip error

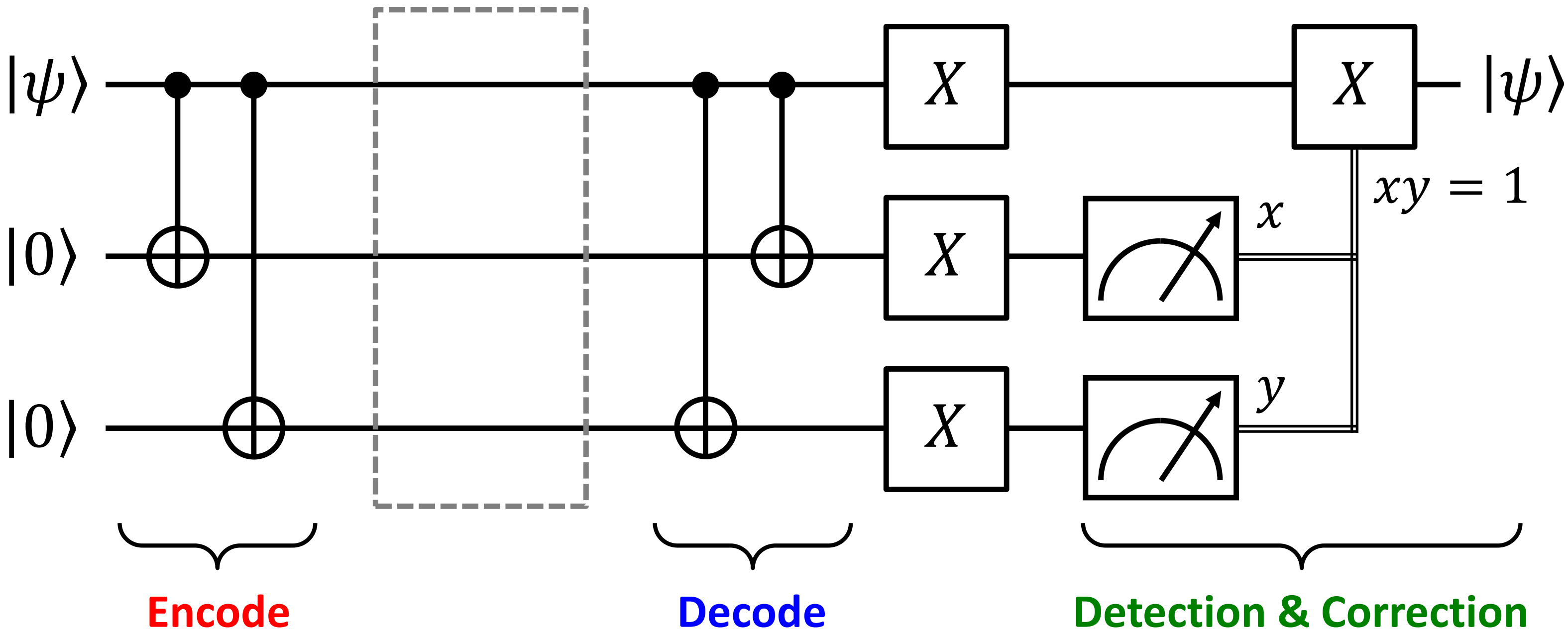


$$|\psi\rangle|00\rangle = (\alpha|0\rangle + \beta|1\rangle)|00\rangle$$

$$|\psi\rangle_L = \alpha|000\rangle + \beta|111\rangle \xrightarrow{\text{BFE}_1} \alpha|100\rangle + \beta|011\rangle$$

$$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$$

Detection & correction of bit-flip error

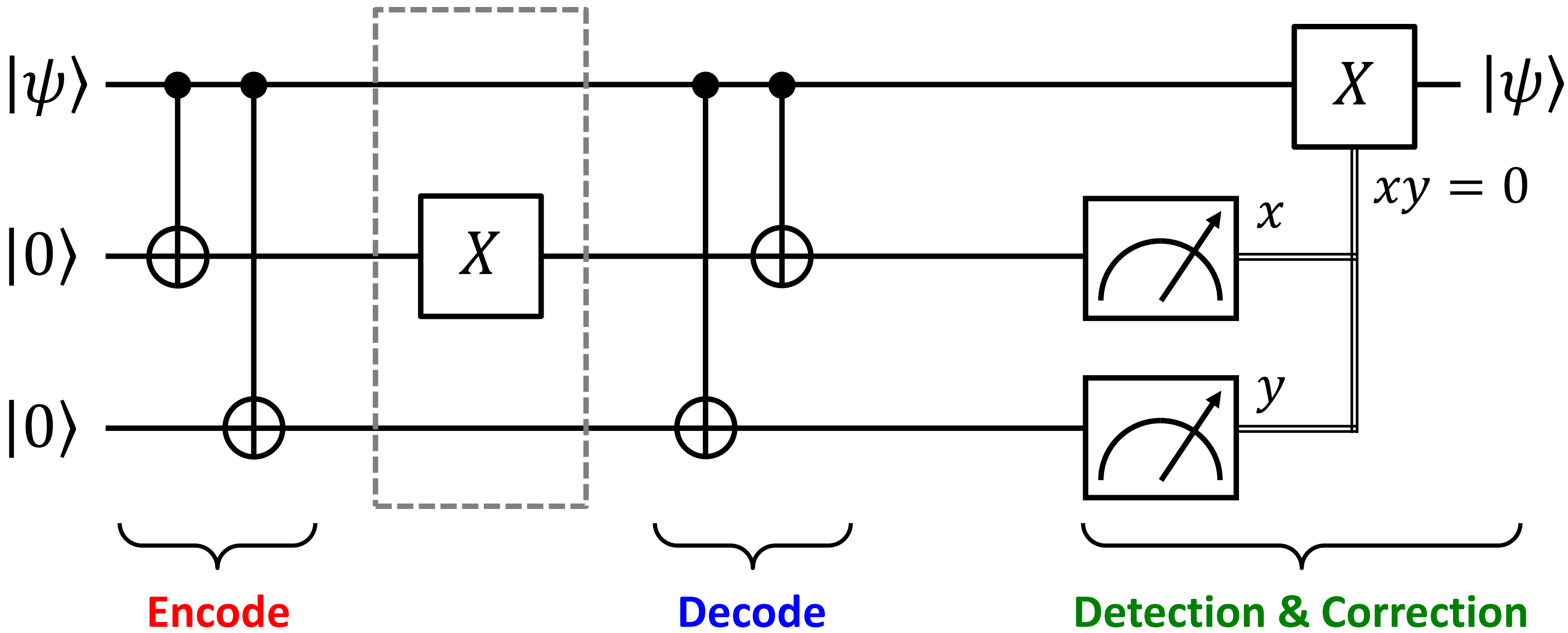


$|\psi\rangle|00\rangle = (\alpha|0\rangle + \beta|1\rangle)|00\rangle$

$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$ Error propagation
→ No need to measure $|\psi\rangle$ itself

$|\psi\rangle_L = \alpha|000\rangle + \beta|111\rangle \xrightarrow{\text{BFE}_1} \alpha|100\rangle + \beta|011\rangle$

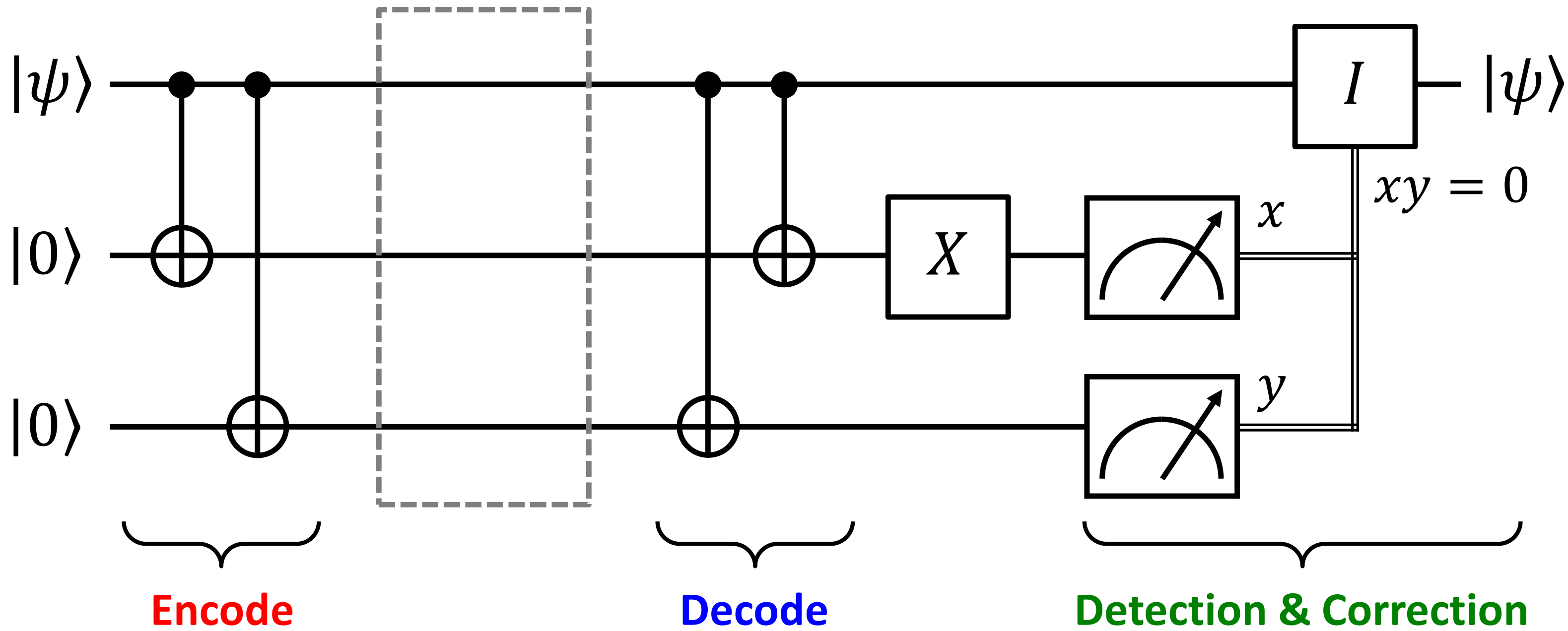
Detection & correction of bit-flip error



$|\psi\rangle|00\rangle = (\alpha|0\rangle + \beta|1\rangle)|00\rangle$ $\qquad$ $(\alpha|0\rangle + \beta|1\rangle)|10\rangle$

$|\psi\rangle_L = \alpha|000\rangle + \beta|111\rangle \xrightarrow{\text{BFE}_2} \alpha|010\rangle + \beta|101\rangle$

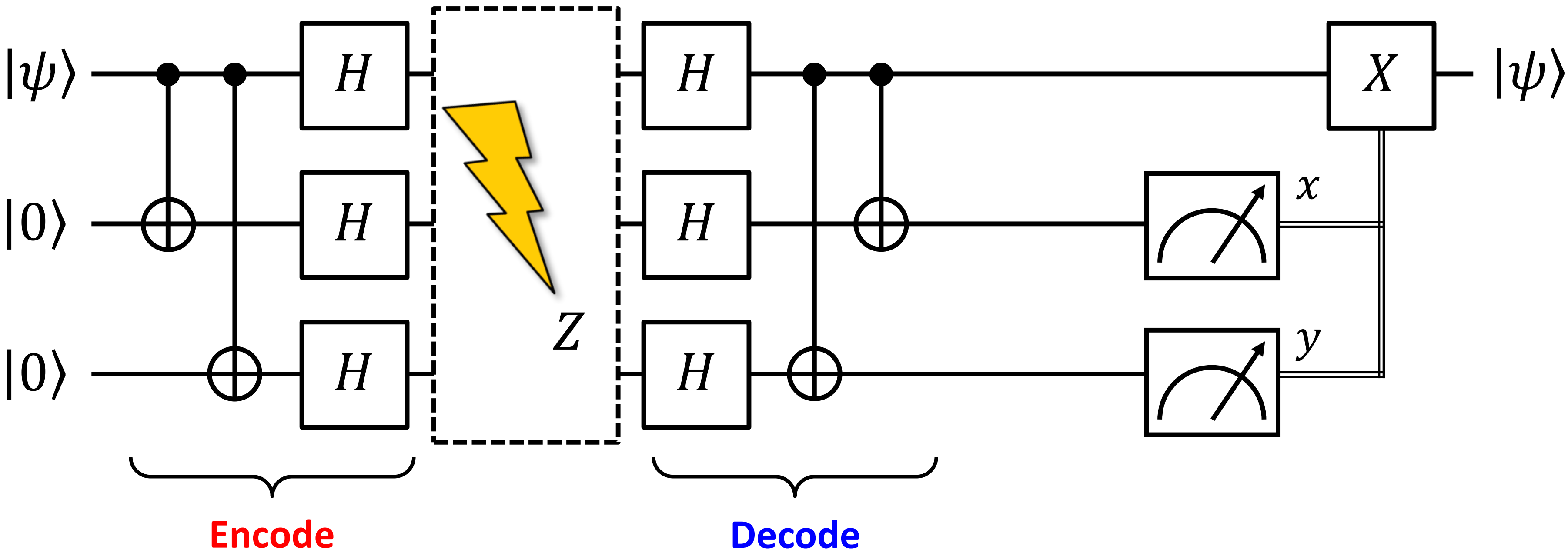
Detection & correction of bit-flip error



$|\psi\rangle|00\rangle = (\alpha|0\rangle + \beta|1\rangle)|00\rangle$ $\qquad$ $(\alpha|0\rangle + \beta|1\rangle)|10\rangle$

$|\psi\rangle_L = \alpha|000\rangle + \beta|111\rangle \xrightarrow{\text{BFE}_2} \alpha|010\rangle + \beta|101\rangle$

Detection & correction of phase-flip error

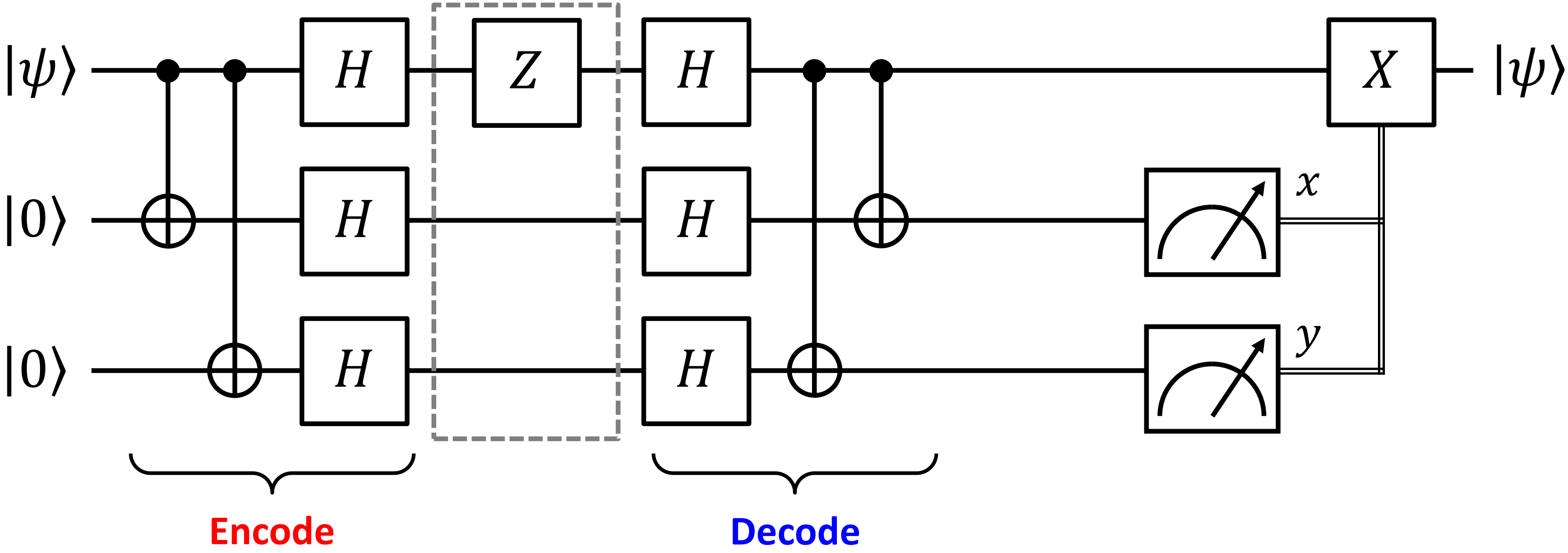


$|\psi\rangle|00\rangle = (\alpha|0\rangle + \beta|1\rangle)|00\rangle$

$\alpha|000\rangle + \beta|111\rangle \longrightarrow |\psi\rangle_L = \alpha|+++\rangle + \beta|---\rangle$

