超伝導量子ビット

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2021年12月22日

参考文献

- Appl. Phys. Rev. **6**, 021318 (2019) P. Krantz, M. Kjaergaard, F. Yan, T. P. Orlando, S. Gustavsson, and W. D. Oliver
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- Annu. Rev. Condens. Matter Phys. **11**, 369 (2020) M. Kjaergaard, M. E. Schwartz, J. Braumüller, P. Krantz, J. I.-J. Wang, S. Gustavsson and W. D. Oliver
  - “**Superconducting Qubits: Current State of Play**”
- J. Appl. Phys. **129**, 041102 (2021) S. Kwon, A. Tomonaga, G. L. Bhai, S. J. Devitt and J.-S. Tsai
  - “**Gate-based superconducting quantum computing**”
- Rev. Mod. Phys. **93**, 025005 (2021) A. Blais, A. L. Grimsmo, S. M. Girvin and A. Wallraff
  - “**Circuit quantum electrodynamics**”
- PRX Quantum **2**, 040202 (2021) Y. Y. Gao, M. A. Rol, S. Touzard and C. Wang
  - “**Practical Guide for Building Superconducting Quantum Devices**”

共振器量子電磁力学

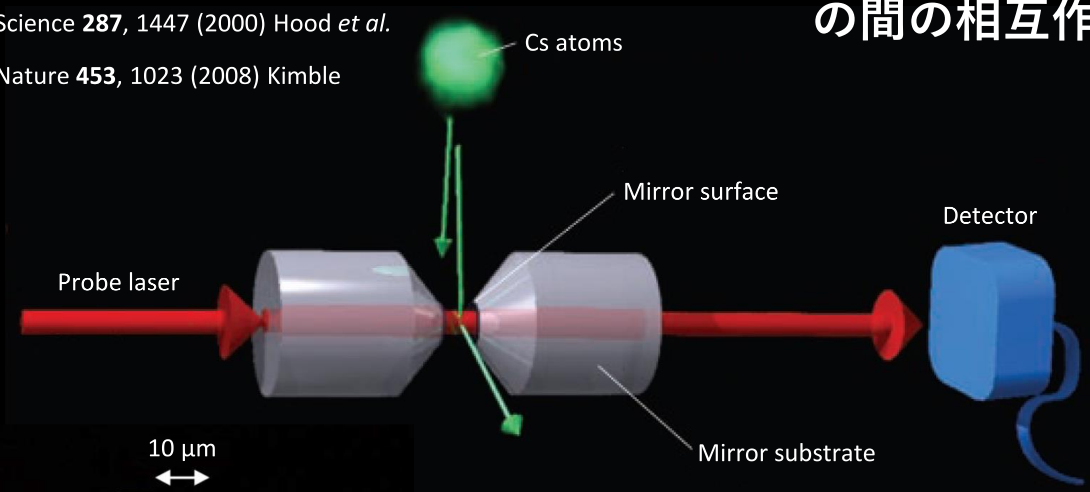
(Cavity Quantum ElectroDynamics, CQED)

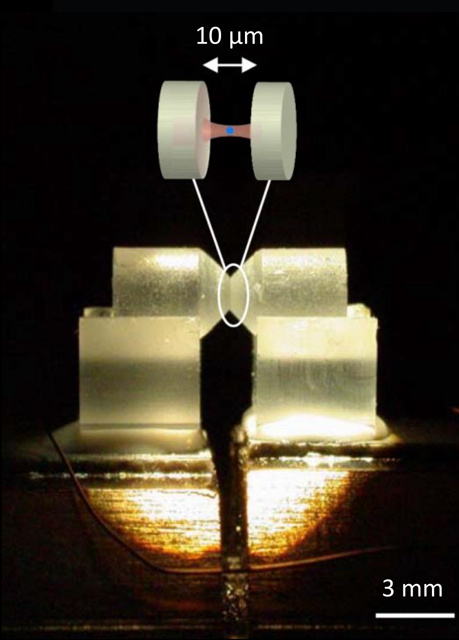
原子と共振器中に閉じ込めた光子の間の相互作用を調べる

Optical cavity: Kimble group (Caltech)

Science **287**, 1447 (2000) Hood *et al.*

Nature **453**, 1023 (2008) Kimble





Serge Haroche
(1944–)

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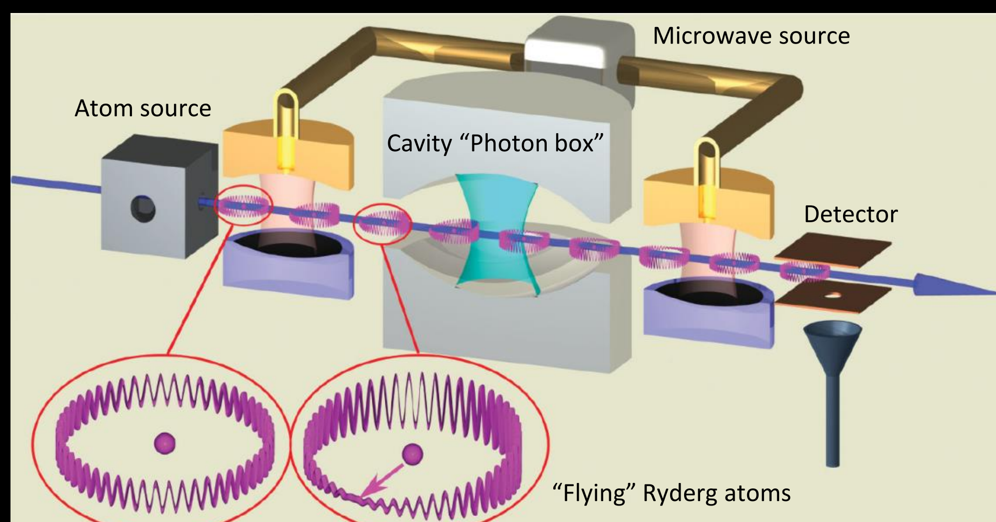
Microwave cavity: Haroche group (ENS)

Rev. Mod. Phys. **85**, 1083 (2013) Haroche



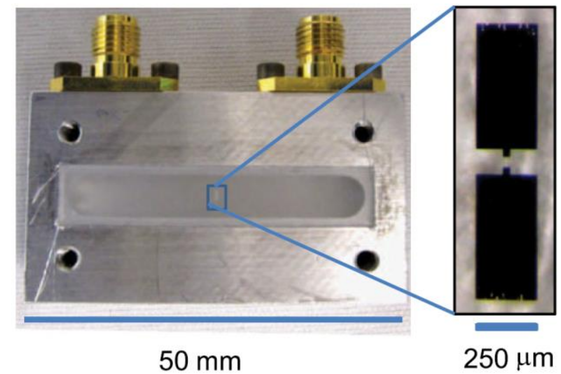
回路量子電磁力学

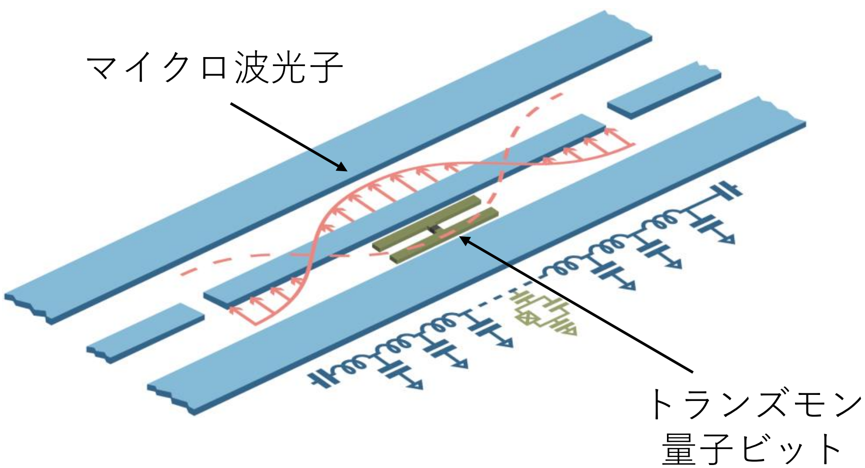
(Circuit Quantum ElectroDynamics, CQED)

超伝導量子回路に基づく人工原子–マイクロ波光子相互作用の制御

- ✓ 系の安定性(人工原子は動かない)
- ✓ 設計自由度の高さ
- ✓ サイズ(〜cm)と集積性

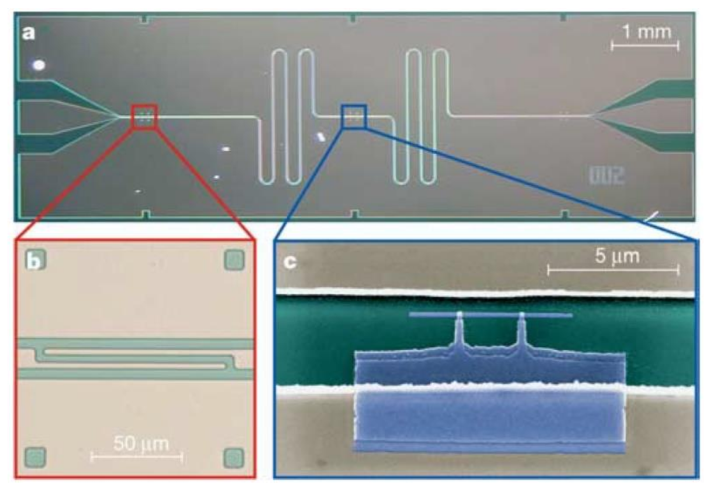
Phys. Rev. Lett. **107**, 240501 (2011) Paik *et al.*





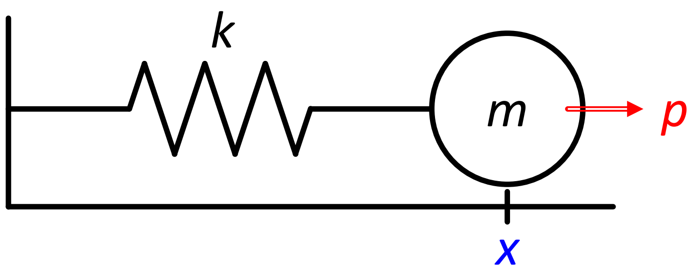
Rev. Mod. Phys. **93**, 025005 (2021) Blais *et al.*

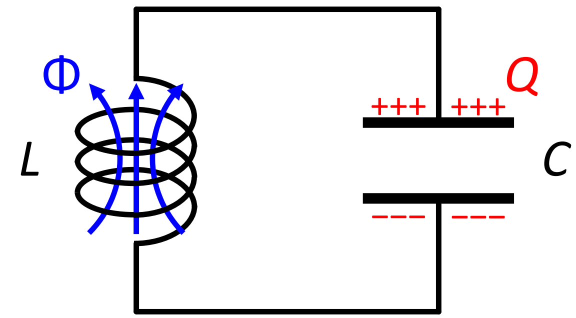
Phys. Rev. A **69**, 062320 (2004) Blais *et al.*