$|\pm\rangle \equiv \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$

Detection & correction of phase-flip error



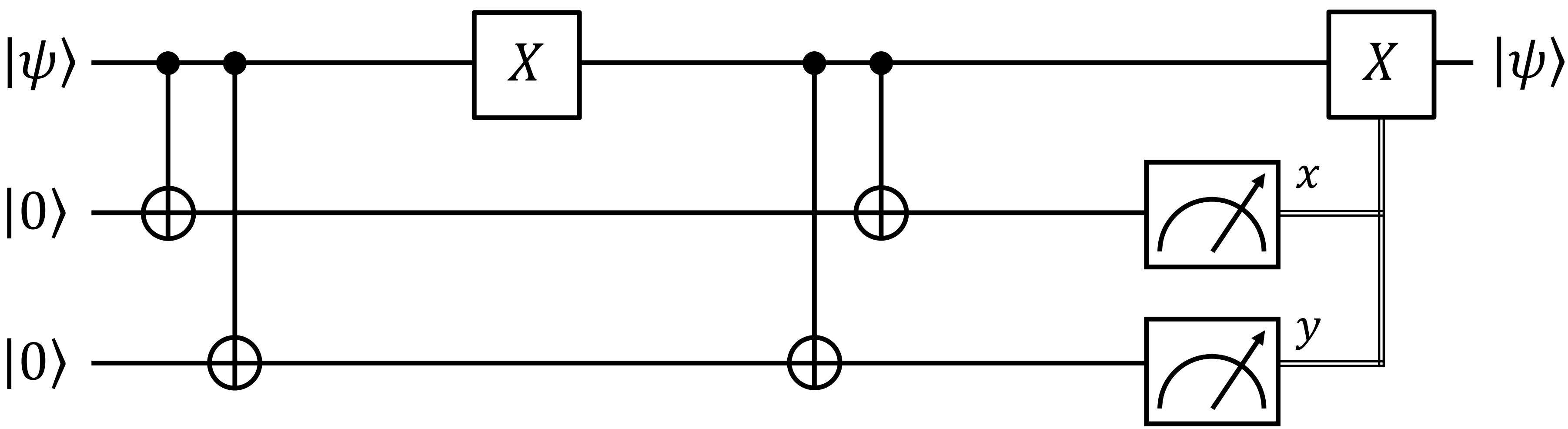
$$|\psi\rangle|00\rangle = (\alpha|0\rangle + \beta|1\rangle)|00\rangle$$

$$\downarrow$$

$$\alpha|000\rangle + \beta|111\rangle \longrightarrow |\psi\rangle_L = \alpha|+++\rangle + \beta|---\rangle$$

$$|\pm\rangle \equiv \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

Detection & correction of phase-flip error



Converted into bit-flip error

$$HZH = X$$

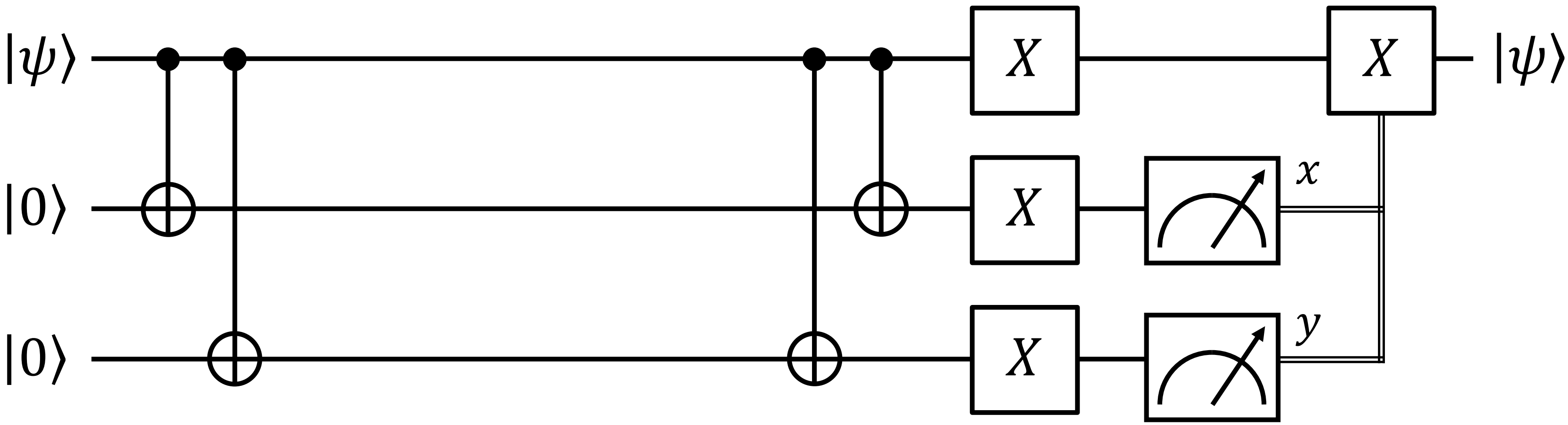
$$HH = I$$

Homework 3

Verify $HZH = X, HH = I$ using

$$\begin{cases} H|a\rangle = \frac{1}{\sqrt{2}} \sum_{b=0,1} (-1)^{a \cdot b} |b\rangle \\ Z|a\rangle = (-1)^a |a\rangle \\ X|a\rangle = |a+1\rangle = |\bar{a}\rangle \end{cases}$$

Detection & correction of phase-flip error



Converted into bit-flip error

$$HZH = X$$

$$HH = I$$

It is inefficient to decode every time…
Can we keep logical qubits throughout computation & error correction?

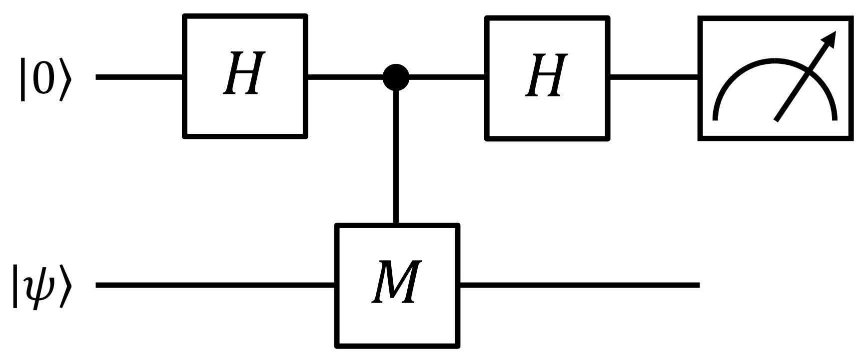
Syndrome measurement

Measurement of Operator M

$M^2 = I$ Self-adjoint

$MP_\pm|\psi\rangle = \pm P_\pm|\psi\rangle$

$P_\pm = \frac{I \pm M}{2}$ Projector: $P_\pm|\psi\rangle$ are eigenstates of M with eigenvalues $\lambda = \pm 1$



$$|0\rangle|\psi\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|\psi\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle M|\psi\rangle)$$

$$\longrightarrow \frac{1}{2}[(|0\rangle + |1\rangle)|\psi\rangle + (|0\rangle - |1\rangle)M|\psi\rangle] \longrightarrow |0\rangle P_+|\psi\rangle + |1\rangle P_-|\psi\rangle$$

Syndrome meas. (Shor code)

Logical qubit $|\psi\rangle_{\mathrm{L}} = \alpha|0\rangle_{\mathrm{L}} + \beta|1\rangle_{\mathrm{L}}$

$$|0\rangle_{\mathrm{L}} = \frac{1}{2\sqrt{2}}(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$$

$$|1\rangle_{\mathrm{L}} = \frac{1}{2\sqrt{2}}(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

Error syndrome

$M_1 = Z_1 Z_2$

$M_2 = Z_2 Z_3$

$M_3 = Z_4 Z_5$

$M_4 = Z_5 Z_6$

$M_5 = Z_7 Z_8$

$M_6 = Z_8 Z_9$

$M_7 = X_1 X_2 X_3 X_4 X_5 X_6$

$M_8 = X_4 X_5 X_6 X_7 X_8 X_9$

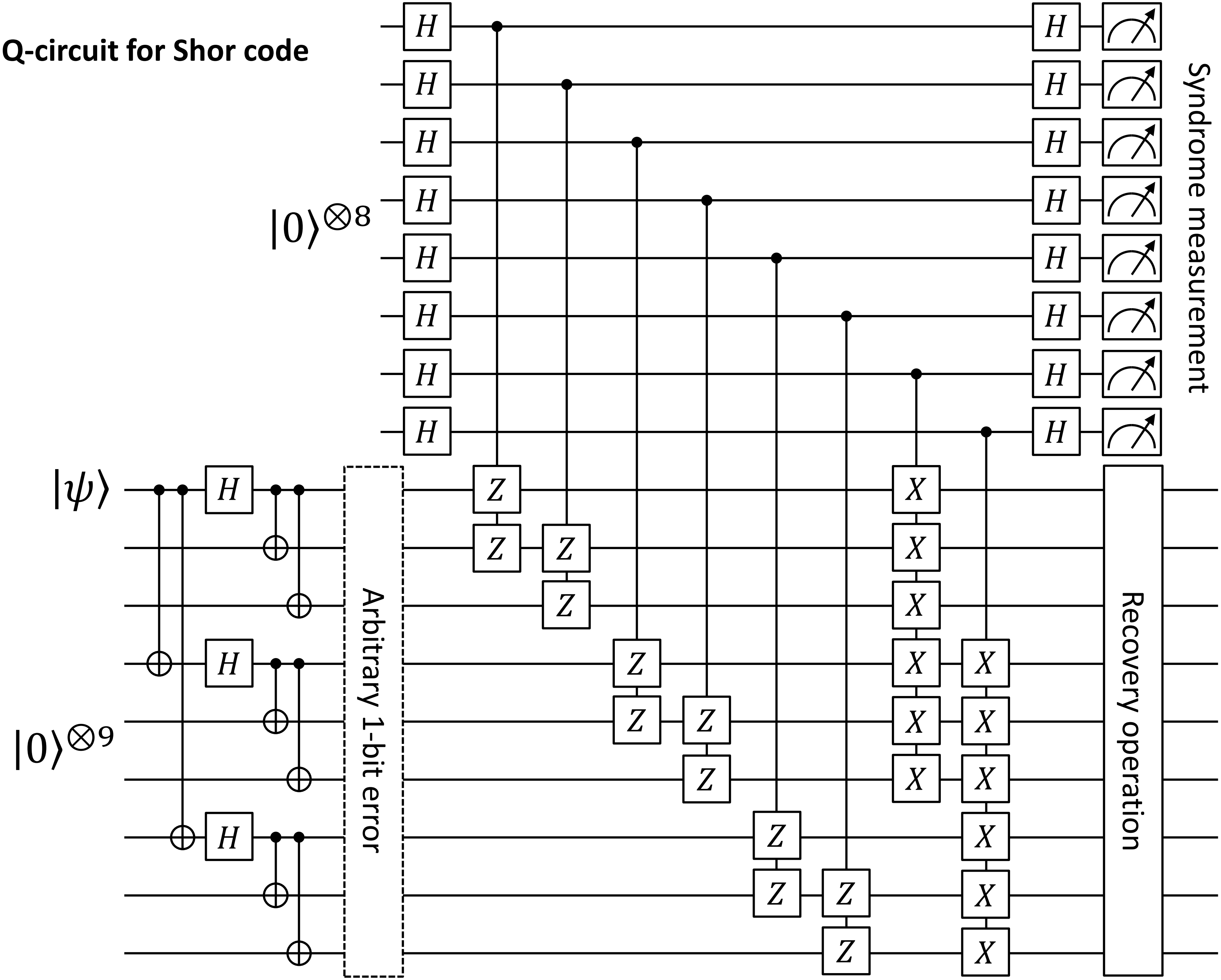
- Logical qubits satisfy $M_i|\psi\rangle_{\mathrm{L}} = |\psi\rangle_{\mathrm{L}}$ with $\lambda = 1$
- $M_i M_j = M_j M_i$ (simultaneous observable)
- At least one of X_i, Y_i, Z_i **anti-commutes** with M_i
- Errors are detected by the **parity** change

Example) Bit-flip on Q1

$$M_1 X_1 |\psi\rangle_{\mathrm{L}} = -X_1 M_1 |\psi\rangle_{\mathrm{L}} = -X_1 |\psi\rangle_{\mathrm{L}}$$

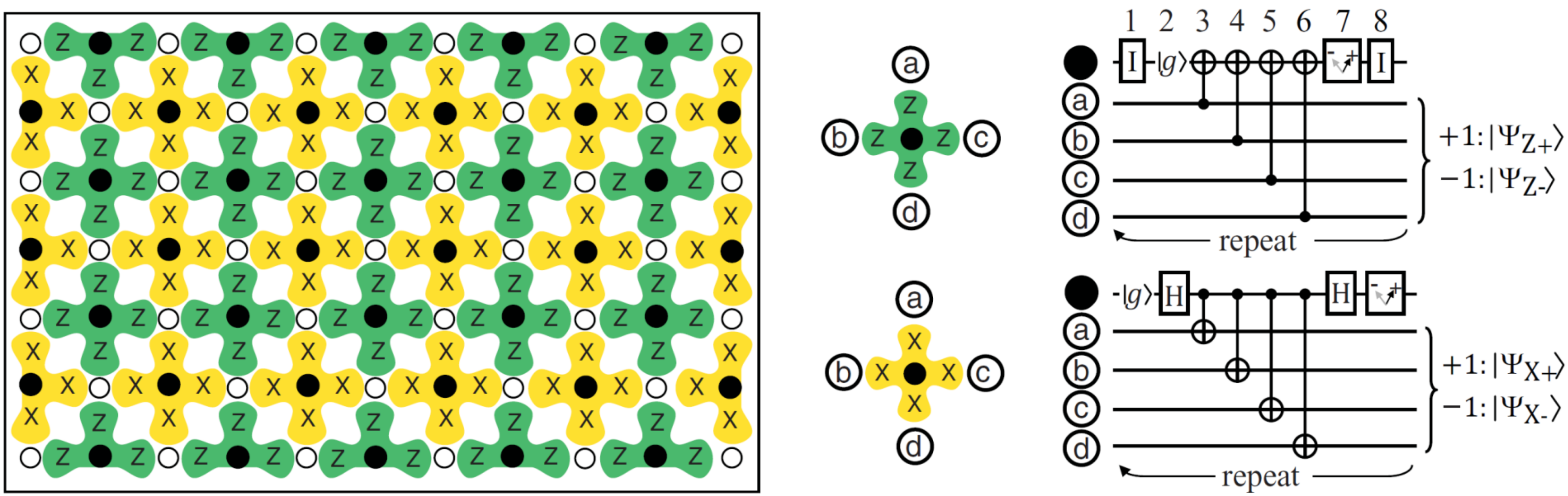
$$M_{i\neq 1} X_1 |\psi\rangle_{\mathrm{L}} = X_1 M_{i\neq 1} |\psi\rangle_{\mathrm{L}} = X_1 |\psi\rangle_{\mathrm{L}}$$

Q-circuit for Shor code



Surface code

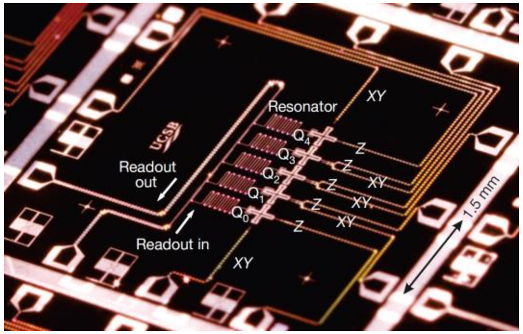
Phys. Rev. A **86**, 032324 (2012) Fowler *et al.*



- 2D lattice
- Nearest-neighbor coupling
- High error-tolerance (~1%)

Superconducting quantum circuits at the surface code threshold for fault tolerance

R. Barends[1]*, J. Kelly[1]*, A. Megrant[1], A. Veitia[2], D. Sank[1], E. Jeffrey[1], T. C. White[1], J. Mutus[1], A. G. Fowler[1,3], B. Campbell[1], Y. Chen[1], Z. Chen[1], B. Chiaro[1], A. Dunsworth[1], C. Neill[1], P. O'Malley[1], P. Roushan[1], A. Vainsencher[1], J. Wenner[1], A. N. Korotkov[2], A. N. Cleland[1] & John M. Martinis[1]



Nature **508**, 500 (2014) Barends *et al.*

Contents

- **Quantum computation**
  - From an electron in a double-well potential to qubit
  - Quantum gates
  - Deutsch–Jozsa algorithm

- **Quantum error correction**
  - DiVincenzo's criteria and the need of QEC
  - Spin, spin resonance, and spin relaxation
  - Basics of quantum error correction

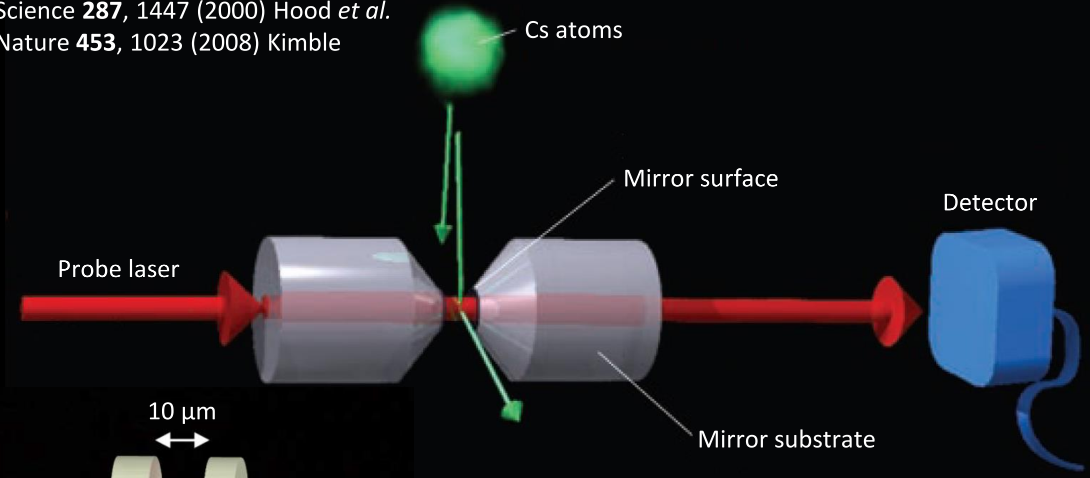
- **Superconducting quantum circuits**
  - Circuit QED and transmon
  - Quantum control
  - Recent experiments by Google and ETH

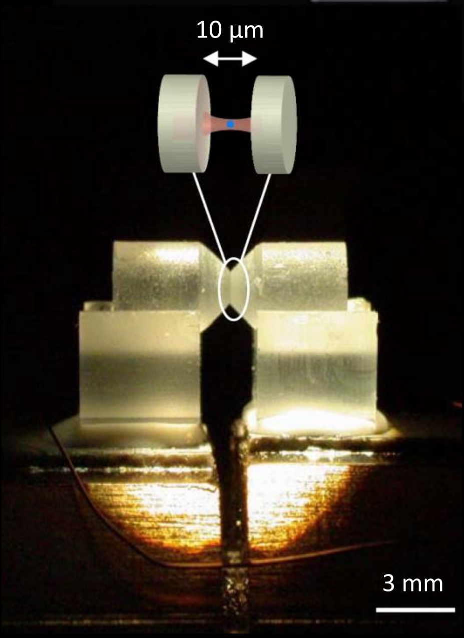
Cavity QED (Quantum ElectroDynamics)

Interaction between an atom & a photon confined in a cavity

Optical cavity: Kimble group (Caltech)

Science **287**, 1447 (2000) Hood *et al.*
Nature **453**, 1023 (2008) Kimble



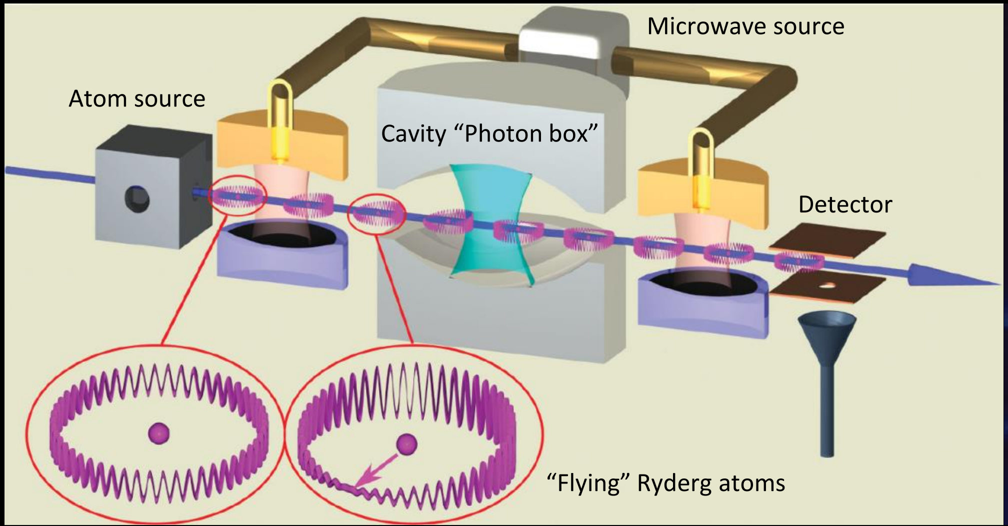


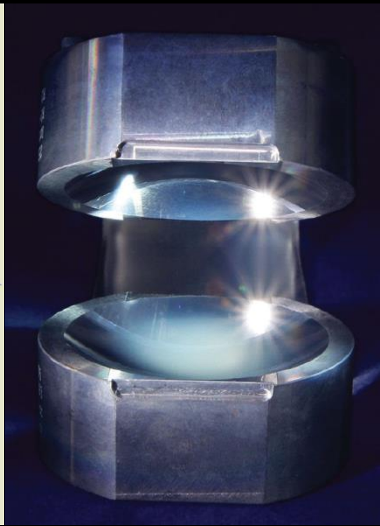
Physics Today **49**, (8) 51 (1996) Haroche & Raimond
"Quantum Computing: Dream or Nightmare?"

Serge Haroche
(1944–)
©Nobel Foundation

Microwave cavity: Haroche group (ENS)

Rev. Mod. Phys. **85**, 1083 (2013) Haroche

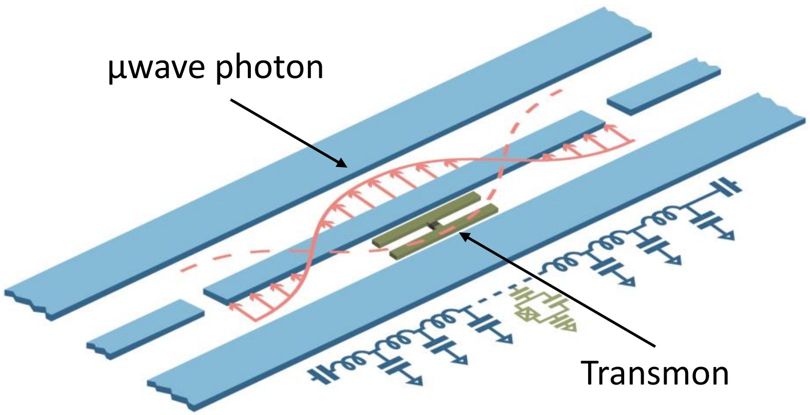




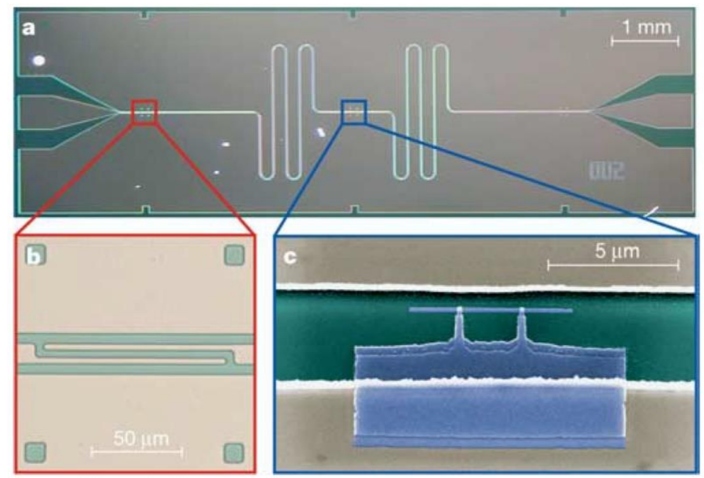
Circuit QED

Artificial atom–μwave photon interaction in superconducting quantum circuits

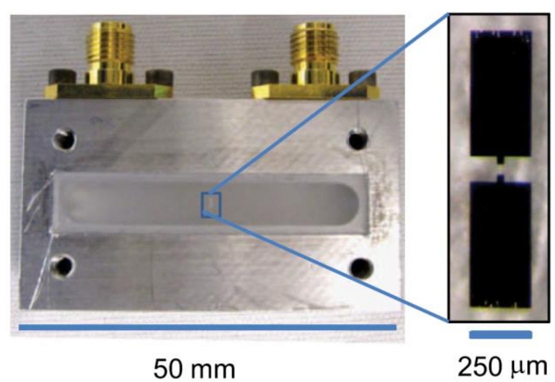
- ✓ System stability (an artificial atom “transmon” doesn’t move)
- ✓ Design flexibility
- ✓ Size & scalability



Rev. Mod. Phys. **93**, 025005 (2021) Blais *et al.*
Phys. Rev. A **69**, 062320 (2004) Blais *et al.*

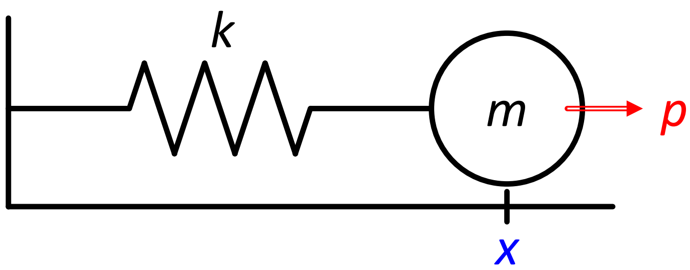


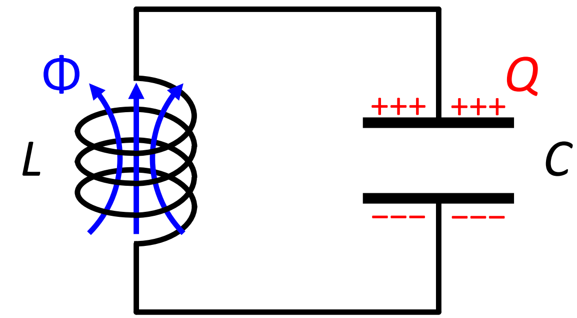
Nature **431**, 162 (2004) Wallraff *et al.*



Phys. Rev. Lett. **107**, 240501 (2011) Paik *et al.*

Harmonic oscillator & LC circuit





$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

Hamiltonian

$$H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$$[\hat{x}, \hat{p}] = i\hbar$$

Quantization

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

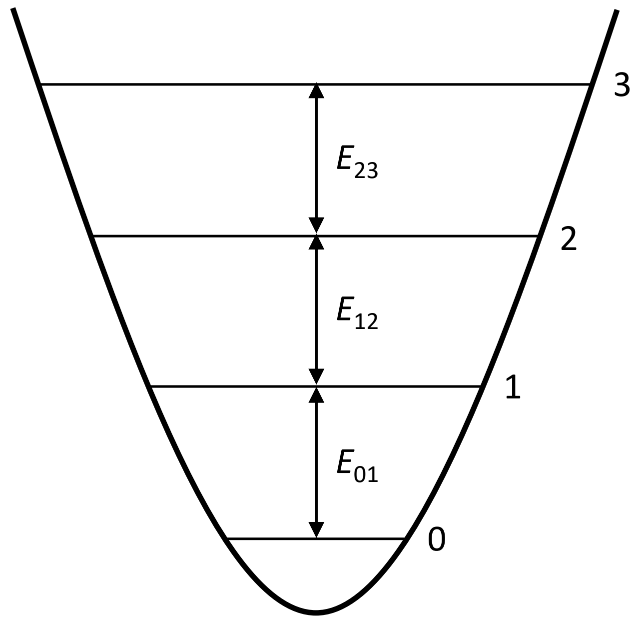
Energy levels

$$\omega = \sqrt{\frac{k}{m}}$$

$$E_n = \hbar\omega\left(\frac{1}{2} + n\right)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

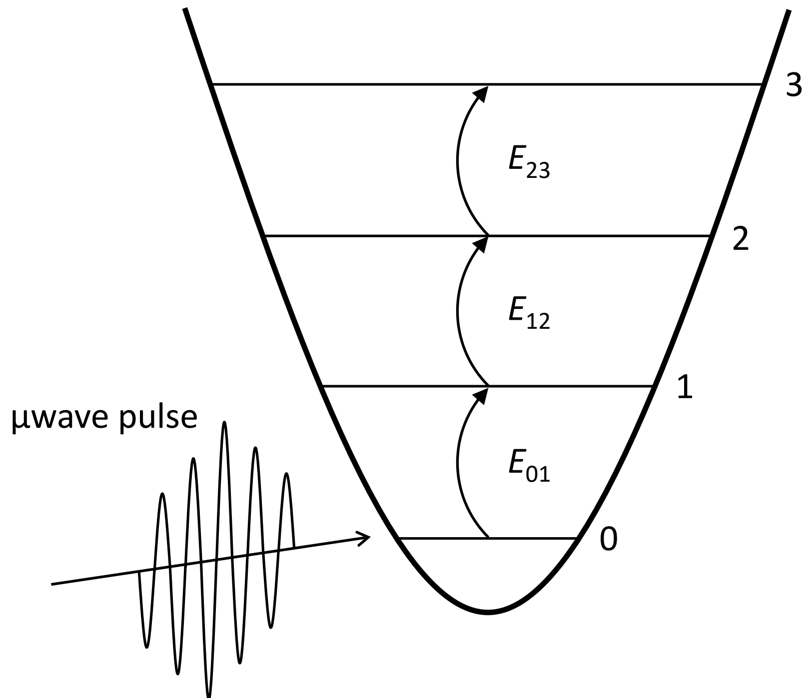
Harmonic oscillator = Qubit?



$$E_{01} = E_{12} = E_{23}$$

$$E_n = \hbar\omega\left(\frac{1}{2} + n\right)$$

Harmonic oscillator = Qubit?



$E_{01} = E_{12} = E_{23}$

$$E_n = \hbar\omega\left(\frac{1}{2} + n\right)$$

Bosonic field

$$[a, a^\dagger] = 1$$

$a^\dagger$: Creation op.

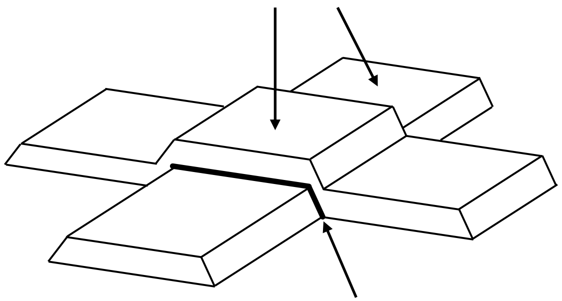
a: Annihilation op.

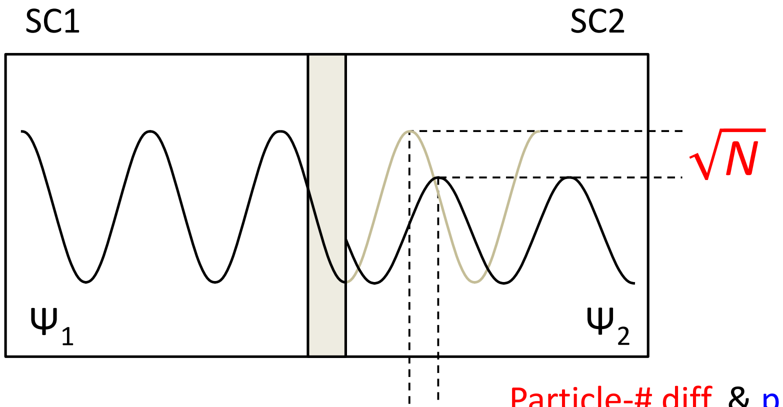
Resonant on all transitions
(Nonselectivity to 2-level)

⇩

Need of anharmonicity

Josephson junction





Particle-# diff. & phase diff. of macroscopic wavefunction

B. Josephson (1940–)

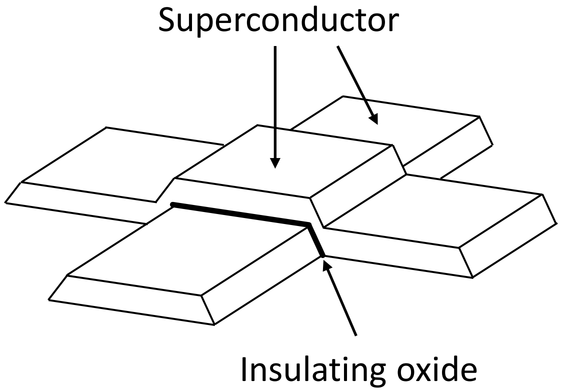
Josephson equation

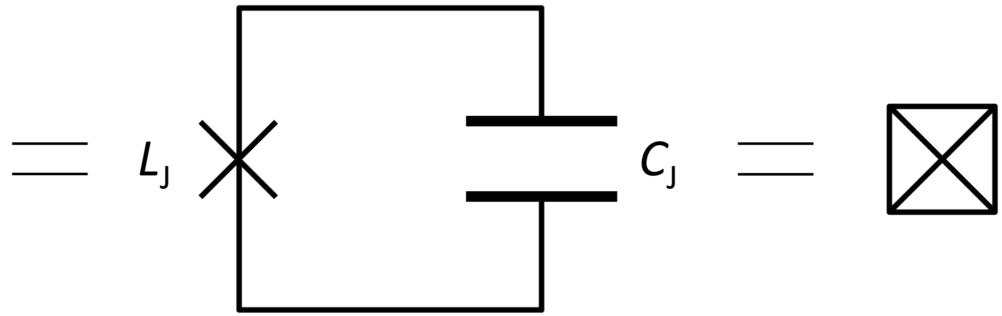
$$\begin{cases} V = -\dfrac{\hbar}{2e}\dfrac{d\varphi}{dt} \\ I = I_c \sin\varphi \end{cases} \longrightarrow V = -\underbrace{\frac{\hbar}{2eI_c}\frac{1}{\sqrt{1-(I/I_c)^2}}}_{L_J}\frac{dI}{dt}$$

(Only) nonlinear, dissipationless inductor

$$\longrightarrow U = -\int IV\,dt = \int \left(\frac{\hbar I_c}{2e}\right)\sin\varphi\,\frac{d\varphi}{dt}\,dt = -E_J\cos\varphi$$

Josephson junction





B. Josephson
(1940–)
© Nobel Foundation

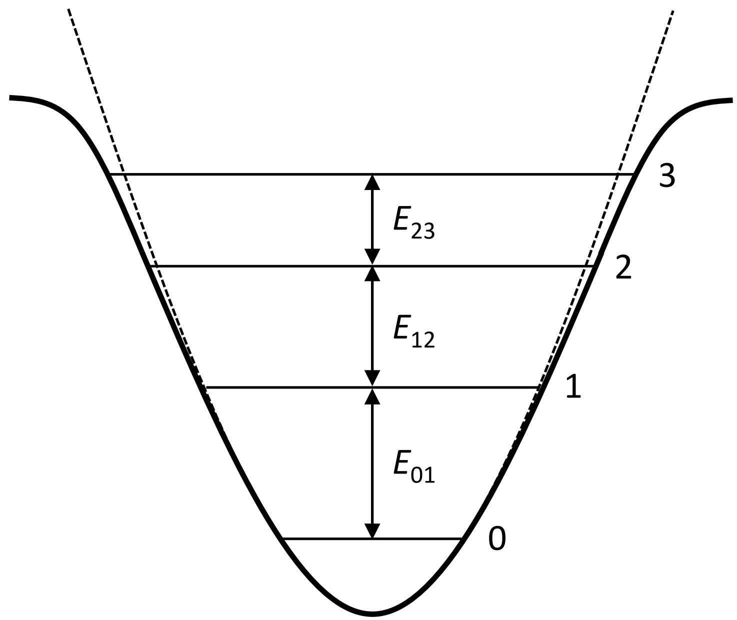
Josephson equation

$$\begin{cases} V = -\dfrac{\hbar}{2e}\dfrac{d\varphi}{dt} \\ I = I_c \sin\varphi \end{cases} \longrightarrow \quad V = -\underbrace{\frac{\hbar}{2eI_c}\frac{1}{\sqrt{1-(I/I_c)^2}}}_{L_J}\frac{dI}{dt}$$

(Only) nonlinear, dissipationless inductor

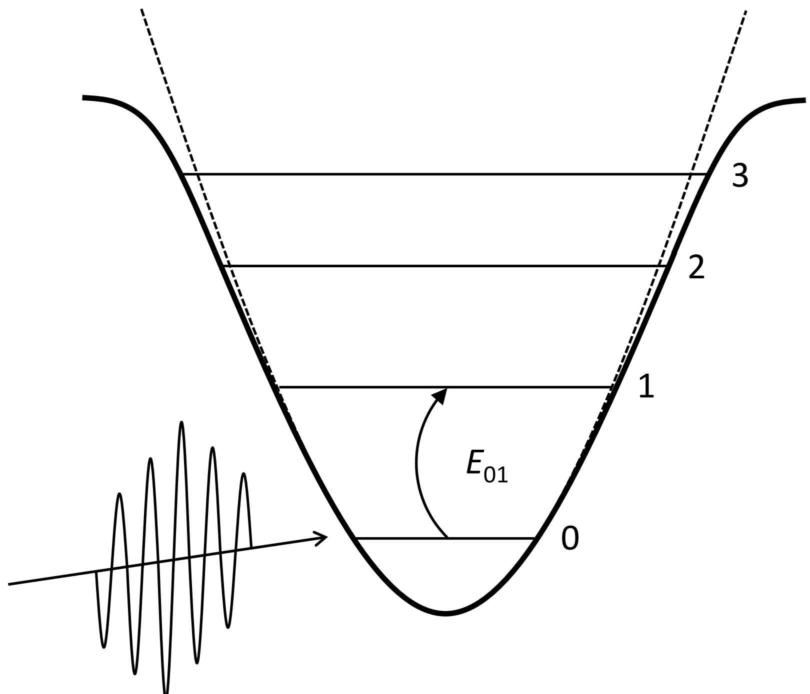
$$\longrightarrow \quad U = -\int IV dt = \int \left(\frac{\hbar I_c}{2e}\right) \sin\varphi \frac{d\varphi}{dt} dt = -E_J \cos\varphi$$