Nature **431**, 162 (2004) Wallraff *et al.*

調和振動子とLC回路





$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

ハミルトニアン

$$H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$$[\hat{x}, \hat{p}] = i\hbar$$

交換関係(量子化)

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

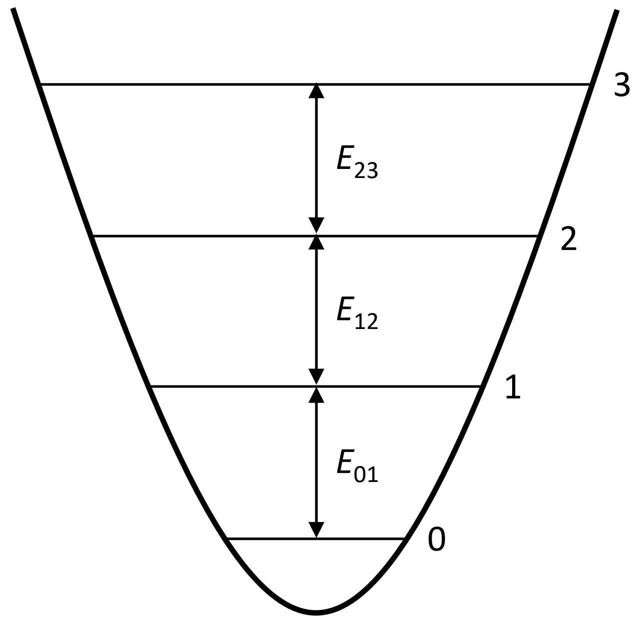
離散エネルギー準位

$$\omega = \sqrt{\frac{k}{m}}$$

$$E_n = \hbar\omega\left(\frac{1}{2} + n\right)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

調和振動子 = 量子ビット?

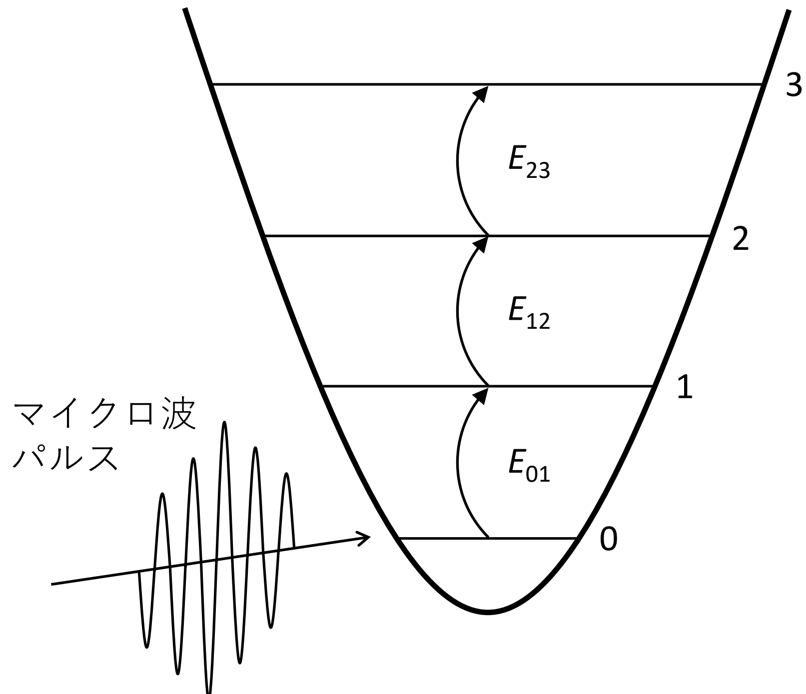


$E_{01} = E_{12} = E_{23}$

離散エネルギー準位

$$E_n = \hbar\omega\left(\frac{1}{2} + n\right)$$

調和振動子 = 量子ビット?



$E_{01} = E_{12} = E_{23}$

離散エネルギー準位

$$E_n = \hbar\omega\left(\frac{1}{2} + n\right)$$

光子場(ボソン)

$$\left[a, a^\dagger\right] = 1$$

$a^\dagger$: 生成演算子

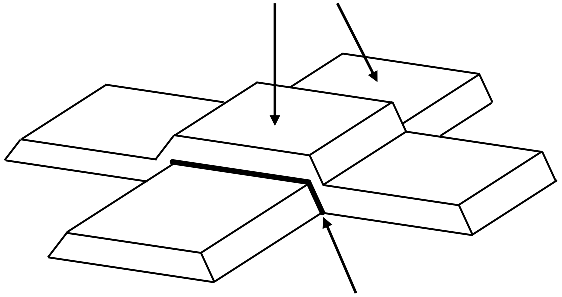
a: 消滅演算子

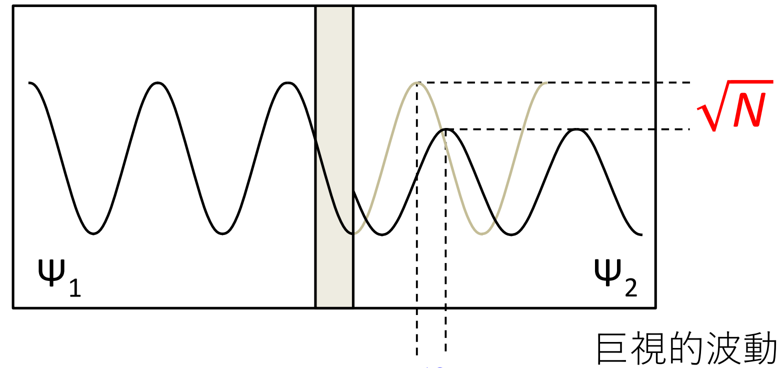
すべての遷移に共鳴
(2準位への選択性無し)

⇩

非調和性の導入

ジョセフソン接合





Brian Josephson
(1940–)

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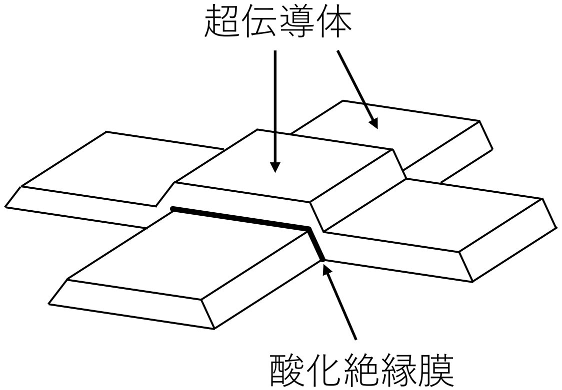
ジョセフソン方程式

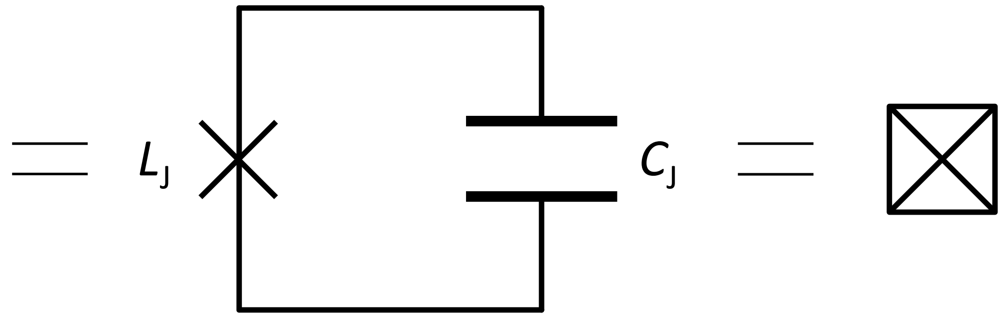
$$\begin{cases} V = -\dfrac{\hbar}{2e}\dfrac{d\varphi}{dt} \\ I = I_c \sin\varphi \end{cases} \longrightarrow V = -\underbrace{\frac{\hbar}{2eI_c}\frac{1}{\sqrt{1-(I/I_c)^2}}}_{L_J}\frac{dI}{dt}$$

(唯一の)非線形非散逸インダクタ

$$\longrightarrow U = -\int IV dt = \int \left(\frac{\hbar I_c}{2e}\right)\sin\varphi \frac{d\varphi}{dt}dt = -E_J\cos\varphi$$

ジョセフソン接合





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(1940–)

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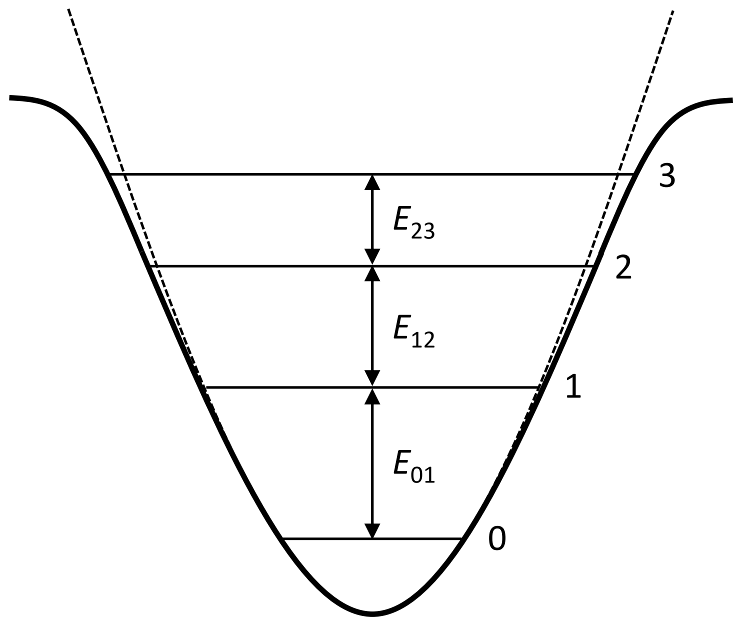
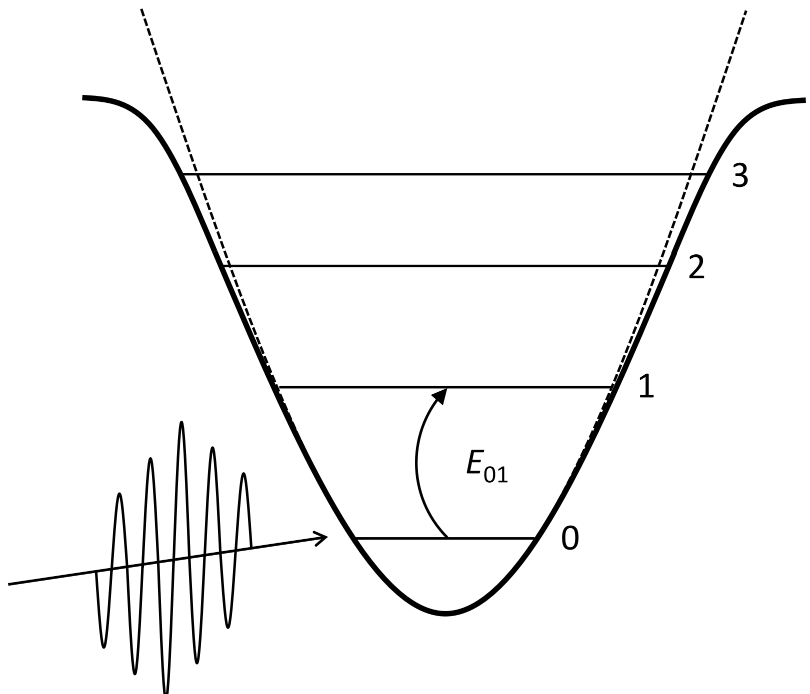
$$\longrightarrow U = -\int IV dt = \int \left(\frac{\hbar I_\mathrm{c}}{2e}\right)\sin\varphi\frac{d\varphi}{dt}dt = -E_\mathrm{J}\cos\varphi$$

非線形調和振動子 = 量子ビット



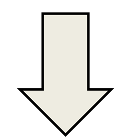
$E_{01} > E_{12} > E_{23}$

非線形調和振動子 = 量子ビット



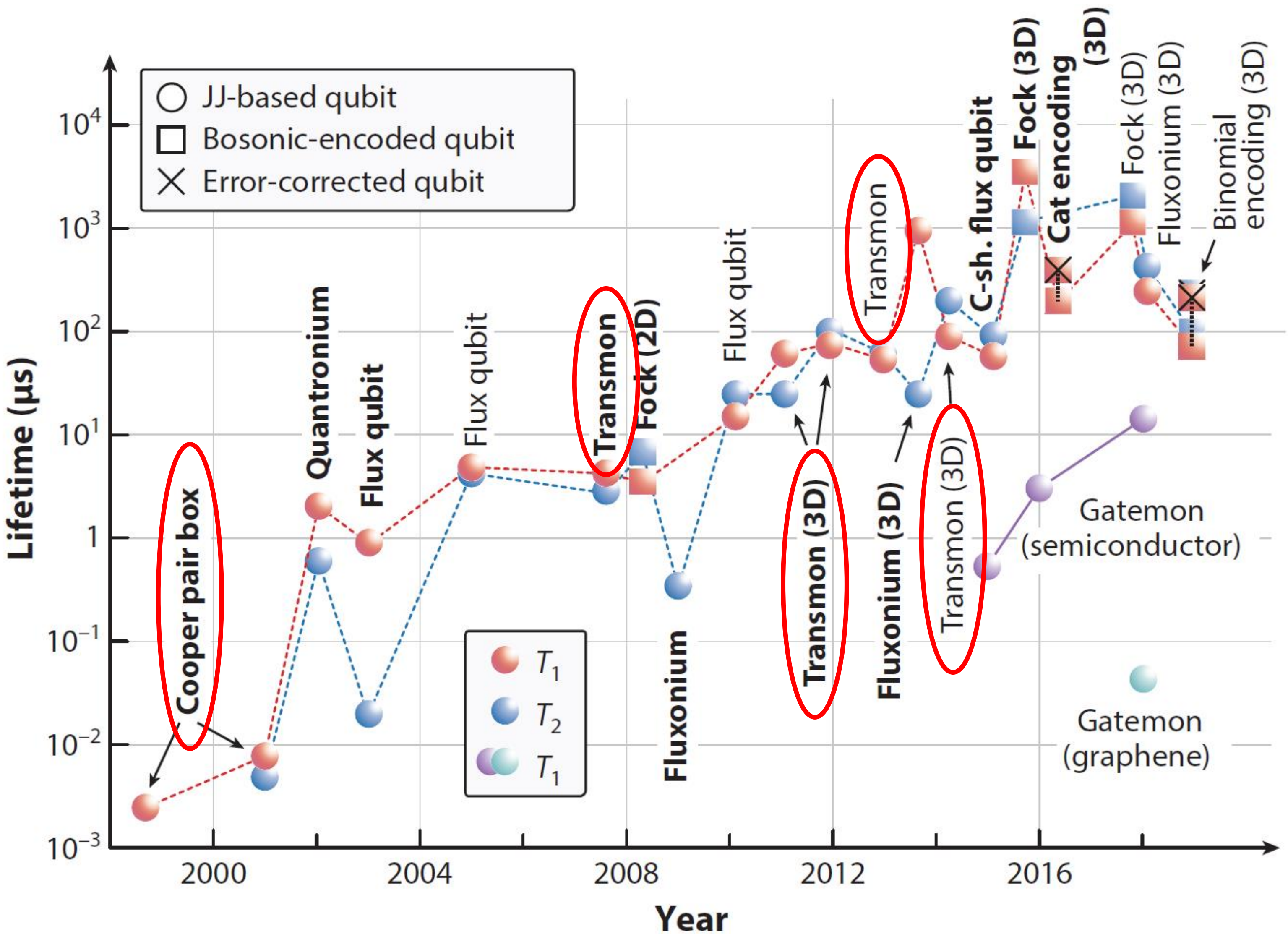
$E_{01} > E_{12} > E_{23}$

2準位を抜き出す



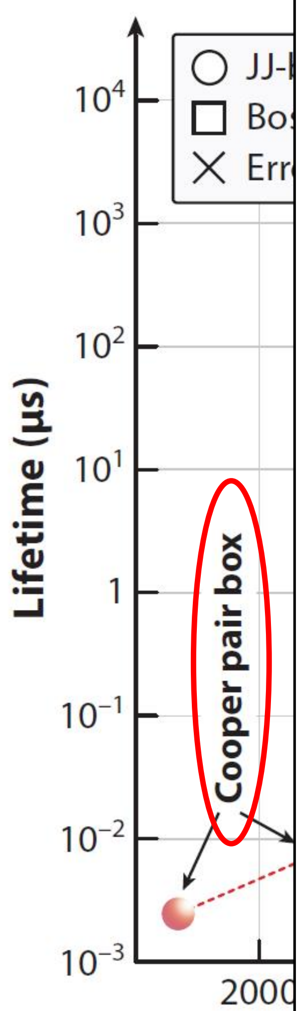
$$[\sigma_x, \sigma_y] = 2i\sigma_z$$

T_1, T_2の変遷

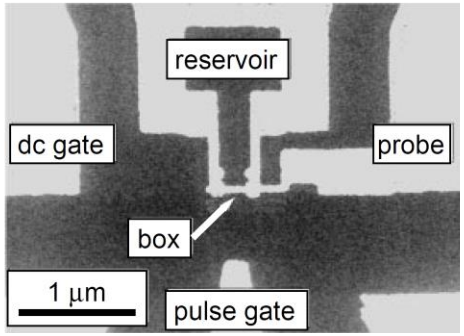


Annu. Rev. Condens. Matter Phys. **11**, 369 (2020) Kjaergaard *et al.*

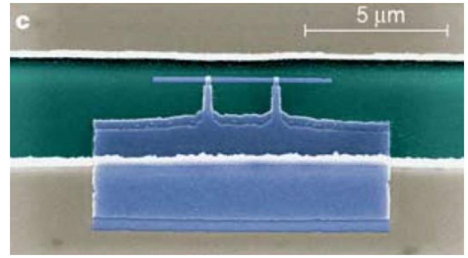
T_1, T_2の変遷



電荷量子ビット: 大型化 → 高コヒーレンス



(CPB) Nature **398**, 786 (1999) Nakamura *et al.*



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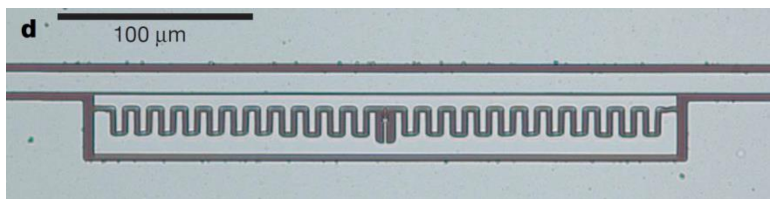
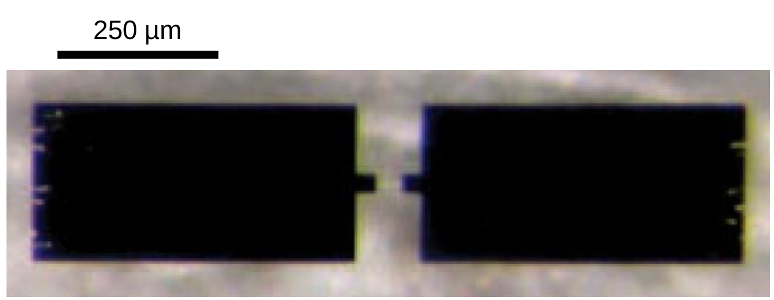
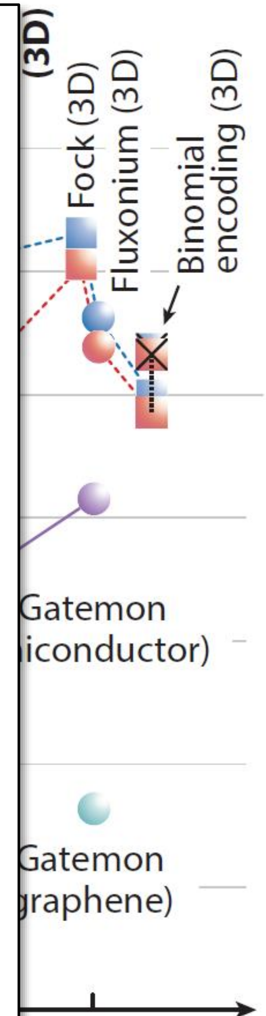


(2D Transmon) Nature **449**, 328 (2007) Houck *et al.*



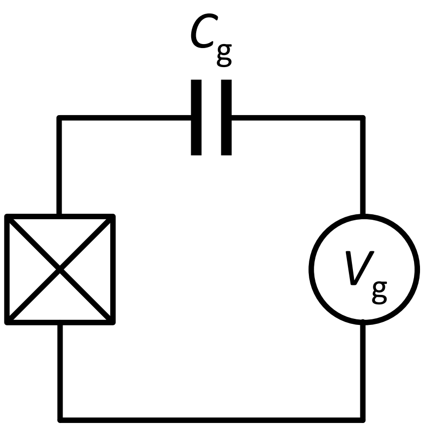
(3D Transmon) Phys. Rev. Lett. **107**, 240501 (2011) Paik *et al.*