Anharmonic oscillator = Qubit



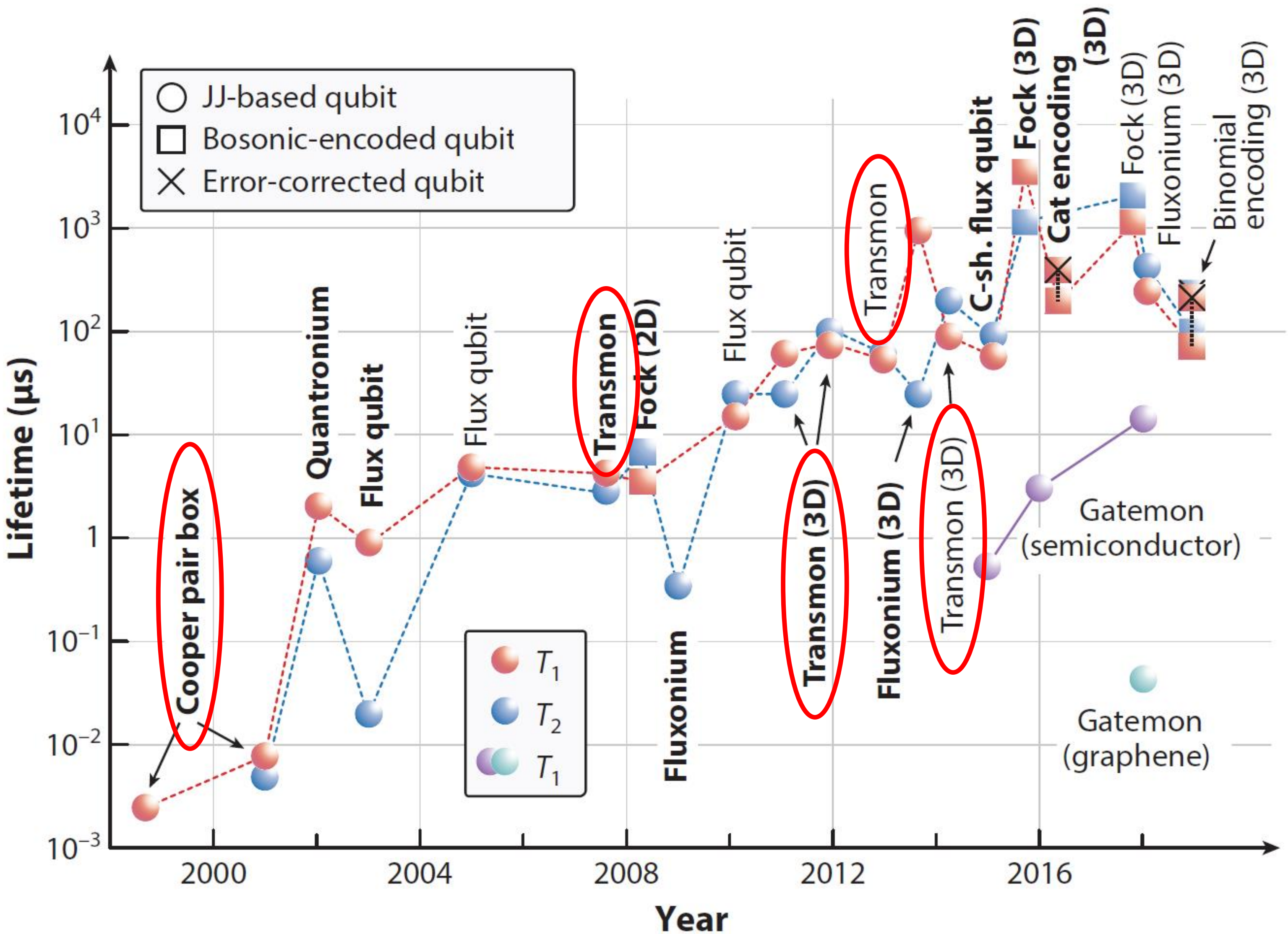
$$E_{01} > E_{12} > E_{23}$$

Anharmonic oscillator = Qubit



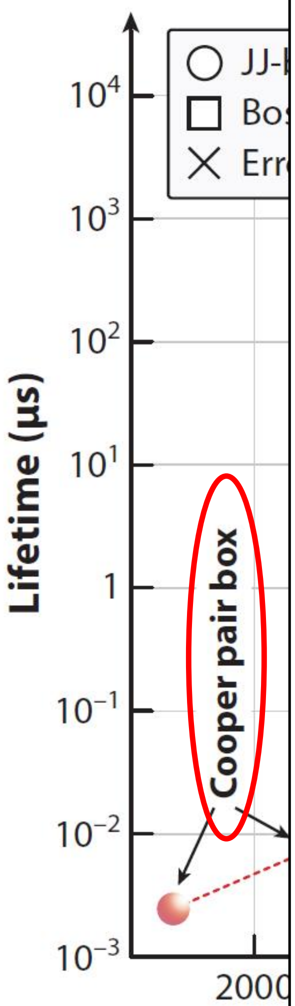
$E_{01} > E_{12} > E_{23}$

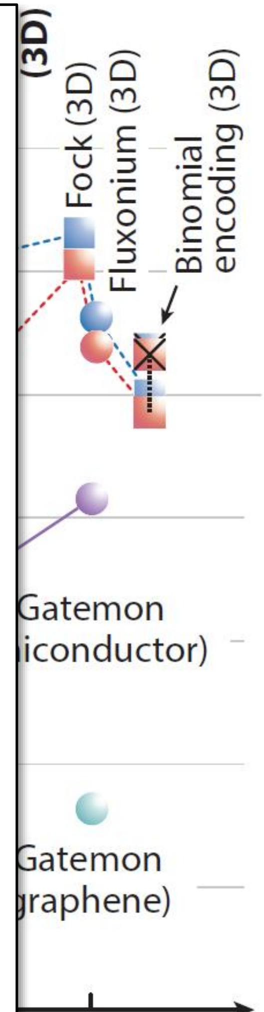
Ever improving T_1 & T_2



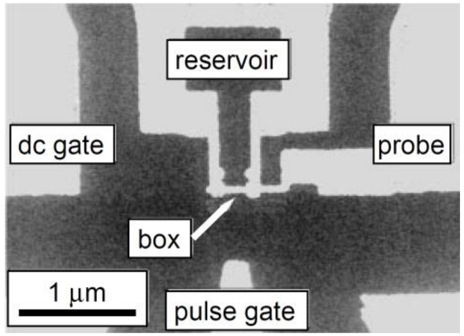
Annu. Rev. Condens. Matter Phys. **11**, 369 (2020) Kjaergaard *et al.*

Ever improving T_1 & T_2

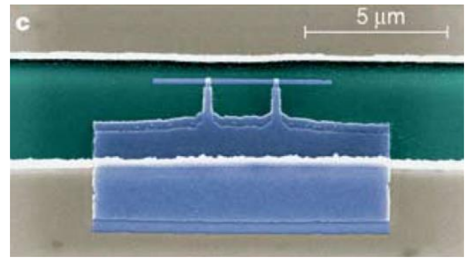




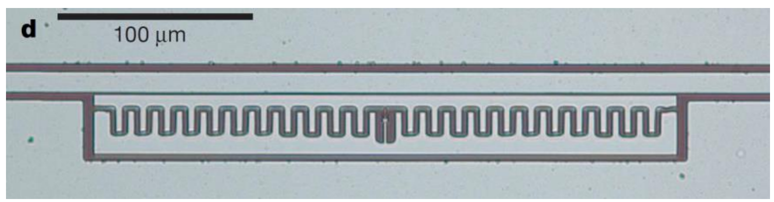
Charge qubit: The bigger, the better coherence



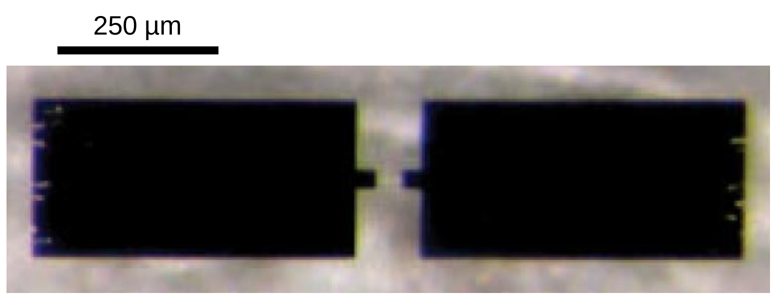
(CPB) Nature **398**, 786 (1999) Nakamura *et al.*



(CPB) Nature **431**, 162 (2004) Wallraff *et al.*



(2D Transmon) Nature **449**, 328 (2007) Houck *et al.*

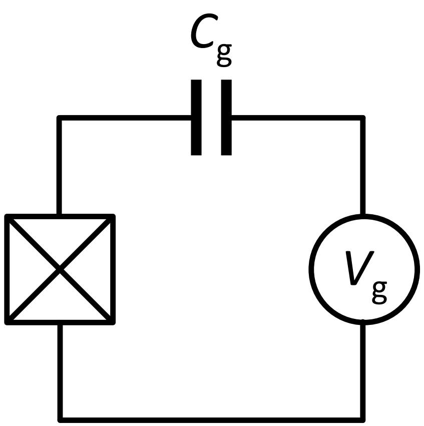


(3D Transmon) Phys. Rev. Lett. **107**, 240501 (2011) Paik *et al.*

Annu. Rev. Condens. Matter Phys. **11**, 369 (2020) Kjaergaard *et al.*

Charge qubit

Cooper-pair box



Hamiltonian

$$H = 4E_C(N - n_g)^2 - E_J \cos\varphi$$

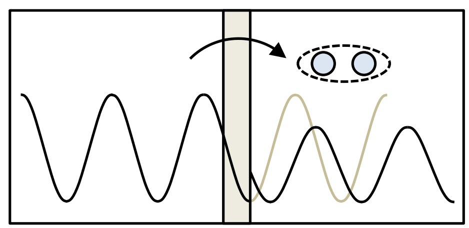
Charging energy

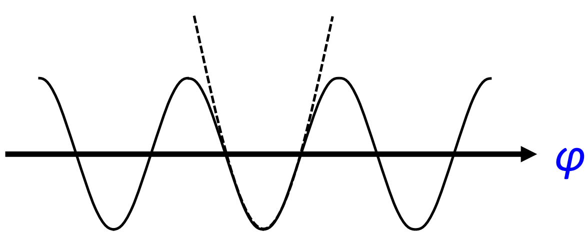
$$\frac{e^2}{2C_\Sigma}$$

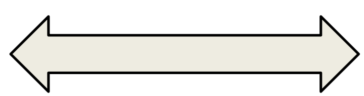
Charge offset

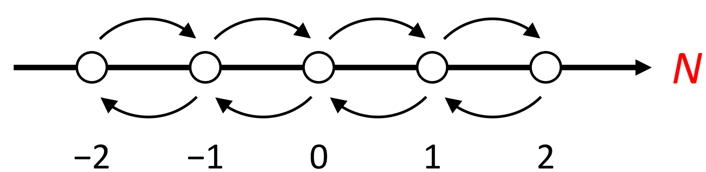
$$[\hat{\varphi}, \hat{N}] = i\hbar$$

$$\frac{1}{2}\sum_{N=-\infty}^{\infty}\left(|N+1\rangle\langle N| + |N\rangle\langle N+1|\right)$$





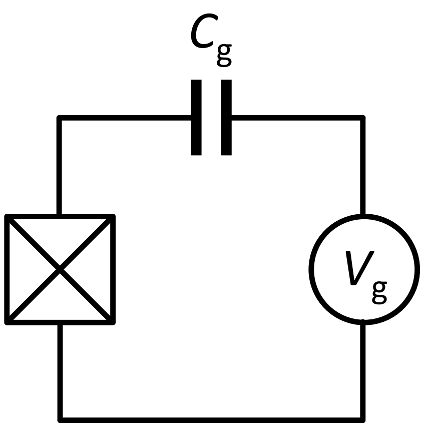




$$|\varphi\rangle = \frac{1}{\sqrt{2\pi}}\sum_{N=-\infty}^{\infty} e^{i\varphi N}|N\rangle$$

Charge qubit

Cooper-pair box



Cooper-pair tunneling = 1D tight-binding model

$$\cos\varphi\,|\varphi\rangle$$

$$= \left(\frac{e^{-i\varphi}+e^{i\varphi}}{2}\right)\frac{1}{\sqrt{2\pi}}\sum_{N=-\infty}^{\infty} e^{i\varphi N}|N\rangle$$

$$= \frac{1}{2\sqrt{2\pi}}\sum_{N=-\infty}^{\infty} e^{i\varphi(N-1)}|N\rangle + e^{i\varphi(N+1)}|N\rangle$$

$$= \frac{1}{2\sqrt{2\pi}}\left(\sum_{N=-\infty}^{\infty} e^{i\varphi N}|N+1\rangle + \sum_{N=-\infty}^{\infty} e^{i\varphi N}|N-1\rangle\right)$$

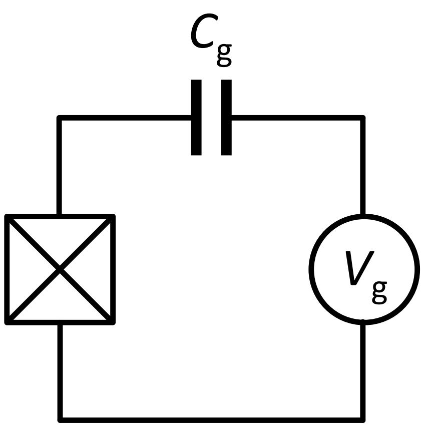
$$= \frac{1}{2\sqrt{2\pi}}\sum_{N=-\infty}^{\infty}\left(\sum_{N'=-\infty}^{\infty} |N'+1\rangle\langle N'| + |N'-1\rangle\langle N'|\right) e^{i\varphi N}|N\rangle$$

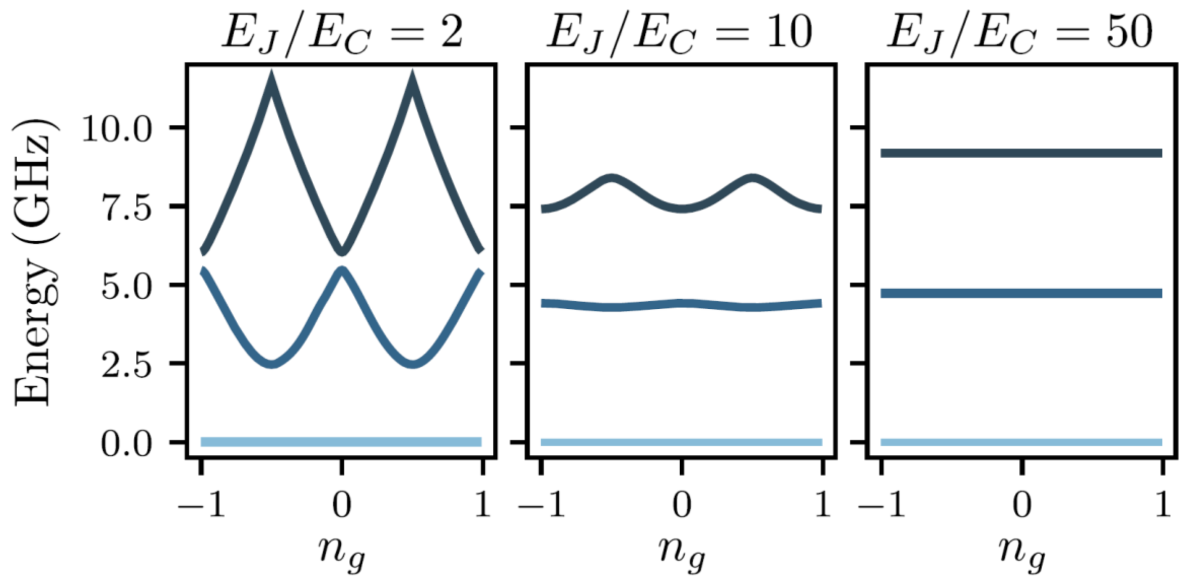
$$= \frac{1}{2}\sum_{N=-\infty}^{\infty}\left(|N+1\rangle\langle N| + |N\rangle\langle N+1|\right)|\varphi\rangle$$

Charge qubit

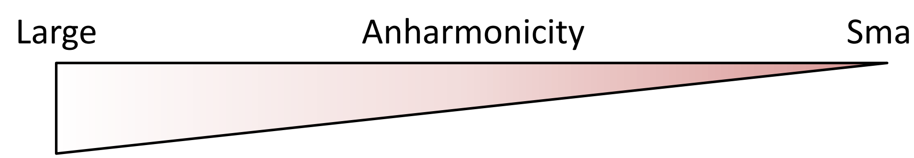
Rev. Mod. Phys. **93**, 025005 (2021) Blais *et al.*

Cooper-pair box





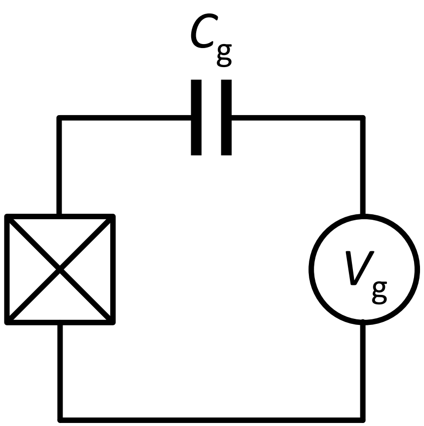
$$H = 4E_{\mathrm{C}}\left(N - n_{\mathrm{g}}\right)^2 - E_{\mathrm{J}}\cos\varphi$$

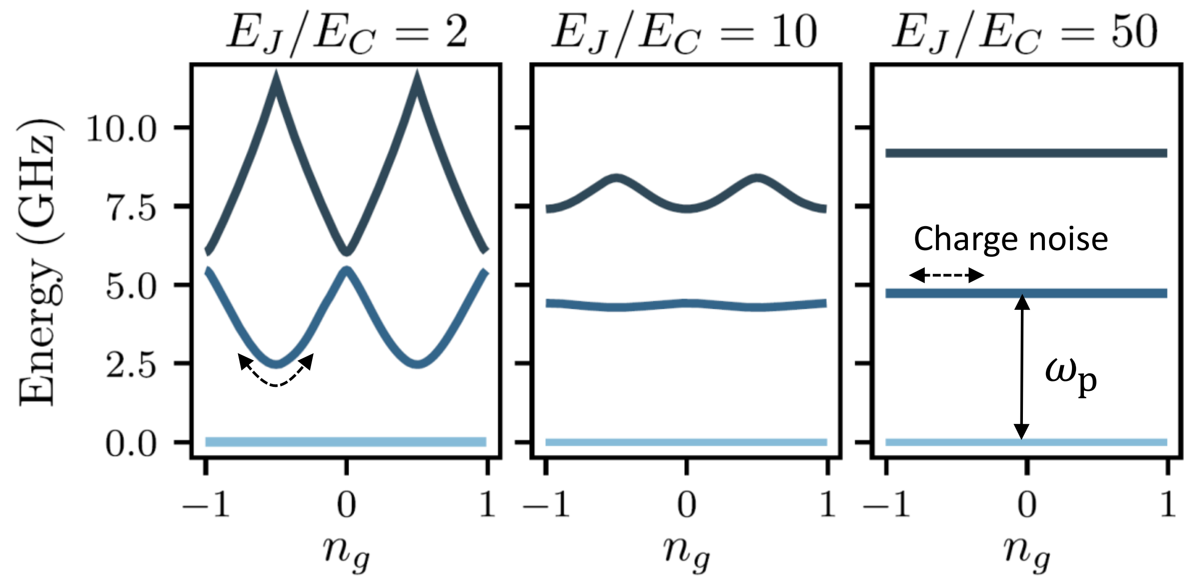


Charge qubit

Rev. Mod. Phys. **93**, 025005 (2021) Blais *et al.*

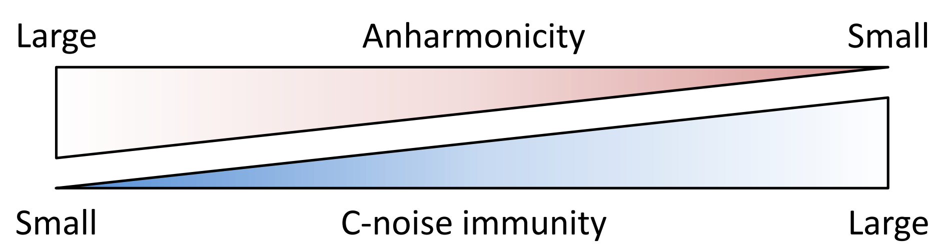
Cooper-pair box





$$H = 4E_C(N - n_g)^2 - E_J \cos\varphi$$

$$\xrightarrow{\frac{E_J}{E_C} \gg 1} \quad H \approx \frac{1}{2}(8E_C)N^2 + \frac{1}{2}E_J\varphi^2$$