Annu. Rev. Condens. Matter Phys. **11**, 369 (2020) Kjaergaard *et al.*

電荷量子ビット

クーパー対箱



ハミルトニアン

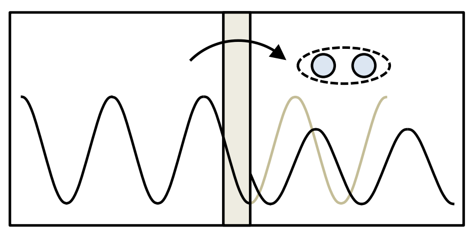
$$H = 4E_C\left(N - n_g\right)^2 - E_J \cos\varphi$$

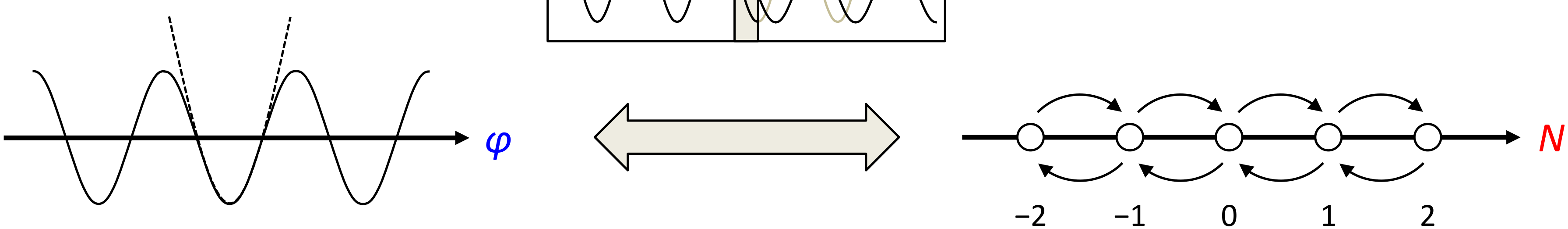
帯電エネルギー $\dfrac{e^2}{2C_\Sigma}$

電荷オフセット

$$[\hat{\varphi}, \hat{N}] = i\hbar$$

$$\frac{1}{2}\sum_{N=-\infty}^{\infty}\left(|N+1\rangle\langle N| + |N\rangle\langle N+1|\right)$$

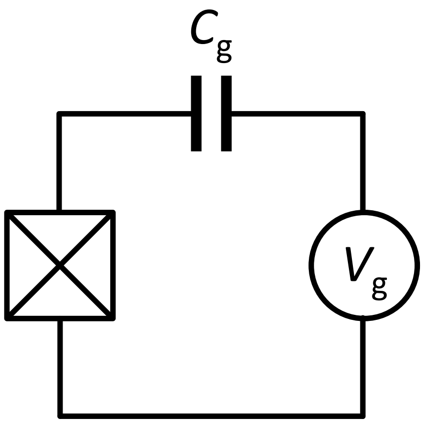




$$|\varphi\rangle = \frac{1}{\sqrt{2\pi}}\sum_{N=-\infty}^{\infty} e^{i\varphi N}|N\rangle$$

電荷量子ビット

クーパー対箱



クーパー対トンネル = 強束縛模型(1D)

$$\cos\varphi\,|\varphi\rangle$$

$$= \left(\frac{e^{-i\varphi}+e^{i\varphi}}{2}\right)\frac{1}{\sqrt{2\pi}}\sum_{N=-\infty}^{\infty} e^{i\varphi N}|N\rangle$$

$$= \frac{1}{2\sqrt{2\pi}}\sum_{N=-\infty}^{\infty} e^{i\varphi(N-1)}|N\rangle + e^{i\varphi(N+1)}|N\rangle$$

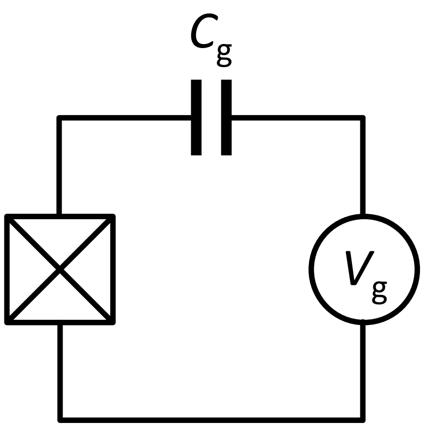
$$= \frac{1}{2\sqrt{2\pi}}\left(\sum_{N=-\infty}^{\infty} e^{i\varphi N}|N+1\rangle + \sum_{N=-\infty}^{\infty} e^{i\varphi N}|N-1\rangle\right)$$

$$= \frac{1}{2\sqrt{2\pi}}\sum_{N=-\infty}^{\infty}\left(\sum_{N'=-\infty}^{\infty}|N'+1\rangle\langle N'| + |N'-1\rangle\langle N'|\right)e^{i\varphi N}|N\rangle$$

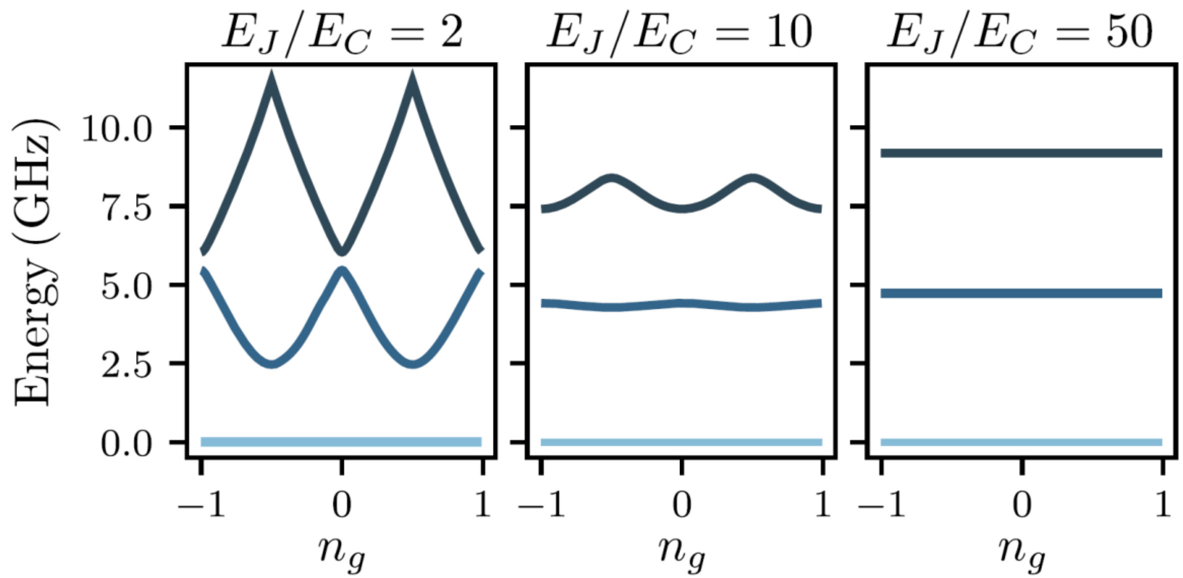
$$= \frac{1}{2}\sum_{N=-\infty}^{\infty}\left(|N+1\rangle\langle N| + |N\rangle\langle N+1|\right)|\varphi\rangle$$

電荷量子ビット

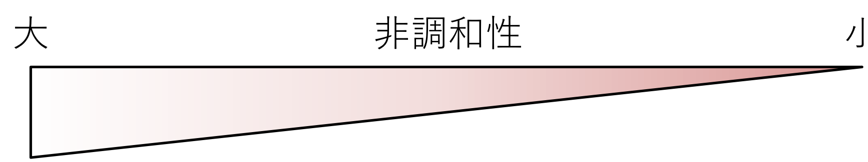
クーパー対箱



Rev. Mod. Phys. **93**, 025005 (2021) Blais *et al.*



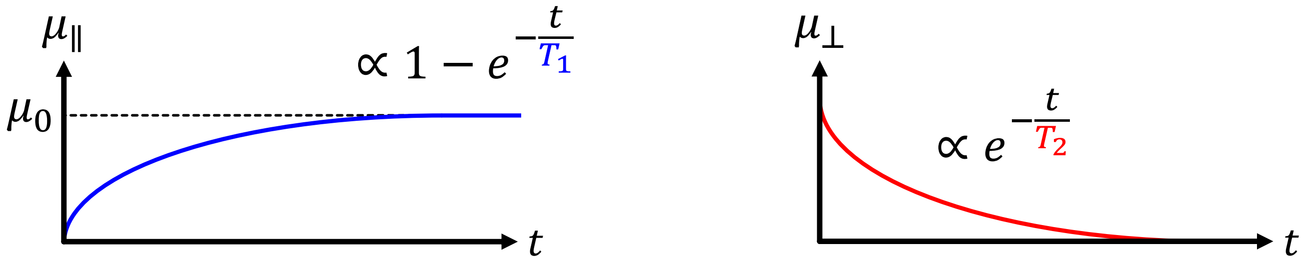
$$H = 4E_C(N - n_g)^2 - E_J \cos\varphi$$

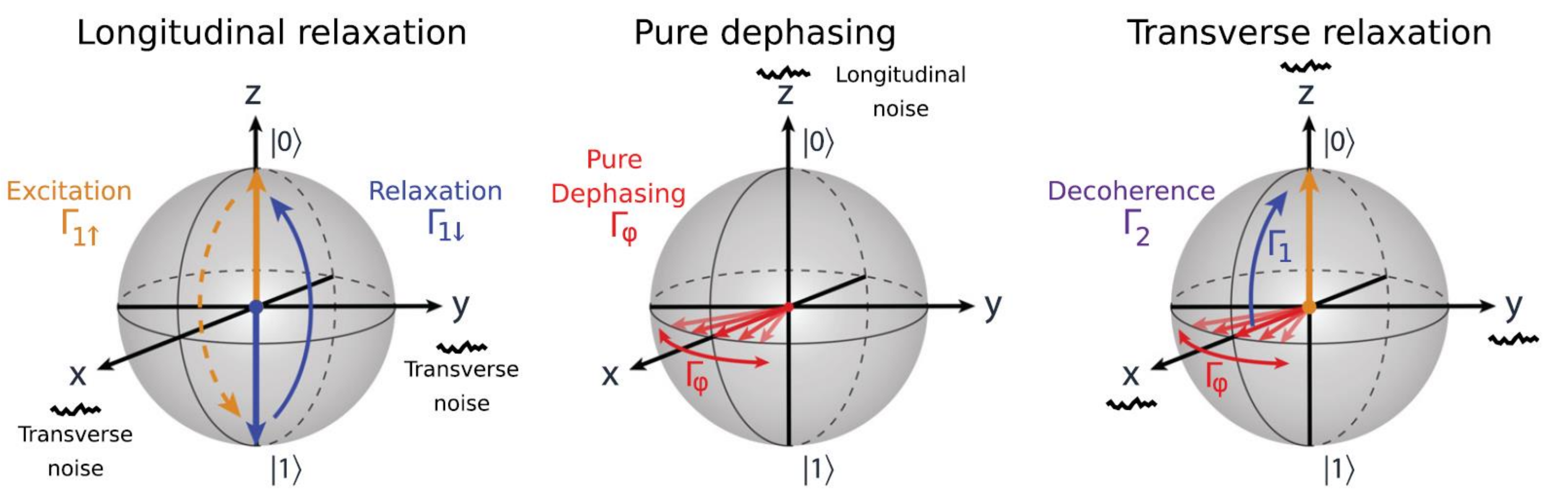


緩和時間: T_1 と T_2

ブロッホ方程式

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\mu} \times \gamma \boldsymbol{B}_0 - \frac{\boldsymbol{\mu}_\parallel - \boldsymbol{\mu}_0}{T_1} - \frac{\boldsymbol{\mu}_\perp}{T_2}$$

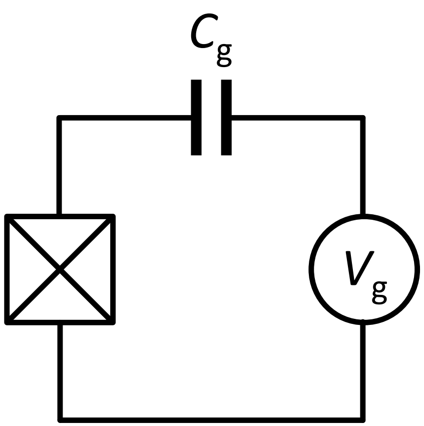




Appl. Phys. Rev. **6**, 021318 (2019) Krantz *et al.*

電荷量子ビット

クーパー対箱



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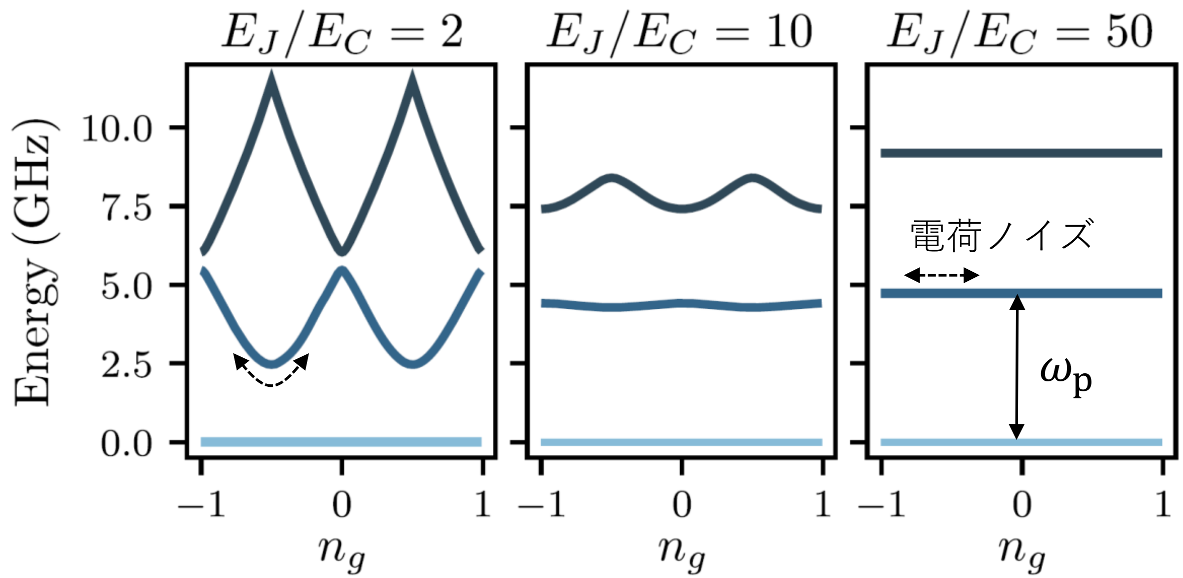


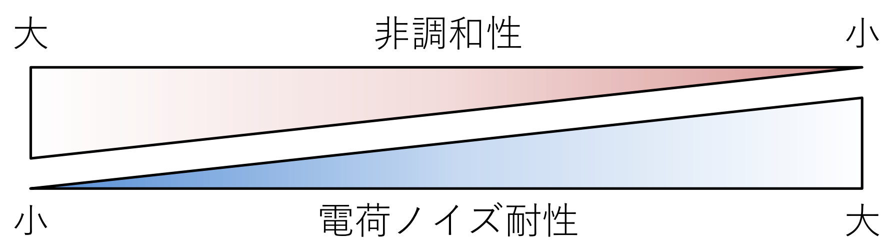
$$H = 4E_C(N - n_g)^2 - E_J \cos\varphi$$

$$\xrightarrow{\frac{E_J}{E_C} \gg 1} \quad H \approx \frac{1}{2}(8E_C)N^2 + \frac{1}{2}E_J\varphi^2$$

$$8E_C \leftrightarrow (C)^{-1}$$

$$E_J \leftrightarrow (L)^{-1}$$

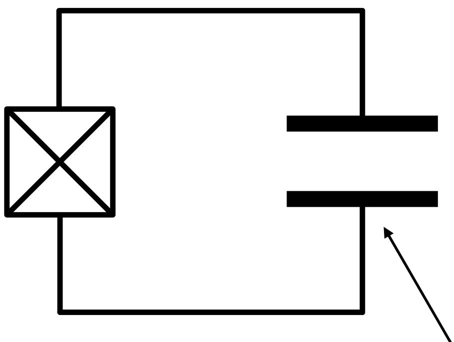
$$\longrightarrow \quad \omega_p = \sqrt{8E_C E_J}$$



電荷量子ビット

トランズモン

(Transmission-line shunted plasma oscillation qubit)



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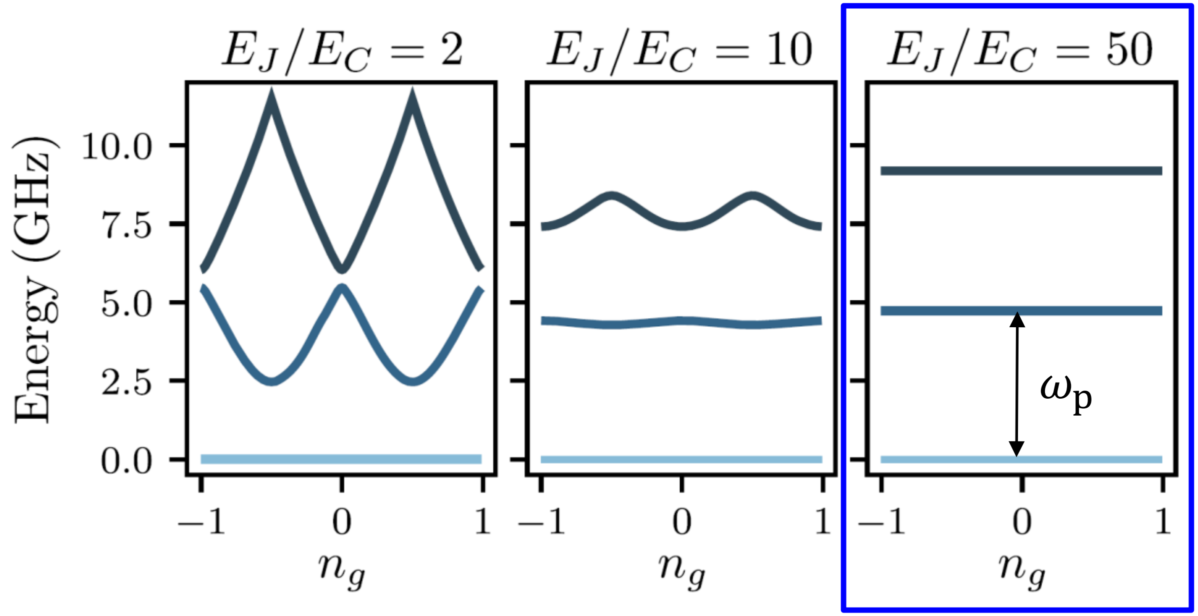
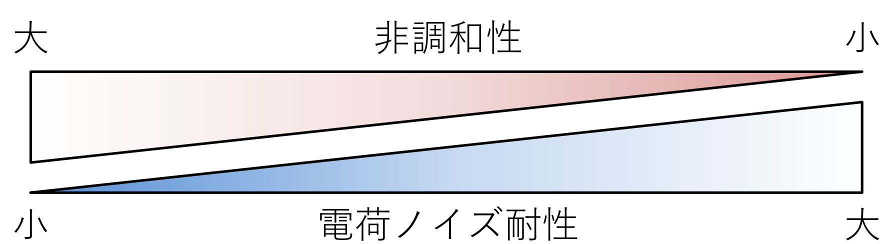


$$H = 4E_C(N - n_g)^2 - E_J \cos\varphi$$

$$\xrightarrow{\frac{E_J}{E_C} \gg 1} \quad H \approx \frac{1}{2}(8E_C)N^2 + \frac{1}{2}E_J\varphi^2$$

$$\xrightarrow[E_J \leftrightarrow (L)^{-1}]{8E_C \leftrightarrow (C)^{-1}} \quad \omega_p = \sqrt{8E_C E_J}$$



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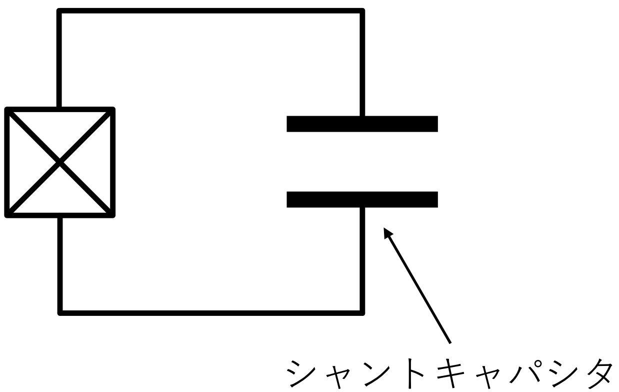