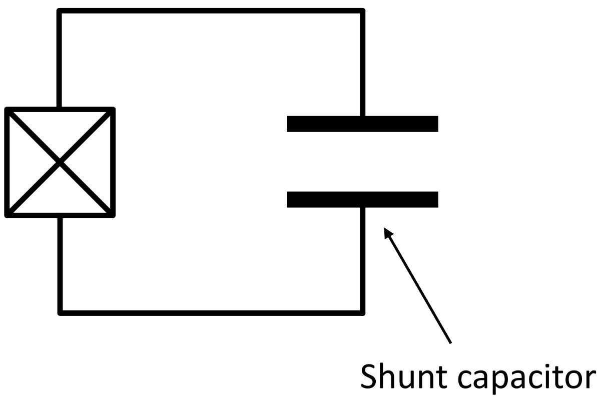
$8E_C \leftrightarrow (C)^{-1}$

$$\xrightarrow{E_J \leftrightarrow (L)^{-1}} \quad \omega_p = \sqrt{8E_C E_J}$$

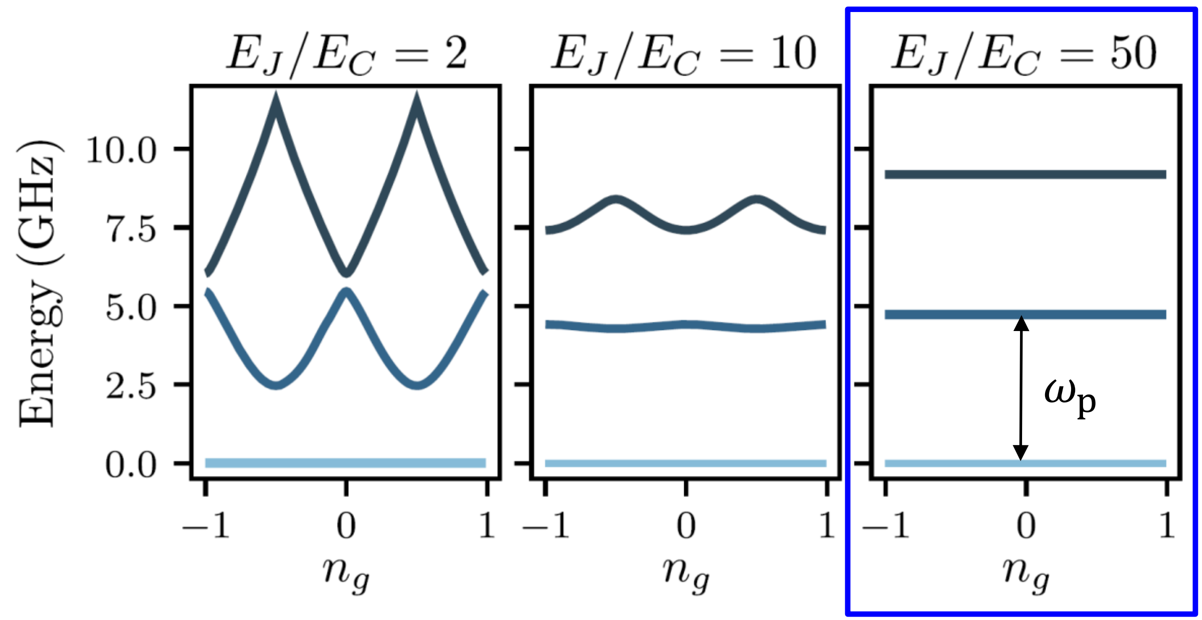
Charge qubit

Transmon

(Transmission-line shunted plasma oscillation qubit)



Rev. Mod. Phys. **93**, 025005 (2021) Blais *et al.*

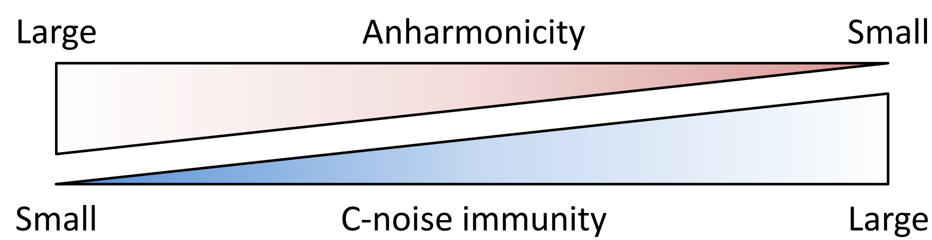


$$H = 4E_C(N - n_g)^2 - E_J \cos\varphi$$

$$\xrightarrow{\frac{E_J}{E_C} \gg 1} \quad H \approx \frac{1}{2}(8E_C)N^2 + \frac{1}{2}E_J\varphi^2$$

$8E_C \leftrightarrow (C)^{-1}$

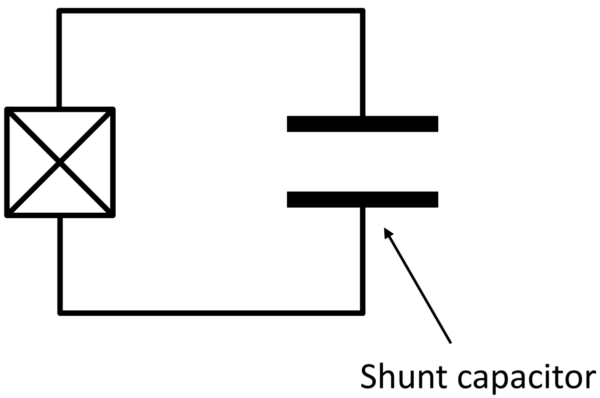
$$\xrightarrow{E_J \leftrightarrow (L)^{-1}} \quad \omega_p = \sqrt{8E_C E_J}$$



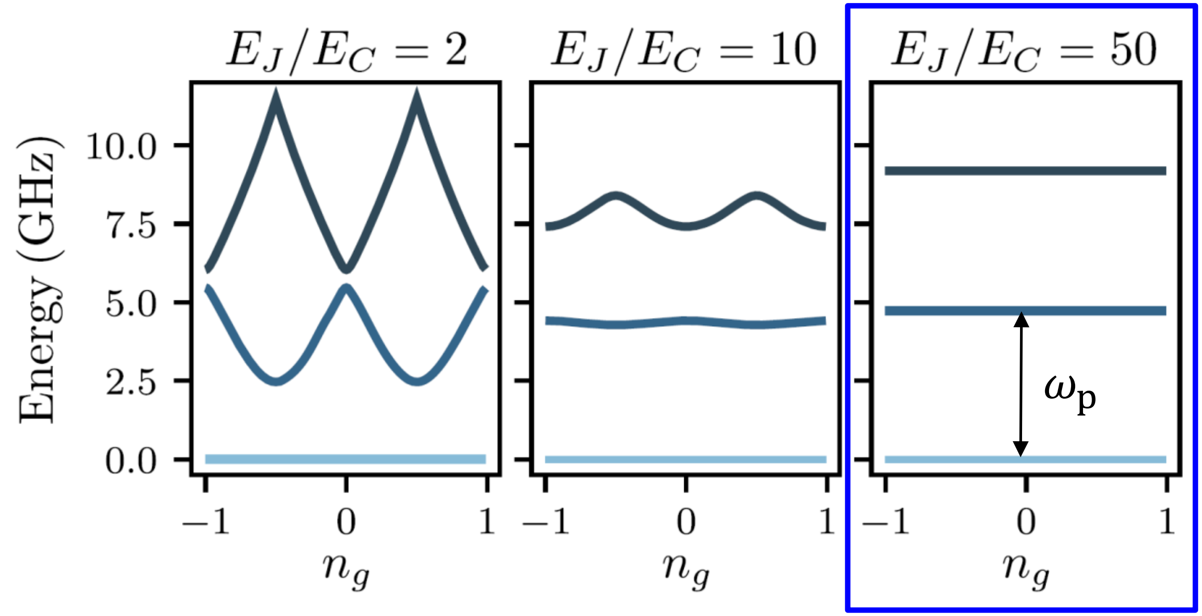
Charge qubit

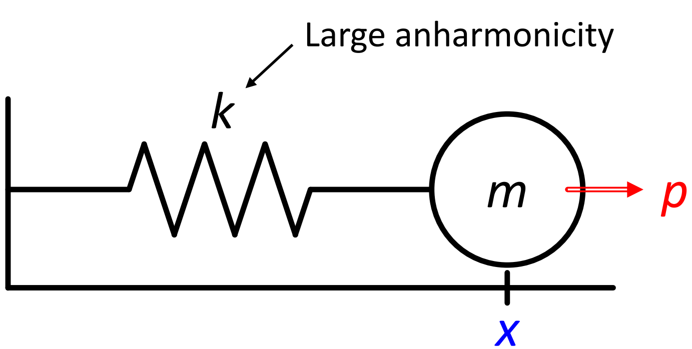
Transmon

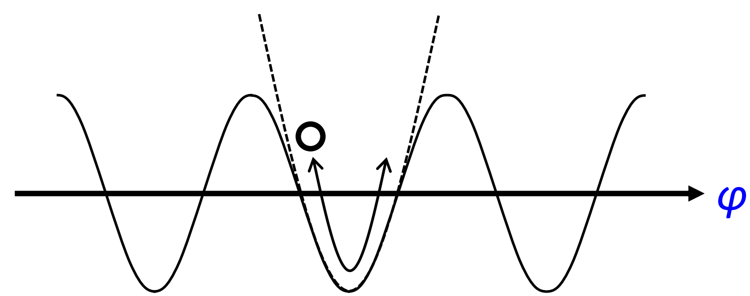
(Transmission-line shunted plasma oscillation qubit)



Rev. Mod. Phys. **93**, 025005 (2021) Blais *et al.*



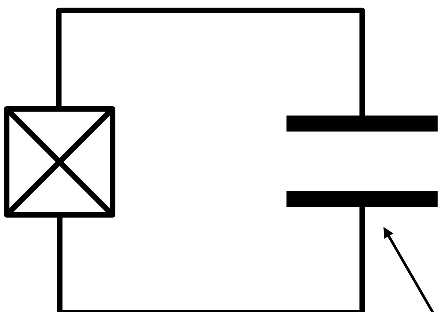




Charge qubit

Transmon

(Transmission-line shunted plasma oscillation qubit)



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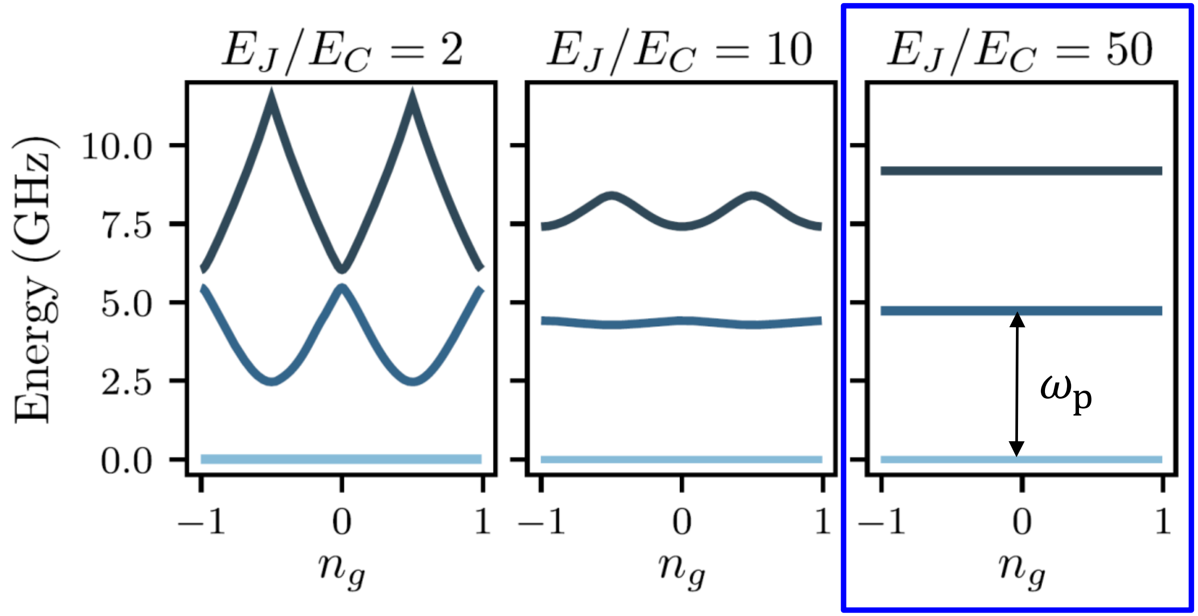
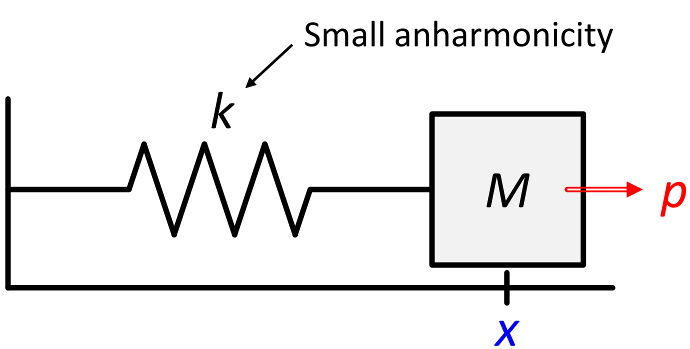
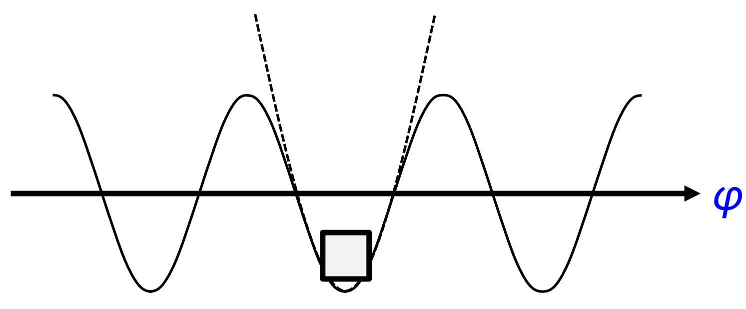




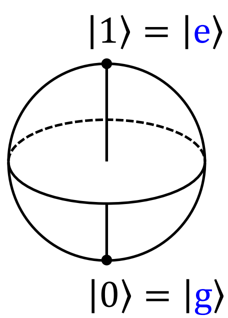


Strong coupling regime

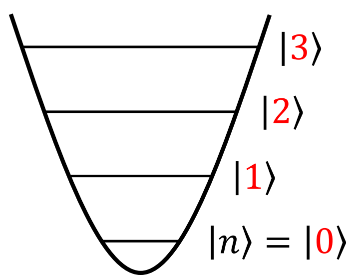
Jaynes–Cummings Hamiltonian

$$H_{\mathrm{JC}} = \omega_{\mathrm{q}} \frac{\sigma_z}{2} + \omega_{\mathrm{r}} a^{\dagger} a + g(\sigma_{+} a + \sigma_{-} a^{\dagger})$$

Qubit



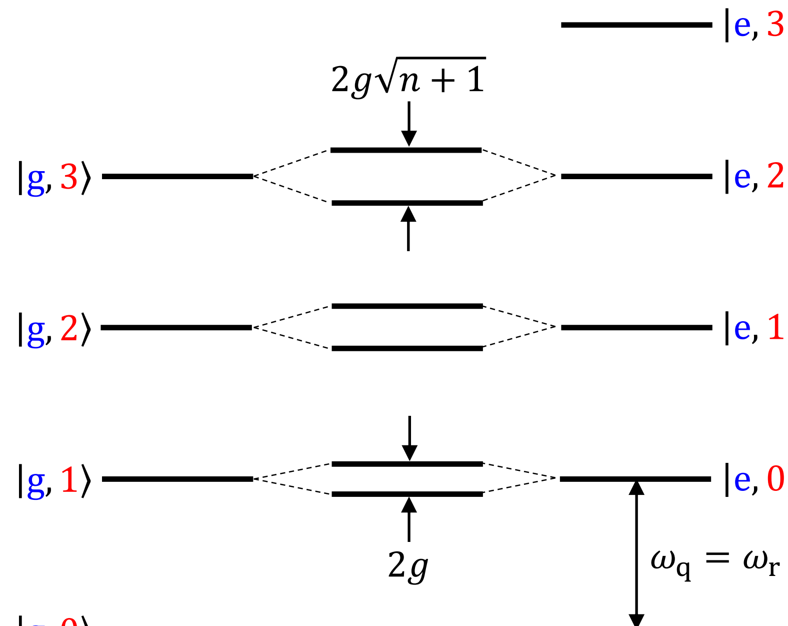
Cavity (Resonator)



Resonance: $\Delta \equiv \omega_{\mathrm{q}} - \omega_{\mathrm{r}} = 0$

$$\begin{cases} \omega_{n\pm,\Delta=0} = \omega_{\mathrm{r}}\left(n + \dfrac{1}{2}\right) \pm g\sqrt{n+1} \\ \left|\Psi_{n\pm,\Delta=0}\right\rangle = \dfrac{1}{\sqrt{2}}\left(|\mathrm{g}, n+1\rangle \pm |\mathrm{e}, n\rangle\right) \end{cases}$$

Vacuum Rabi splitting



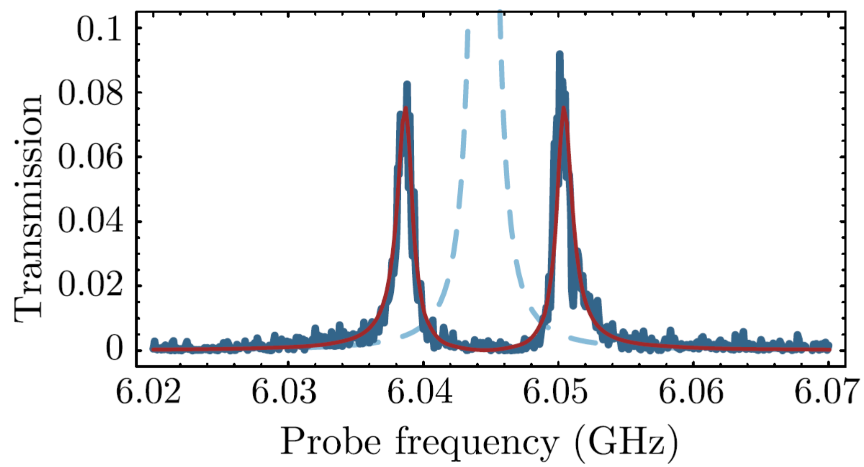
$g \gg \kappa, \gamma$ → Strong coupling