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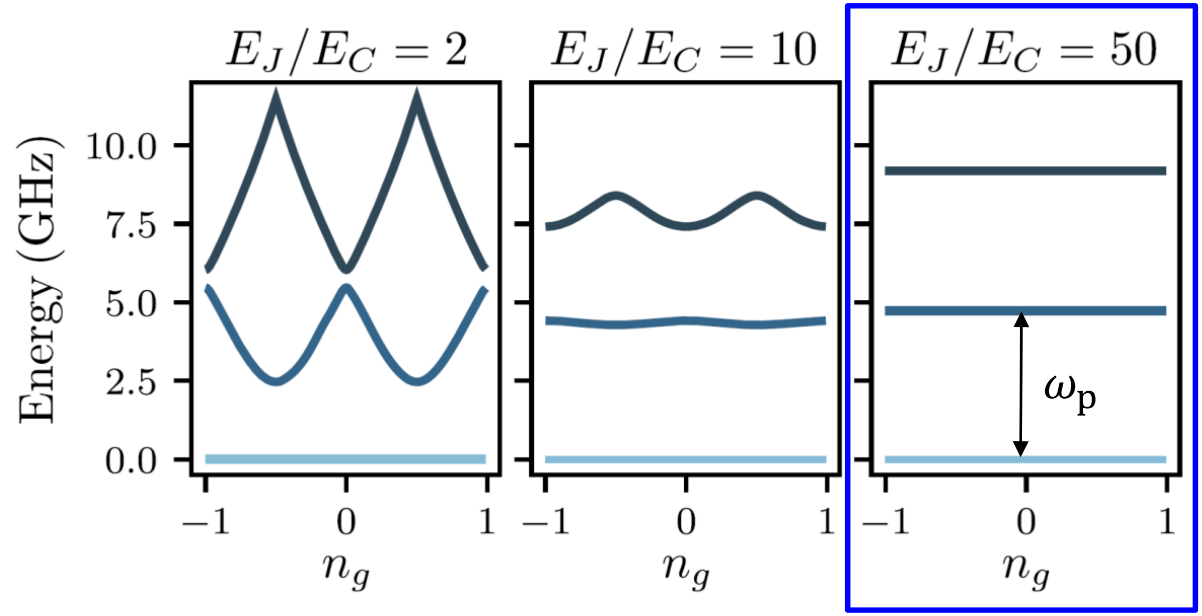

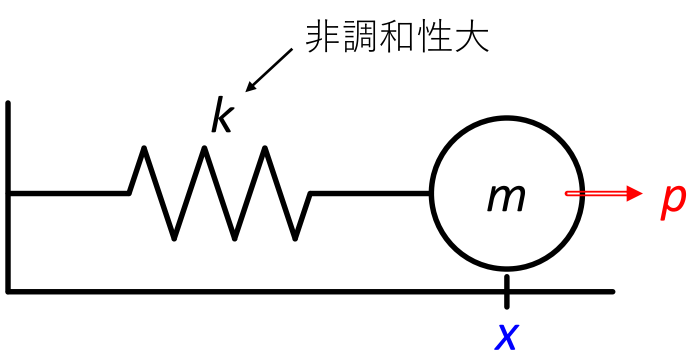

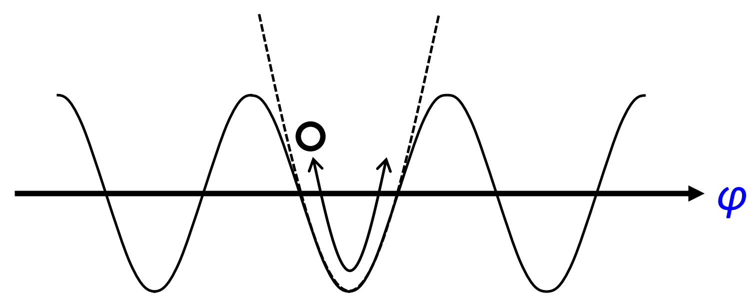

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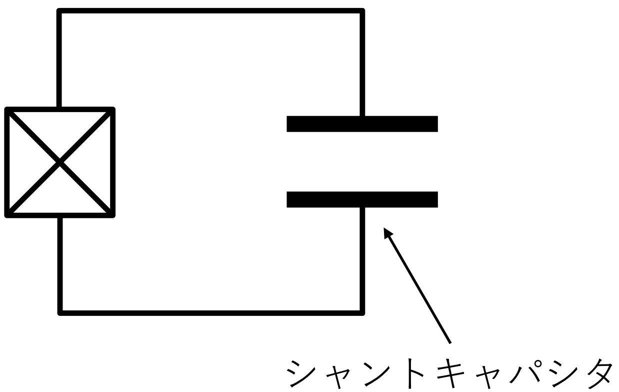

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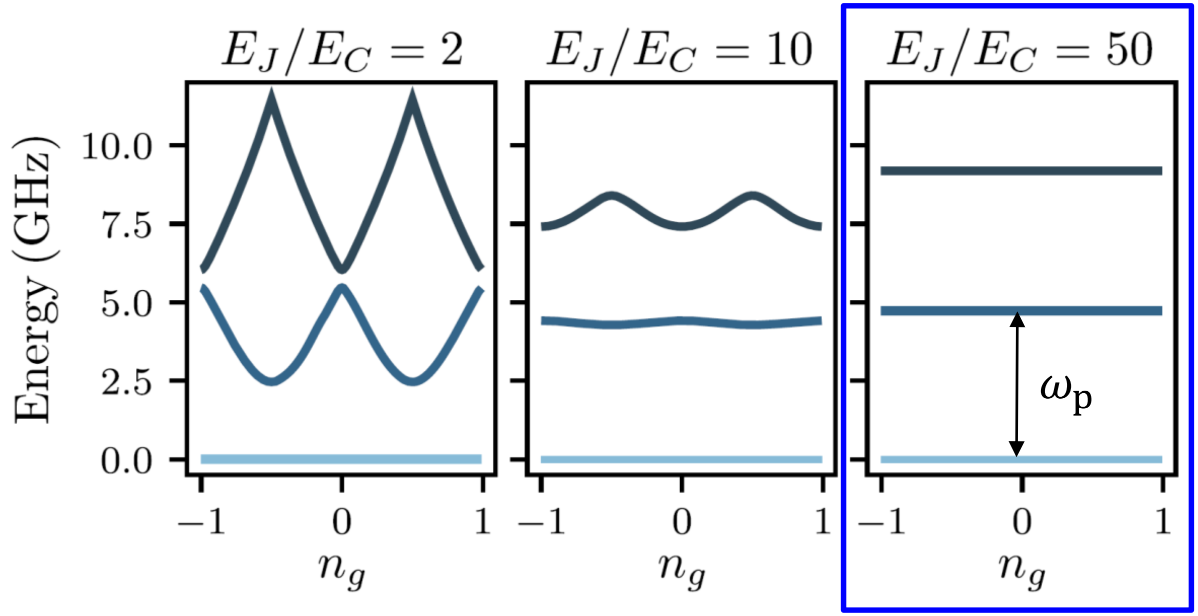
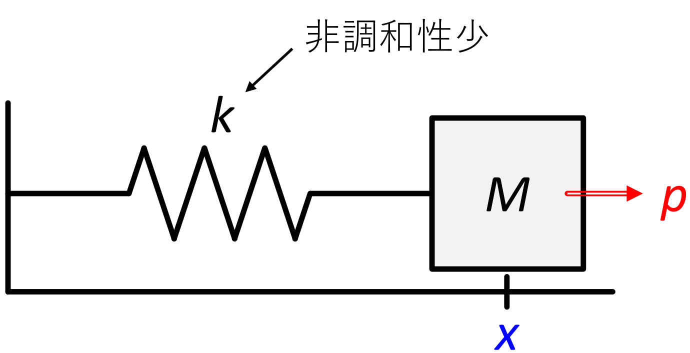
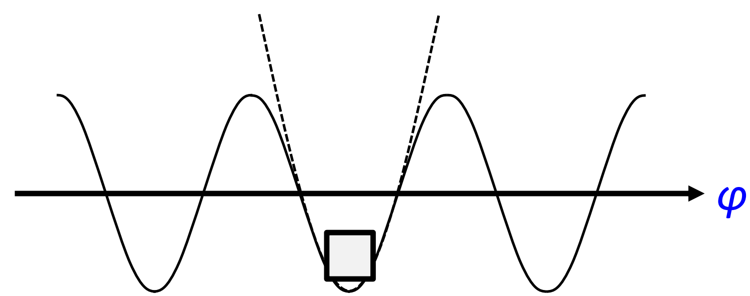






頻出公式

ボゾン(共振器)演算子

$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$a|n\rangle = \sqrt{n}|n-1\rangle$$

$$a^\dagger a|n\rangle = n|n\rangle$$

$$\left[a, a^\dagger\right] = 1$$

$$\left[a^\dagger a, a^\dagger\right] = a^\dagger$$

$$\left[a^\dagger a, a\right] = -a$$

2準位系(原子, 量子ビット)演算子

$$\left[\sigma_x, \sigma_y\right] = 2i\sigma_z$$

$$\left[\sigma_y, \sigma_z\right] = 2i\sigma_x$$

$$[\sigma_z, \sigma_x] = 2i\sigma_y$$

$$[\sigma_z, \sigma_\pm] = \pm 2\sigma_\pm$$

$$[\sigma_+, \sigma_-] = \sigma_z$$

$$\sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$$

$$\sigma_+\sigma_- = \frac{1}{2}(1 + \sigma_z)$$

$$\sigma_-\sigma_+ = \frac{1}{2}(1 - \sigma_z)$$

キャンベル–ベーカー–ハウスドルフ(Campbell–Baker–Hausdorff)公式

$$e^A B e^{-A} = B + [A, B] + \frac{1}{2!}\big[A, [A, B]\big] + \frac{1}{3!}\Big[A, \big[A, [A, B]\big]\Big] + \cdots$$

ジェインズ–カミングス模型

(Jaynes–Cummings model)

CQEDハミルトニアン

$$H_{1\mathrm{q}} = H_0 + H_{\mathrm{qr}} \qquad H_0 = \omega_{\mathrm{q}}\frac{\sigma_z}{2} + \omega_{\mathrm{r}} a^\dagger a \qquad H_{\mathrm{qr}} = g\sigma_x(a + a^\dagger)$$

量子ビット　共振器　結合

回転座標系(H_0)

$$H_{1\mathrm{q}}^{\mathrm{rot}} = e^{iH_0 t} g\sigma_x(a + a^\dagger)\, e^{-iH_0 t}$$

$$= \boxed{g(\sigma_+ a e^{i(\omega_{\mathrm{q}}-\omega_{\mathrm{r}})t} + \sigma_- a^\dagger e^{-i(\omega_{\mathrm{q}}-\omega_{\mathrm{r}})t}} + \cancel{\sigma_+ a^\dagger e^{i(\omega_{\mathrm{q}}+\omega_{\mathrm{r}})t}} + \cancel{\sigma_- a e^{-i(\omega_{\mathrm{q}}+\omega_{\mathrm{r}})t}})$$

回転波近似(高速で回転する成分を無視)

$$H_{\mathrm{qr}} \approx g(\sigma_+ a + \sigma_- a^\dagger)$$

ジェインズ–カミングスハミルトニアン

$$H_{\mathrm{JC}} = \omega_{\mathrm{q}}\frac{\sigma_z}{2} + \omega_{\mathrm{r}} a^\dagger a + g(\sigma_+ a + \sigma_- a^\dagger)$$

ジェインズ–カミングス模型

計算

$$H_{1\mathrm{q}}^{\mathrm{rot}} = e^{iH_0 t} g\sigma_x (a + a^\dagger)\, e^{-iH_0 t}$$

$$H_0 = \omega_{\mathrm{q}} \frac{\sigma_z}{2} + \omega_{\mathrm{r}} a^\dagger a$$

CBH公式: 量子ビット部分

$$A = i\omega_{\mathrm{q}} \frac{\sigma_z}{2} t \qquad B = \sigma_x$$

$$[A, B] = i\frac{\omega_{\mathrm{q}} t}{2}\,[\sigma_z, \sigma_x] = i\frac{\omega_{\mathrm{q}} t}{2}(2i\sigma_y) = -\omega_{\mathrm{q}} t \sigma_y$$

$$[\sigma_i, \sigma_{i+1}] = 2i\sigma_{i+2}$$

$$\big[A, [A, B]\big] = \left(i\frac{\omega_{\mathrm{q}} t}{2}\right)(-\omega_{\mathrm{q}} t)\big[\sigma_z, \sigma_y\big] = \left(i\frac{\omega_{\mathrm{q}} t}{2}\right)(-\omega_{\mathrm{q}} t)(-2i\sigma_x) = -\big(\omega_{\mathrm{q}} t\big)^2 \sigma_x$$

$$\Big[A, \big[A, [A, B]\big]\Big] = -\left(i\frac{\omega_{\mathrm{q}} t}{2}\right)\big(\omega_{\mathrm{q}} t\big)^2\,[\sigma_z, \sigma_x] = \big(\omega_{\mathrm{q}} t\big)^3 \sigma_y$$

$$\longrightarrow \begin{cases} \big[A, \ldots [A, B]\big] = (-1)^{n+1}\big(\omega_{\mathrm{q}} t\big)^{2n+1}\sigma_y \\ \big[A, \ldots [A, B]\big] = (-1)^{n}\big(\omega_{\mathrm{q}} t\big)^{2n}\sigma_x \end{cases}$$

ジェインズ–カミングス模型

CBH公式: 量子ビット部分(続き)

$$e^{i\omega_{\mathrm{q}}\frac{\sigma_z}{2}t}\sigma_x e^{-i\omega_{\mathrm{q}}\frac{\sigma_z}{2}t} = -\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\left(\omega_{\mathrm{q}}t\right)^{2n+1}\sigma_y + \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\left(\omega_{\mathrm{q}}t\right)^{2n}\sigma_x$$

$$= \sigma_x \cos\omega_q t - \sigma_y \sin\omega_q t$$

CBH公式: 共振器部分

$$A = i\omega_{\mathrm{r}}a^\dagger a t \qquad B = a + a^\dagger$$

$$[A,B] = i\omega_{\mathrm{r}}t\left[a^\dagger a, a + a^\dagger\right] = i\omega_{\mathrm{r}}t(-a + a^\dagger)$$

$$\left[A,[A,B]\right] = (i\omega_{\mathrm{r}}t)^2\left[a^\dagger a, -a + a^\dagger\right] = (i\omega_{\mathrm{r}}t)^2(a + a^\dagger)$$

$$\longrightarrow \left[A, \ldots [A,B]\right] = (-i\omega_{\mathrm{r}}t)^n a + (i\omega_{\mathrm{r}}t)^n a^\dagger$$

$$\left[a^\dagger a, a\right] = -a$$
$$\left[a^\dagger a, a^\dagger\right] = a^\dagger$$

$$e^{i\omega_{\mathrm{r}}a^\dagger a t}\left(a + a^\dagger\right)e^{-i\omega_{\mathrm{r}}a^\dagger a t} = \left(\sum_{n=0}^{\infty}\frac{(-i\omega_{\mathrm{r}}t)^n}{n!}\right)a + \left(\sum_{n=0}^{\infty}\frac{(i\omega_{\mathrm{r}}t)^n}{n!}\right)a^\dagger = e^{-i\omega_{\mathrm{r}}t}a + e^{i\omega_{\mathrm{r}}t}a^\dagger$$

ジェインズ–カミングス模型

$$e^{iH_0 t} g\sigma_x\left(a + a^\dagger\right)e^{-iH_0 t}$$

$$= g(\sigma_x \cos\omega_q t - \sigma_y \sin\omega_q t)(ae^{-i\omega_\mathrm{r}t} + a^\dagger e^{i\omega_\mathrm{r}t})$$

$$= g(\sigma_+ a e^{i(\omega_\mathrm{q}-\omega_\mathrm{r})t} + \sigma_- a^\dagger e^{-i(\omega_\mathrm{q}-\omega_\mathrm{r})t} + \sigma_+ a^\dagger e^{i(\omega_\mathrm{q}+\omega_\mathrm{r})t} + \sigma_- a e^{-i(\omega_\mathrm{q}+\omega_\mathrm{r})t})$$

3行目が2行目に一致することを確認

$$\sigma_+ a e^{i(\omega_\mathrm{q}-\omega_\mathrm{r})t} + \sigma_- a^\dagger e^{-i(\omega_\mathrm{q}-\omega_\mathrm{r})t} + \sigma_+ a^\dagger e^{i(\omega_\mathrm{q}+\omega_\mathrm{r})t} + \sigma_- a e^{-i(\omega_\mathrm{q}+\omega_\mathrm{r})t}$$

$$= \frac{1}{2}\left(\sigma_x + i\sigma_y\right)a\left(\cos\omega_\mathrm{q}t + i\sin\omega_\mathrm{q}t\right)e^{-i\omega_\mathrm{r}t} + \frac{1}{2}(\sigma_x - i\sigma_y)a^\dagger(\cos\omega_\mathrm{q}t - i\sin\omega_\mathrm{q}t)e^{i\omega_\mathrm{r}t}$$

$$+ \frac{1}{2}\left(\sigma_x + i\sigma_y\right)a^\dagger\left(\cos\omega_\mathrm{q}t + i\sin\omega_\mathrm{q}t\right)e^{i\omega_\mathrm{r}t} + \frac{1}{2}\left(\sigma_x - i\sigma_y\right)a(\cos\omega_\mathrm{q}t - i\sin\omega_\mathrm{q}t)e^{-i\omega_\mathrm{r}t}$$

$$= \frac{1}{2}\{\left(\sigma_x + i\sigma_y\right)\left(\cos\omega_\mathrm{q}t + i\sin\omega_\mathrm{q}t\right) + \left(\sigma_x - i\sigma_y\right)\left(\cos\omega_\mathrm{q}t - i\sin\omega_\mathrm{q}t\right)\}ae^{-i\omega_\mathrm{r}t}$$

$$+ \frac{1}{2}\{\left(\sigma_x - i\sigma_y\right)\left(\cos\omega_\mathrm{q}t - i\sin\omega_\mathrm{q}t\right) + \left(\sigma_x + i\sigma_y\right)\left(\cos\omega_\mathrm{q}t + i\sin\omega_\mathrm{q}t\right)\}a^\dagger e^{i\omega_\mathrm{r}t}$$

$$= (\sigma_x \cos\omega_q t - \sigma_y \sin\omega_q t)(ae^{-i\omega_\mathrm{r}t} + a^\dagger e^{i\omega_\mathrm{r}t})$$

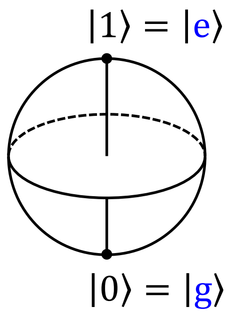
強結合領域

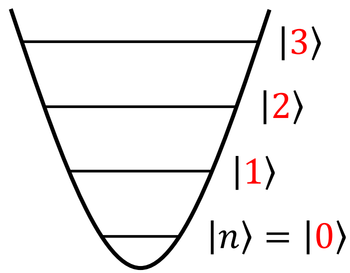
(Strong coupling regime)

$$H_{\mathrm{JC}} = \omega_{\mathrm{q}} \frac{\sigma_z}{2} + \omega_{\mathrm{r}} a^{\dagger} a + g(\sigma_+ a + \sigma_- a^{\dagger})$$

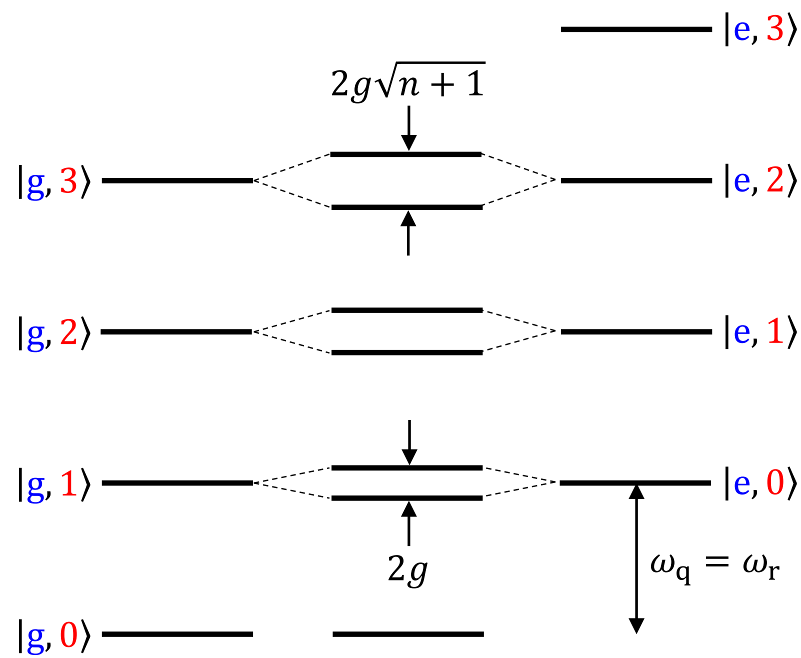
量子ビット

共振器





真空ラビ分裂



共鳴: $\Delta \equiv \omega_{\mathrm{q}} - \omega_{\mathrm{r}} = 0$

$$\begin{cases} \omega_{n\pm,\Delta=0} = \omega_{\mathrm{r}}\left(n + \frac{1}{2}\right) \pm g\sqrt{n+1} \\ \left|\Psi_{n\pm,\Delta=0}\right\rangle = \frac{1}{\sqrt{2}}\left(|\mathrm{g}, n+1\rangle \pm |\mathrm{e}, n\rangle\right) \end{cases}$$