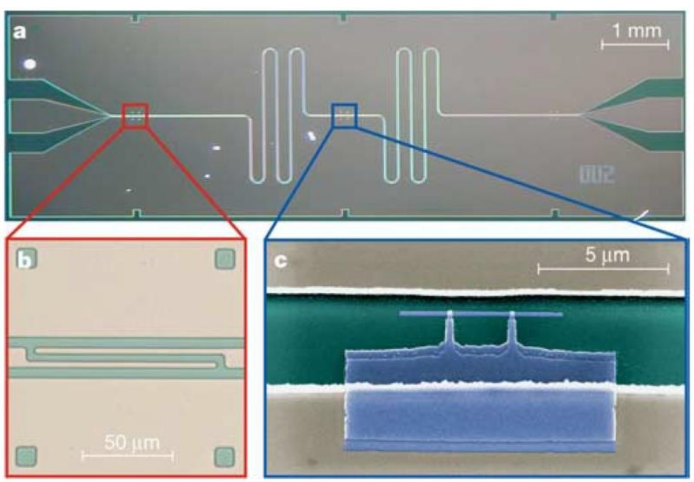
Observation of vacuum Rabi splitting

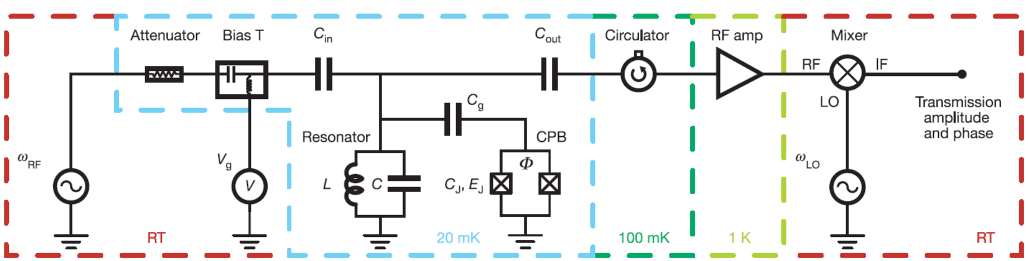
Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics

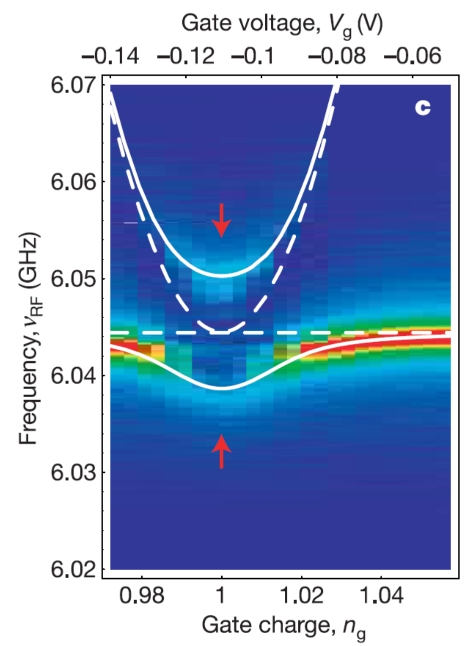
A. Wallraff[1], D. I. Schuster[1], A. Blais[1], L. Frunzio[1], R.- S. Huang[1,2], J. Majer[1], S. Kumar[1], S. M. Girvin[1] & R. J. Schoelkopf[1]

Nature **431**, 162 (2004) Wallraff *et al.*







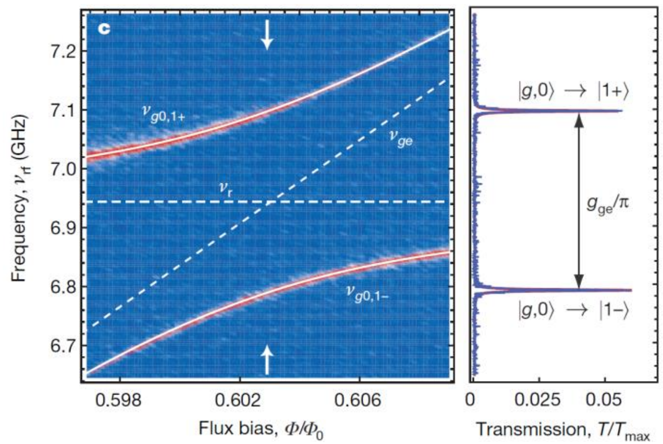


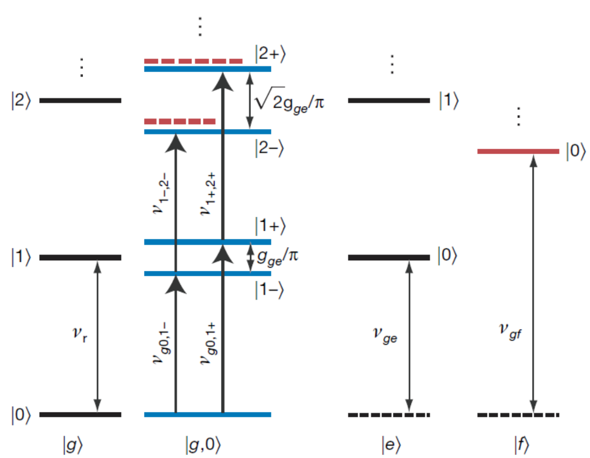
Observation of vacuum Rabi splitting

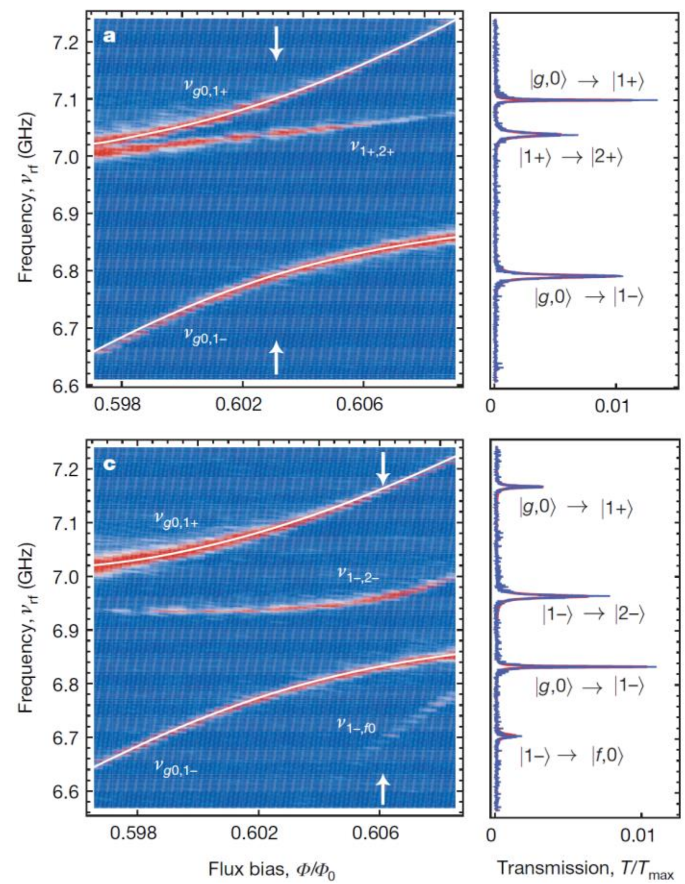
Climbing the Jaynes–Cummings ladder and observing its $\sqrt{n}$ nonlinearity in a cavity QED system

J. M. Fink[1], M. Göppl[1], M. Baur[1], R. Bianchetti[1], P. J. Leek[1], A. Blais[2] & A. Wallraff[1]

Nature **454**, 315 (2008) Fink *et al.*







Contents

Device parameters

	Fixed-frequency		Frequency-tunable	
Company/Institution	IBM[*1]	RIKEN[*3]	Google[*4]	ETH[*6]
Number of qubits on a chip	127	64	74	17
Qubit frequency (GHz)	5.06	7.88	5.98/5.97[*5]	3.95/4.73[*7]
Anharmonicity (GHz)	−0.307	−0.385	−0.265	−0.177
Resonator frequency (GHz)	6.51[*2]	9.44	4.79	6.98[*8]
T_1 (μs)	98.2	24.8	20.6	32.5
T_2 (μs)	93.6	32.2	30.9	47.0

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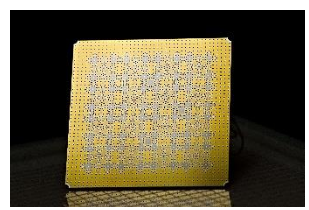

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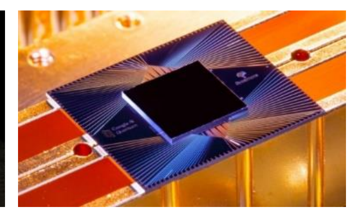

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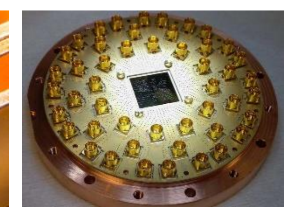

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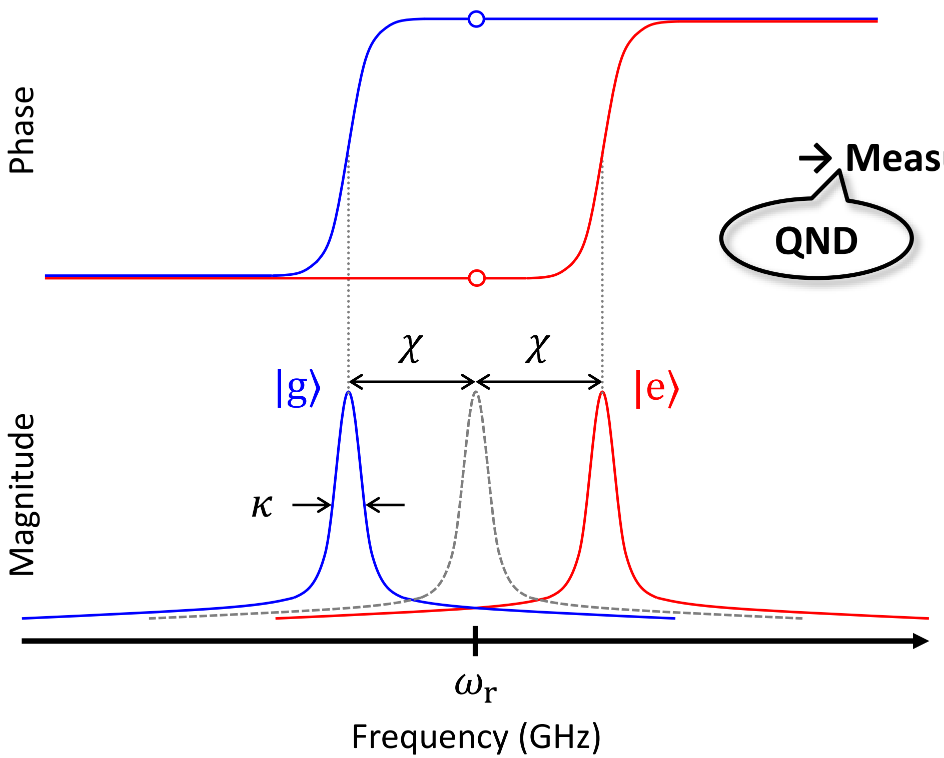
*1: https://quantum-computing.ibm.com/services/resources?system=ibm_washington (Avg. Calibrated regularly)
*2: Nature **567**, 209 (2019) Havlicek *et al.* (Avg. resonator freq. of a 5Q device)
*3: Avg. of 56–63Q (depending on the parameters)
*4: arXiv:2207.06431v2 Google Quantum AI (Avg. of 49Q)
*5: Operating freq./Freq. at readout
*6: Nature **605**, 669 (2022) Krinner *et al.* (Avg. of 17Q except qubit & readout frequencies)
*7: Idle freq./Freq. at readout. Avg. of 9Q (data qubit)
*8: Avg. of 9Q (data qubit)

Readout in the dispersive regime

$$H_{\mathrm{JC}} = \omega_{\mathrm{q}} \frac{\sigma_z}{2} + \omega_{\mathrm{r}} a^\dagger a + g(\sigma_+ a + \sigma_- a^\dagger)$$

$$\xrightarrow{|\Delta| = |\omega_{\mathrm{q}} - \omega_{\mathrm{r}}| \gg g, \kappa} \quad H_{\mathrm{JC}}^{\mathrm{disp}} = (\omega_{\mathrm{q}} + \chi)\frac{\sigma_z}{2} + (\omega_{\mathrm{r}} + \chi\sigma_z) a^\dagger a$$

$$\chi = \frac{g^2}{\Delta}$$



→ **Measurement of qubit state via cavity**

QND

e.g.

f_q = 8 GHz

f_r = 10 GHz

κ = 10 MHz (Q = 1000)

g = 100 MHz

2χ = 10 MHz

1Q rotation gate

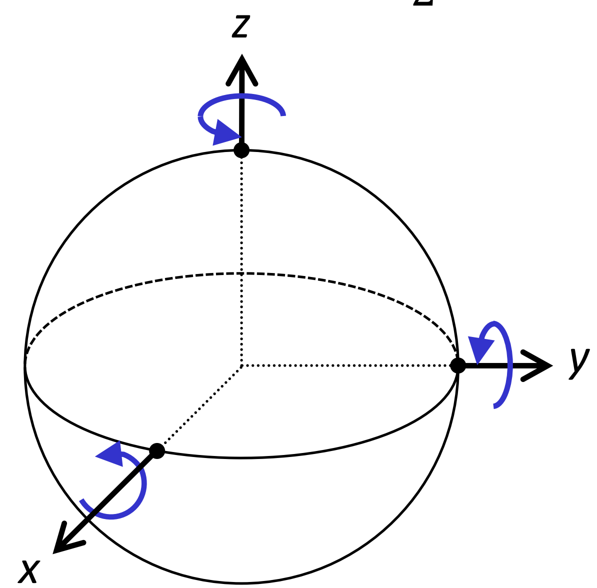
Hamiltonian of external fields (μwave pulse)

$$\Omega_{\mathrm{R}} = \frac{2E_d(t)g}{\Delta}$$

$$H_{\mathrm{d}}(t) = E_d(t)\left(a^\dagger e^{-i\omega_{\mathrm{d}}t} + a e^{i\omega_{\mathrm{d}}t}\right)$$

$$\longrightarrow H_{1\mathrm{q}}^{\mathrm{rot}} = (\omega_{\mathrm{r}} - \omega_{\mathrm{d}} + \chi\sigma_z)a^\dagger a + \left(\omega_{\mathrm{q}} - \omega_{\mathrm{d}} + \chi\right)\frac{\sigma_z}{2} + E_d(t)\left(a^\dagger + a\right) + \Omega_{\mathrm{R}}\frac{\sigma_x}{2}$$

$$R_x\left(\varphi = \frac{\Omega_{\mathrm{R}}\tau}{2}\right) = e^{-i\Omega_{\mathrm{R}}\frac{\sigma_x}{2}\tau} = \begin{pmatrix} \cos\left(\frac{\Omega_{\mathrm{R}}\tau}{2}\right) & -i\sin\left(\frac{\Omega_{\mathrm{R}}\tau}{2}\right) \\ -i\sin\left(\frac{\Omega_{\mathrm{R}}\tau}{2}\right) & \cos\left(\frac{\Omega_{\mathrm{R}}\tau}{2}\right) \end{pmatrix}$$



$$R_y(\varphi) = e^{-i\varphi\sigma_y} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$

$$R_z(\varphi) = e^{-i\varphi\sigma_z} = \begin{pmatrix} e^{-i\varphi} & 0 \\ 0 & e^{i\varphi} \end{pmatrix}$$

In experiments, the rotation axis is set by the LO phase

"Virtual" z-rotation is realized by shifting the LO phase

ZY decomposition

Arbitrary 1Q gates can be realized by a combination of *z* & *y* rotations

$$U = \begin{pmatrix} e^{i(\alpha-\beta/2-\delta/2)}\cos\frac{\gamma}{2} & -e^{i(\alpha-\beta/2+\delta/2)}\sin\frac{\gamma}{2} \\ e^{i(\alpha+\beta/2-\delta/2)}\sin\frac{\gamma}{2} & e^{i(\alpha+\beta/2+\delta/2)}\cos\frac{\gamma}{2} \end{pmatrix}$$

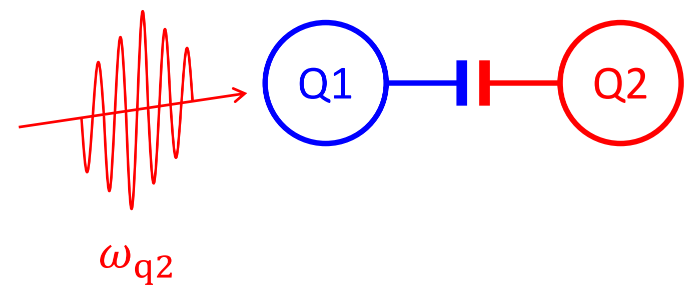
$$= e^{i\alpha}\begin{pmatrix} e^{-i\beta/2} & 0 \\ 0 & e^{i\beta/2} \end{pmatrix}\begin{pmatrix} \cos\frac{\gamma}{2} & -\sin\frac{\gamma}{2} \\ \sin\frac{\gamma}{2} & \cos\frac{\gamma}{2} \end{pmatrix}\begin{pmatrix} e^{-i\delta/2} & 0 \\ 0 & e^{i\delta/2} \end{pmatrix}$$

→ Decomposition is not unique

2Q cross-resonance (CR) gate

Hamiltonian of coupled 2Q system

$$H_{2q} = \underbrace{\omega_{q1}\frac{\sigma_z^1}{2} + \omega_{q2}\frac{\sigma_z^2}{2}}_{\mathrm{H_{qq}}} + \underbrace{J(\sigma_+^1\sigma_-^2 + \sigma_-^1\sigma_+^2)}_{H_J}$$



Rotating frame ($H_d^{\mathrm{rot,CR}}$)

$$H_d^{\mathrm{rot,CR}} = \omega_d\left(\frac{\sigma_z^1}{2} + \frac{\sigma_z^2}{2}\right)$$

$$\Delta_{qq} \equiv \omega_{q1} - \omega_{q2}$$

$$\Delta_{qq} \gg J$$

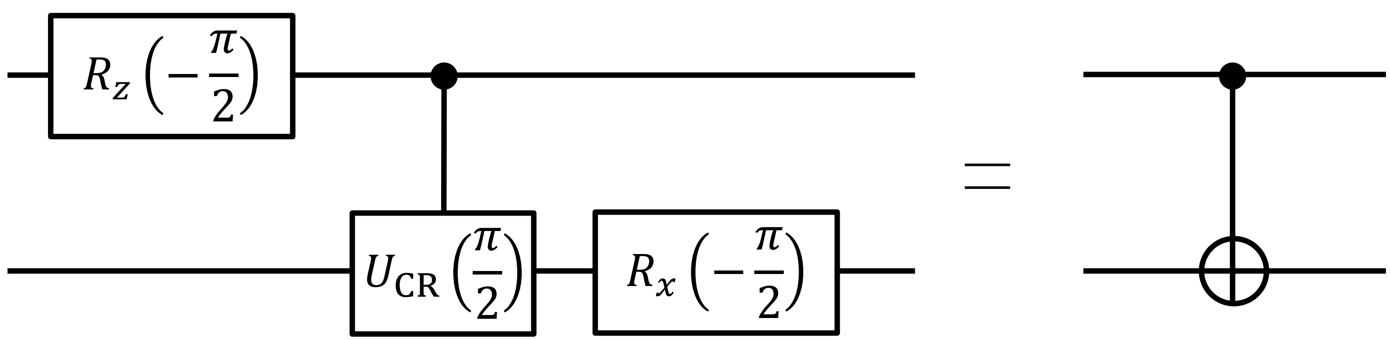
$$\Omega_{CR} = \frac{E_q(t)J}{\Delta_{qq}}$$

$$\longrightarrow H_{2q}^{\mathrm{rot}} = \left(\omega_{q1} - \omega_d + \frac{J^2}{\Delta_{qq}}\right)\frac{\sigma_z^1}{2} + \left(\omega_{q2} - \omega_d - \frac{J^2}{\Delta_{qq}}\right)\frac{\sigma_z^2}{2} + \underbrace{E_q(t)\sigma_x^1 + \Omega_{CR}\sigma_z^1\sigma_x^2}_{\text{Similar to 1Q gate}}$$

CNOT from CR

$$\omega_{\mathrm{d}} = \omega_{\mathrm{q2}} - \frac{J^2}{\Delta_{\mathrm{qq}}} \longrightarrow H_{\mathrm{d}}^{\mathrm{rot}} = \Omega_{\mathrm{CR}} \sigma_z^1 \sigma_x^2$$

$$\longrightarrow U_{\mathrm{CR}}(\theta = \Omega_{\mathrm{CR}}\tau) = e^{-iH_{\mathrm{d}}^{\mathrm{rot}}\tau} = \begin{pmatrix} \cos\theta & -i\sin\theta & 0 & 0 \\ -i\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & i\sin\theta \\ 0 & 0 & i\sin\theta & \cos\theta \end{pmatrix}$$



$$U_{\mathrm{CR}}\left(\frac{\pi}{2}\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i & 0 & 0 \\ -i & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & i & 1 \end{pmatrix}$$

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Contents

Quantum supremacy experiment by Google

Article

Quantum supremacy using a programmable superconducting processor

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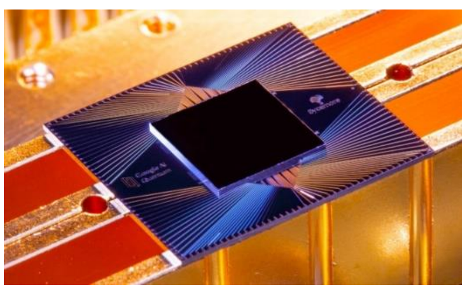
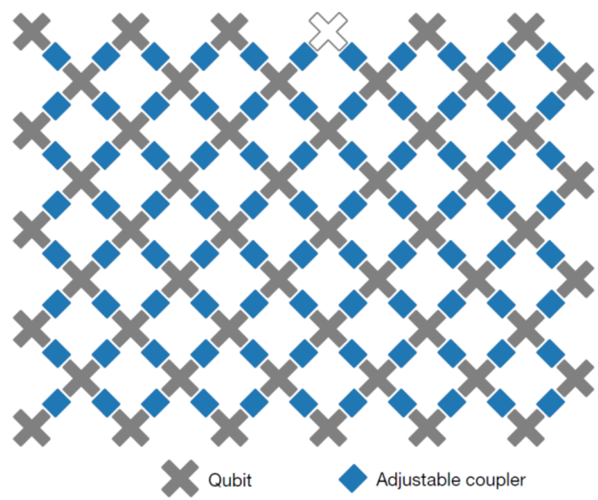
77 authors



John Martinis



53 qubits

Nature **574**, 505 (2019) Arute *et al.*

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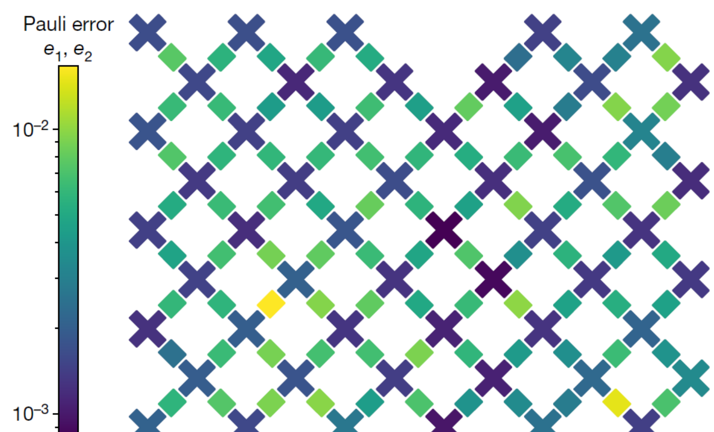
Pauli and measurement errors

Average error	Isolated	Simultaneous
Single-qubit (e_1)	0.15%	0.16%
Two-qubit (e_2)	0.36%	0.62%
Two-qubit, cycle (e_{2c})	0.65%	0.93%
Readout (e_r)	3.1%	3.8%



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53 qubits

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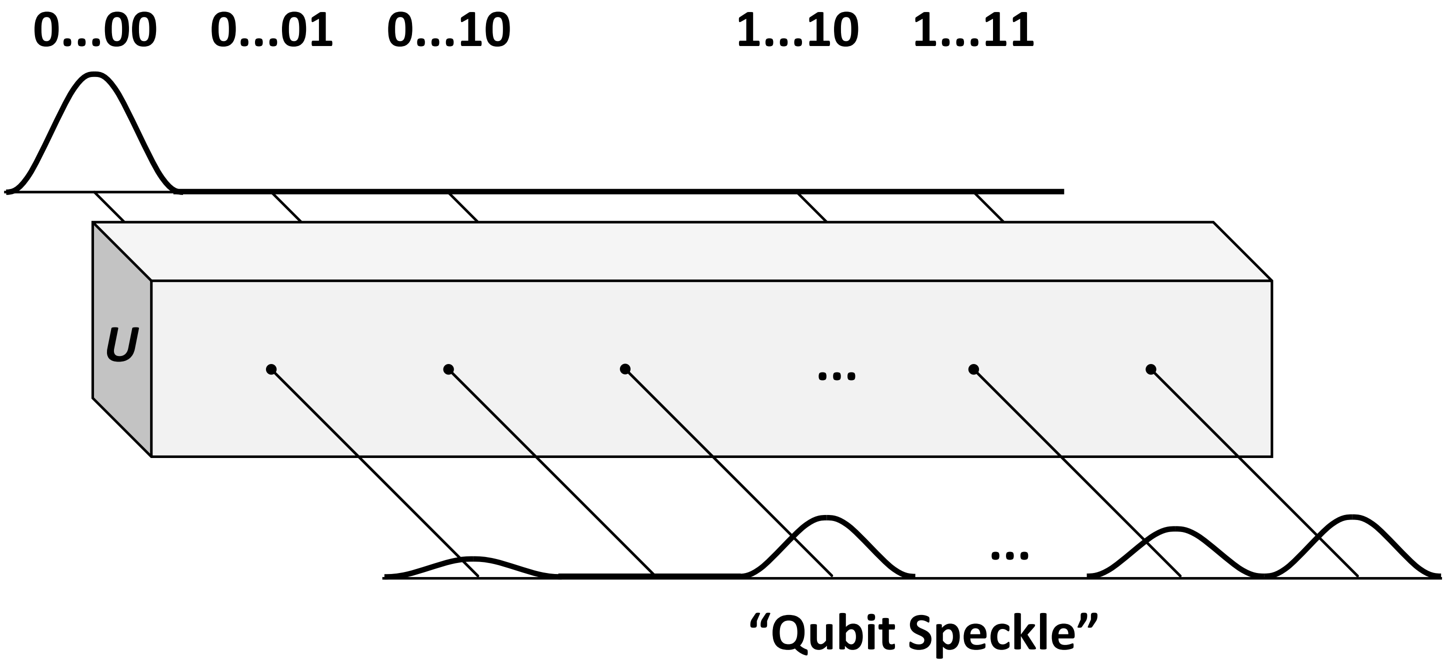
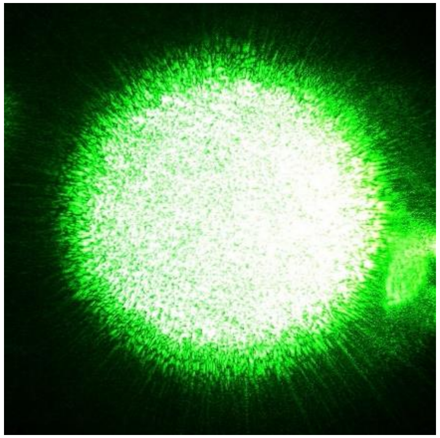


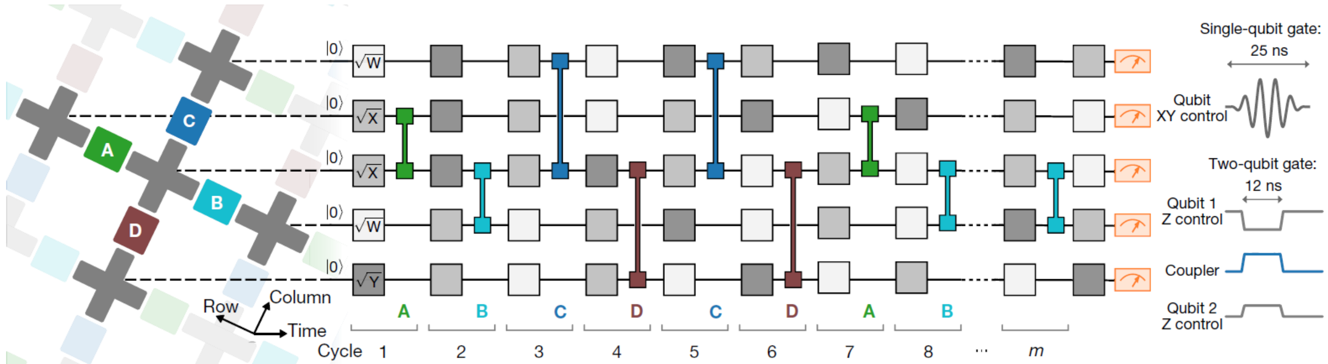
Laser speckle

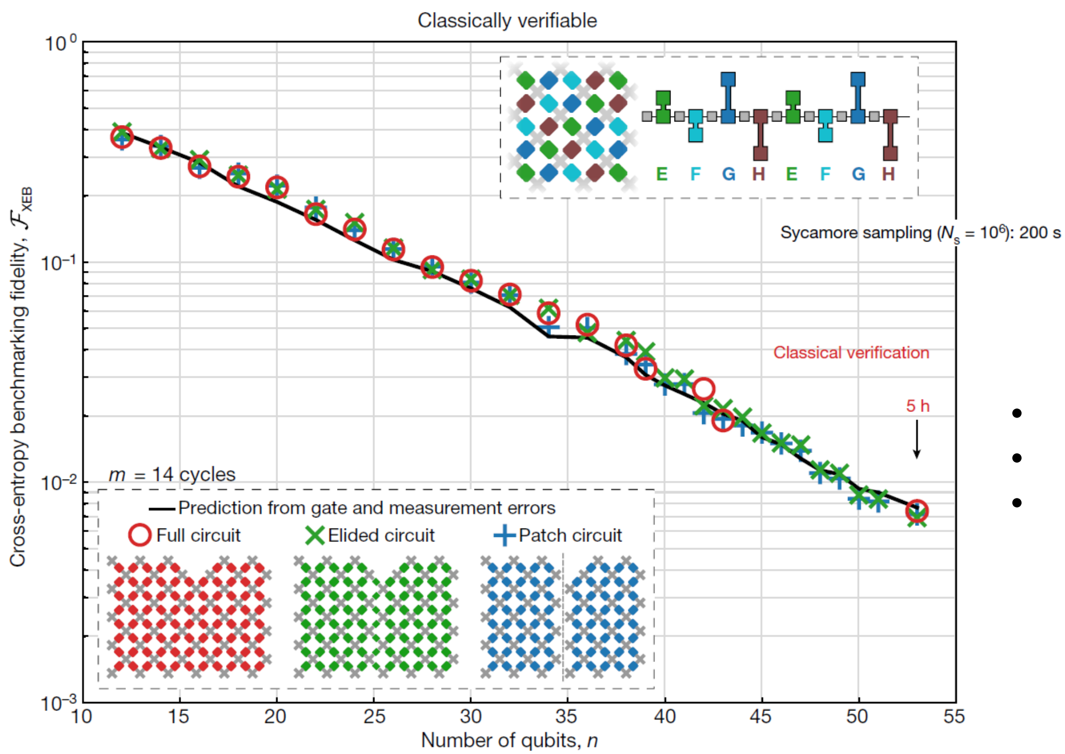


https://en.wikipedia.org/wiki/Speckle_(interference)

Successive application of random 1Q & 2Q gates results in a reproducible (if no errors) but complicated interference pattern that is, for sufficiently many qubits and gates, intractable by classical computers

Quantum supremacy experiment by Google



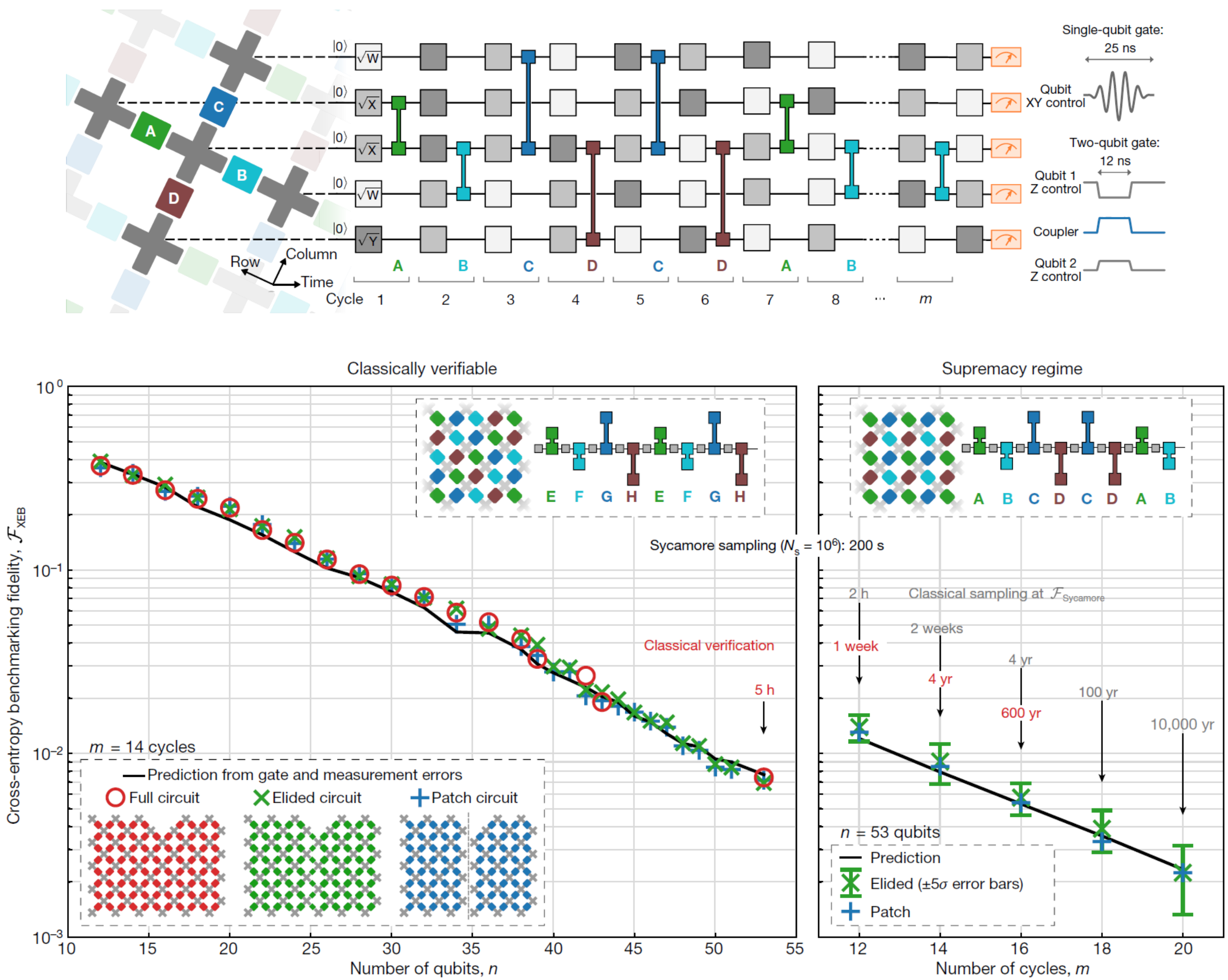


$$\mathcal{F}_{\mathrm{XEB}} = 2^n \langle P(x_i) \rangle_i - 1$$

- 1 → no error, 0 → any error
- $P(x_i)$ computed by classical computers
- Average over many trials

Nature **574**, 505 (2019) Arute *et al.*

Quantum supremacy experiment by Google



Nature **574**, 505 (2019) Arute *et al.*

The race goes on

Quantum vs. Classical, Quantum vs. Quantum

Google

Quantum Computer

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Science

Ordinary computers can beat Google's quantum computer after all

If the quantum computing era dawned 3 years ago, its rising sun may have ducked behind a cloud. In 2019, Google researchers claimed they had...

1 day ago

Solving the Sampling Problem of the Sycamore Quantum Circuits

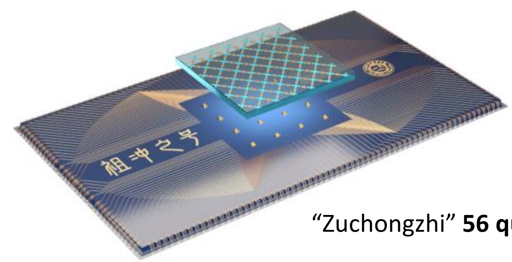
Feng Pan,[1,2] Keyang Chen,[1,3] and Pan Zhang[1,4,5,*]

"If our algorithm could be implemented with high efficiency on a modern supercomputer with ExaFLOPS performance, we estimate that ideally, the simulation would cost a few dozens of seconds, which is faster than Google's quantum hardware"

Phys. Rev. Lett. **129**, 090502 (2022) Pan *et al.*

Strong Quantum Computational Advantage Using a Superconducting Quantum Processor

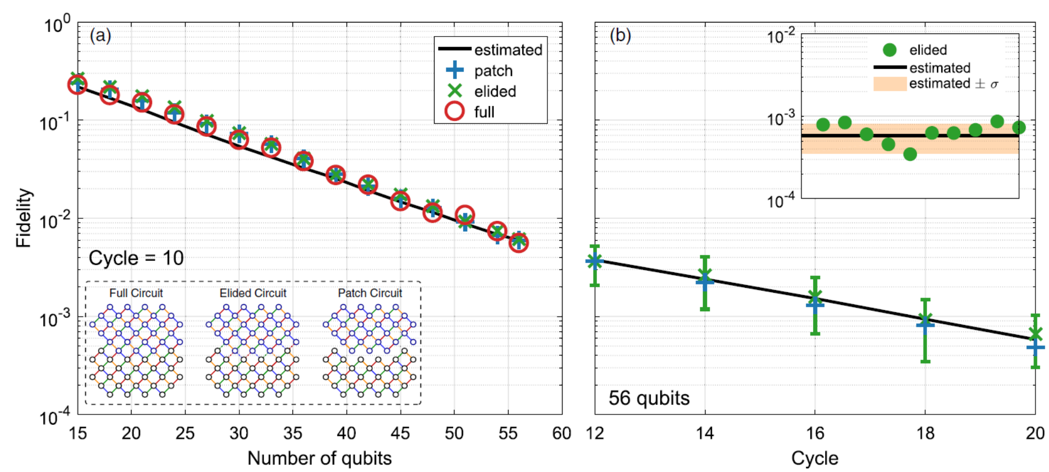
Yulin Wu,[1,2,3] Wan-Su Bao,[4] Sirui Cao,[1,2,3] Fusheng Chen,[1,2,3] Ming-Cheng Chen,[1,2,3] Xiawei Chen,[2] Tung-Hsun Chung,[1,2,3] Hui Deng,[1,2,3] Yajie Du,[2] Daojin Fan,[1,2,3] Ming Gong,[1,2,3] Cheng Guo,[1,2,3] Chu Guo,[1,2,3] Shaojun Guo,[1,2,3] Lianchen Han,[1,2,3] Linyin Hong,[5] He-Liang Huang,[1,2,3,4] Yong-Heng Huo,[1,2,3] Liping Li,[2] Na Li,[1,2,3] Shaowei Li,[1,2,3] Yuan Li,[1,2,3] Futian Liang,[1,2,3] Chun Lin,[6] Jin Lin,[1,2,3] Haoran Qian,[1,2,3] Dan Qiao,[2] Hao Rong,[1,2,3] Hong Su,[1,2,3] Lihua Sun,[1,2,3] Liangyuan Wang,[2] Shiyu Wang,[1,2,3] Dachao Wu,[1,2,3] Yu Xu,[1,2,3] Kai Yan,[2] Weifeng Yang,[5] Yang Yang,[2] Yangsen Ye,[1,2,3] Jianghan Yin,[2] Chong Ying,[1,2,3] Jiale Yu,[1,2,3] Chen Zha,[1,2,3] Cha Zhang,[1,2,3] Haibin Zhang,[2] Kaili Zhang,[1,2,3] Yiming Zhang,[1,2,3] Han Zhao,[2] Youwei Zhao,[1,2,3] Liang Zhou,[5] Qingling Zhu,[1,2,3] Chao-Yang Lu,[1,2,3] Cheng-Zhi Peng,[1,2,3] Xiaobo Zhu,[1,2,3] and Jian-Wei Pan[1,2,3]



"Zuchongzhi" **56 qubits**

Phys. Rev. Lett. **127**, 180501 (2021) Wu *et al.* (**54 authors**)

"The computational cost of the classical simulation of this task is estimated to be 2–3 orders of magnitude higher than the previous work on 53-qubit Sycamore processor"



Error correction experiment by ETH

Article

Realizing repeated quantum error correction in a distance-three surface code

9 (= d^2) data qubits, 8 (= d^2-1) auxiliary qubits, 1 (= $\lfloor(d-1)/2\rfloor$) correctable error

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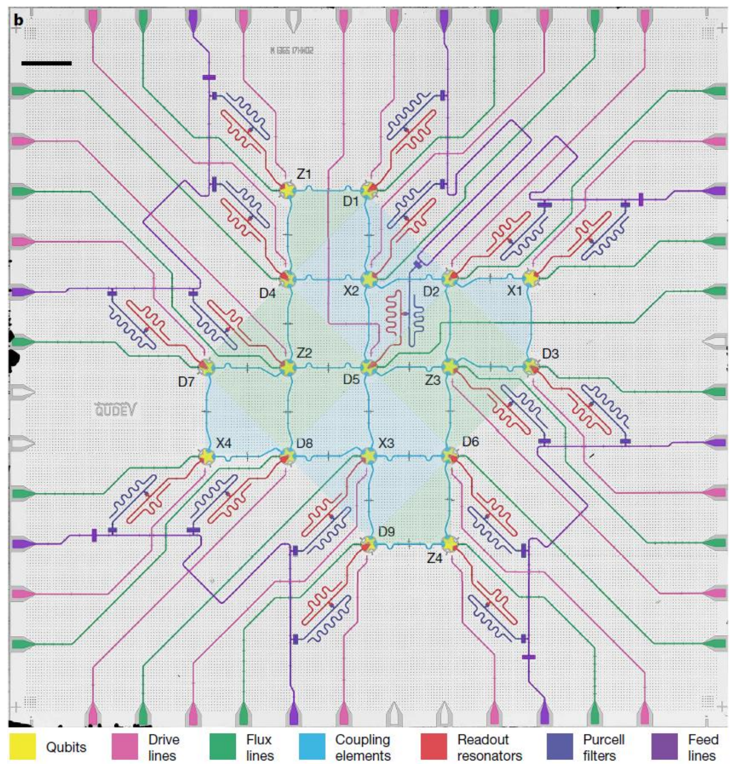
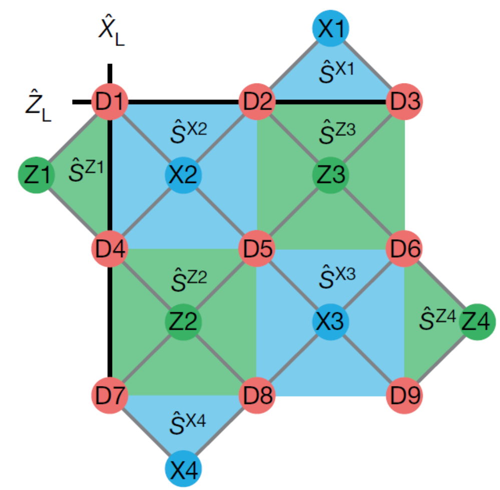
Andreas Wallraff

https://qudev.phys.ethz.ch/Andreas-Wallraff





Nature **605**, 669 (2022) Krinner *et al.*

See also: Phys. Rev. Lett. **129**, 030501 (2022) Zhao *et al.*; arXiv:2203.07205 Sundaresan *et al.*; arXiv:2207.06431v2 Google Quantum AI

Error correction experiment by ETH

Z & X stabilizers (Error syndrome)

$$S^{Z1} = Z_1 Z_4$$

$$S^{Z2} = Z_4 Z_5 Z_7 Z_8$$

$$S^{Z3} = Z_2 Z_3 Z_5 Z_6$$

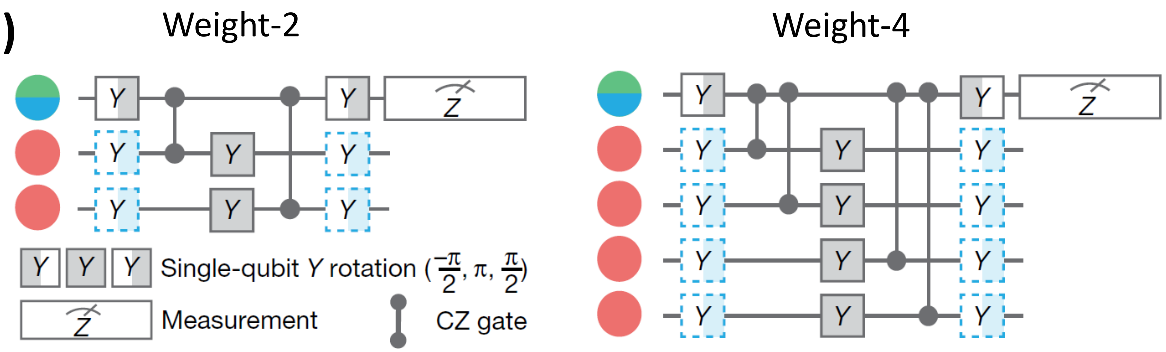
$$S^{Z4} = Z_6 Z_9$$

$$S^{X1} = X_2 X_3$$

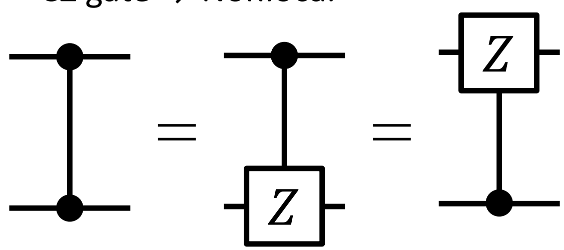
$$S^{X2} = X_1 X_2 X_4 X_5$$

$$S^{X3} = X_5 X_6 X_8 X_9$$

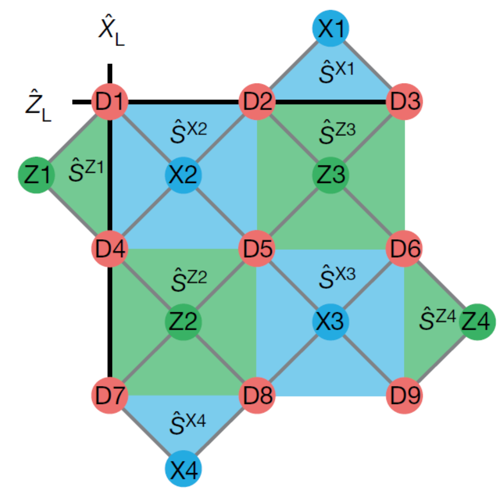
$$S^{X4} = X_7 X_8$$



CZ gate → Nonlocal



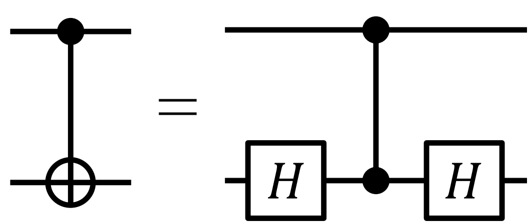
$$\text{CZ} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



Logical operators

$$Z_L = Z_1 Z_2 Z_3$$

$$X_L = X_1 X_4 X_7$$



$$X = HZH$$

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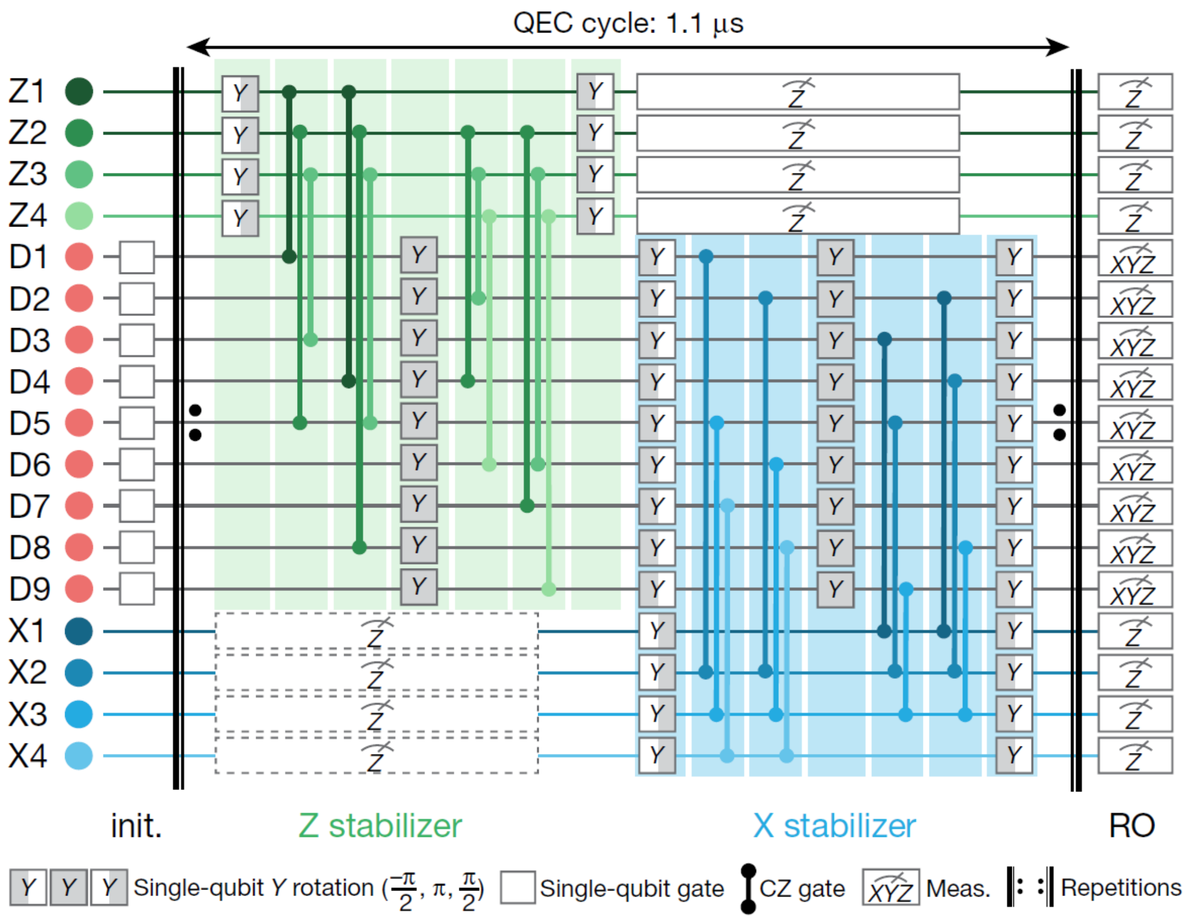
Error correction experiment by ETH



- Logical state $|0\rangle_L, |1\rangle_L, |\pm\rangle_L = (|0\rangle_L \pm |1\rangle_L)/\sqrt{2}$ preparation
  → Prepare $|0\rangle^{\otimes 9}, X_L|0\rangle^{\otimes 9}, |+\rangle^{\otimes 9}, Z_L|+\rangle^{\otimes 9}$ & run the QEC cycle once (w/o meas.)

- "Leakage" detection/rejection in every cycle

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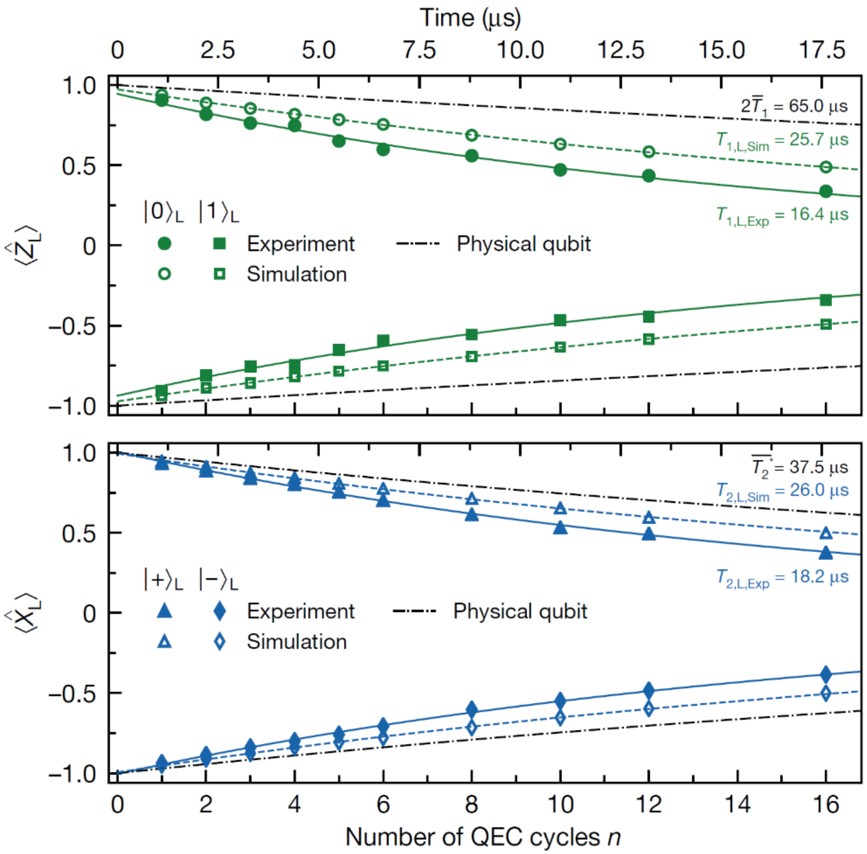
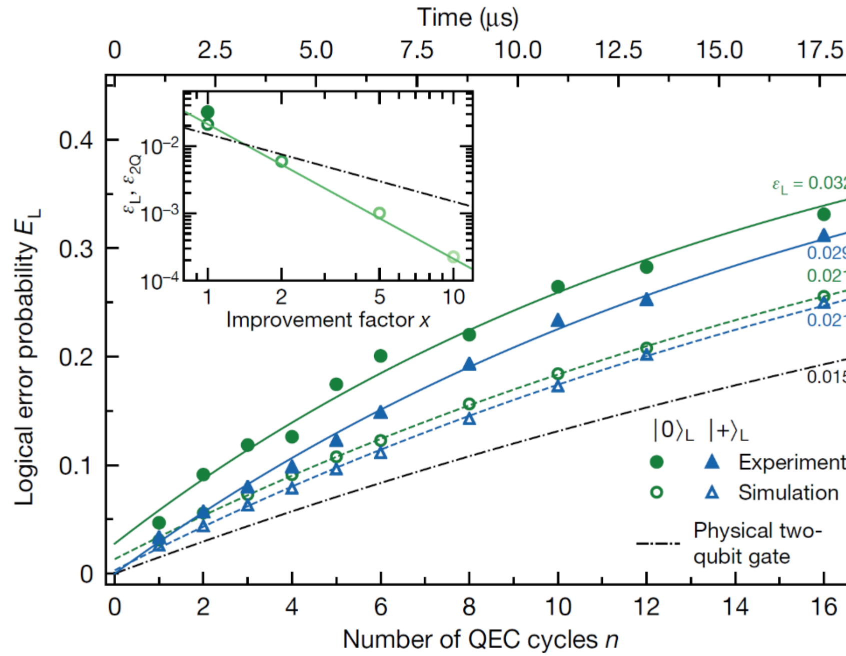
Error correction experiment by ETH





- Error correction in postprocessing
- With $x \approx 2$, “break-even” may be achieved

Nature **605**, 669 (2022) Krinner *et al.*

Summary and references

- **Quantum computation and quantum error correction**
  - The key ingredients of QC are quantum parallelism and quantum interference, which are both susceptible to noise
  - QEC protects quantum states by creating larger quantum states, and detects errors via parity measurements without destroying them

- **Superconducting qubits**
  - Circuit QED offers a scalable approach to quantum computing in the microwave domain, as recently demonstrated by various research groups & companies worldwide
  - My symposium talk

- ***"Quantum Computation and Quantum Information"***
  - Michael A. Nielsen & Isaac L. Chuang (Cambridge University Press, 2000)

- ***"A quantum engineer's guide to superconducting qubits"***
  - Appl. Phys. Rev. **6**, 021318 (2019) Krantz *et al.*