$g \gg \kappa, \gamma$ → **強結合**

強結合領域

固有方程式($\Delta \approx 0$)

$$H_{\mathrm{JC}}|\Psi\rangle = \omega|\Psi\rangle$$

$$|\mathrm{e}, n\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$|\mathrm{g}, n+1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$\longrightarrow |\Psi\rangle = \cos\alpha\,|\mathrm{e}, n\rangle + \sin\alpha\,|\mathrm{g}, n+1\rangle = \begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}$$

$$\longrightarrow H_{\mathrm{JC}}|\Psi\rangle = \cos\alpha\left(\omega_{\mathrm{q}}\frac{\sigma_z}{2} + \omega_{\mathrm{r}}a^\dagger a + g\sigma_+ a + g\sigma_- a^\dagger\right)|\mathrm{e}, n\rangle$$

$$+\sin\alpha\left(\omega_{\mathrm{q}}\frac{\sigma_z}{2} + \omega_{\mathrm{r}}a^\dagger a + g\sigma_+ a + g\sigma_- a^\dagger\right)|\mathrm{g}, n+1\rangle$$

$$= \cos\alpha\left(\frac{\omega_{\mathrm{q}}}{2}|\mathrm{e}, n\rangle + n\omega_{\mathrm{r}}|\mathrm{e}, n\rangle + 0 + g\sqrt{n+1}|\mathrm{g}, n+1\rangle\right)$$

$$+\sin\alpha\left(-\frac{\omega_{\mathrm{q}}}{2}|\mathrm{g}, n+1\rangle + (n+1)\omega_{\mathrm{r}}|\mathrm{g}, n+1\rangle + g\sqrt{n+1}|\mathrm{e}, n\rangle + 0\right)$$

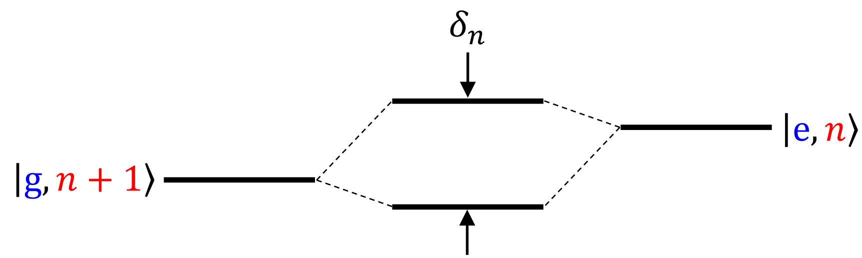
$$= \begin{pmatrix}\frac{\omega_{\mathrm{q}}}{2} + n\omega_{\mathrm{r}} & g\sqrt{n+1}\\ g\sqrt{n+1} & -\frac{\omega_{\mathrm{q}}}{2} + (n+1)\omega_{\mathrm{r}}\end{pmatrix}\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix} = \omega\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}$$

強結合領域

$$\begin{pmatrix} \omega - \frac{\omega_\mathrm{q}}{2} - n\omega_\mathrm{r} & -g\sqrt{n+1} \\ -g\sqrt{n+1} & \omega + \frac{\omega_\mathrm{q}}{2} - (n+1)\omega_\mathrm{r} \end{pmatrix} \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} = 0$$

$$\longrightarrow \left(\omega - \frac{\omega_\mathrm{q}}{2} - n\omega_\mathrm{r}\right)\left(\omega + \frac{\omega_\mathrm{q}}{2} - (n+1)\omega_\mathrm{r}\right) - g^2(n+1) = 0$$

$$\longrightarrow \omega_{n\pm} = \omega_\mathrm{r}\left(n + \frac{1}{2}\right) \pm \frac{\delta_n}{2}$$



$$\longrightarrow \begin{cases} |\Psi_{n+}\rangle = \cos\alpha_n\,|\mathrm{e}, n\rangle + \sin\alpha_n\,|\mathrm{g}, n+1\rangle \\ |\Psi_{n-}\rangle = \sin\alpha_n\,|\mathrm{e}, n\rangle - \cos\alpha_n\,|\mathrm{g}, n+1\rangle \end{cases}$$

$$\delta_n = \sqrt{4g^2(n+1) + \Delta^2}$$

$$\Delta = \omega_\mathrm{q} - \omega_\mathrm{r}$$

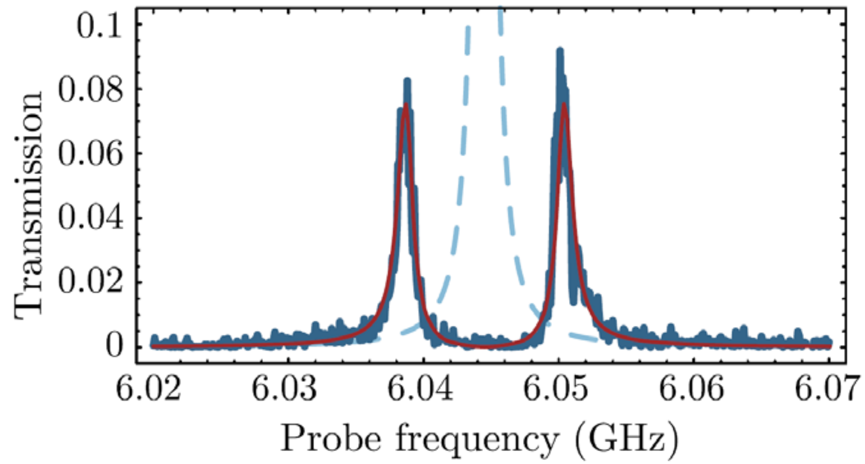
$$\cos\alpha_n = \frac{2g\sqrt{n+1}}{\sqrt{4g^2(n+1) + (\delta_n - \Delta)^2}} \qquad \sin\alpha_n = \frac{\delta_n - \Delta}{\sqrt{4g^2(n+1) + (\delta_n - \Delta)^2}}$$

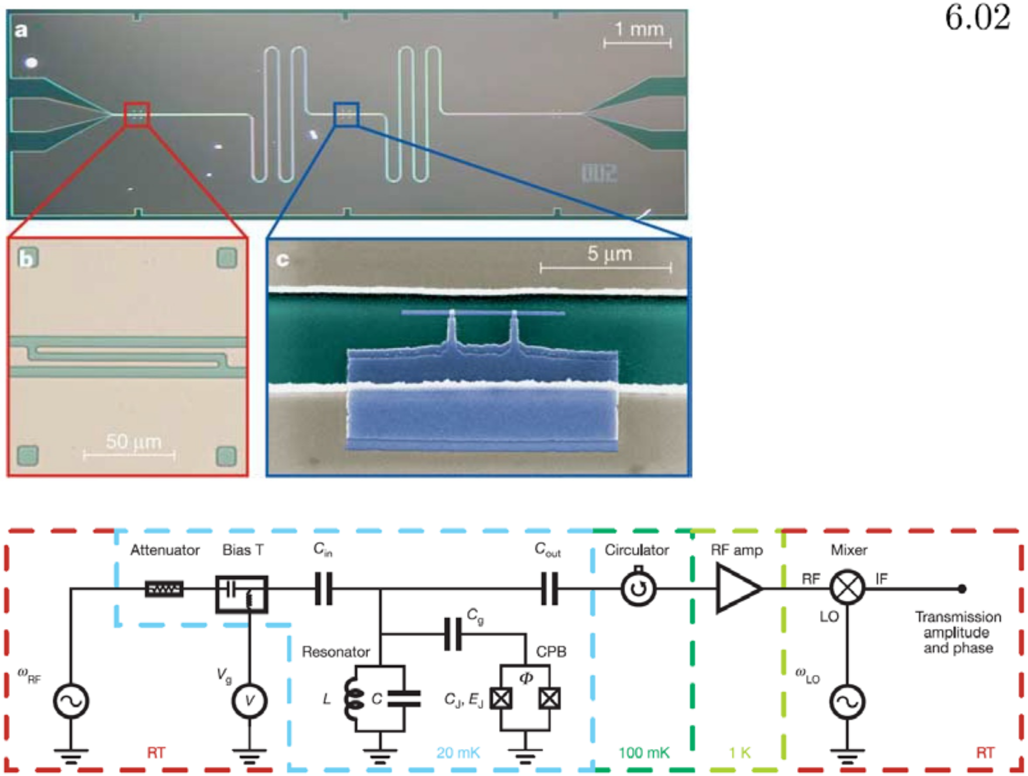
真空ラビ分裂の観測

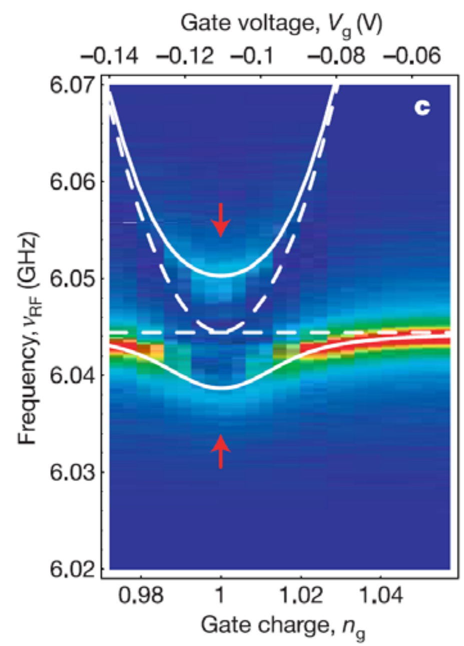
Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics

A. Wallraff[1], D. I. Schuster[1], A. Blais[1], L. Frunzio[1], R.- S. Huang[1,2], J. Majer[1], S. Kumar[1], S. M. Girvin[1] & R. J. Schoelkopf[1]

Nature **431**, 162 (2004) Wallraff *et al.*





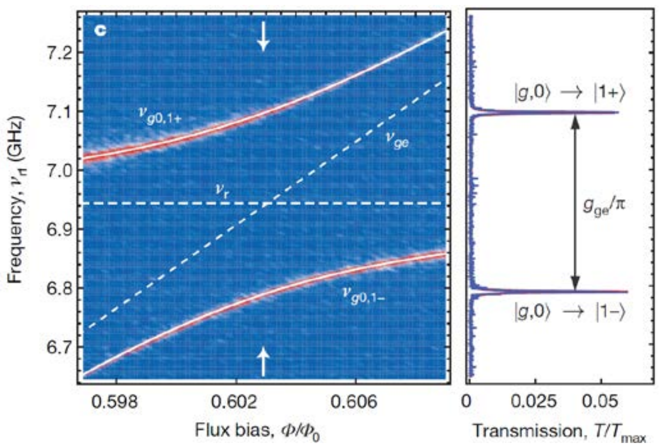


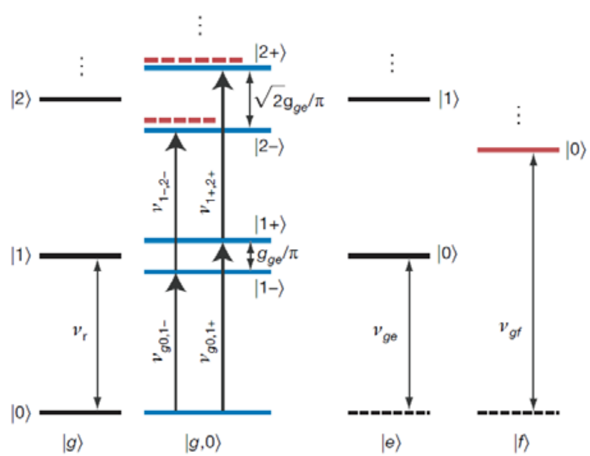
真空ラビ分裂の観測

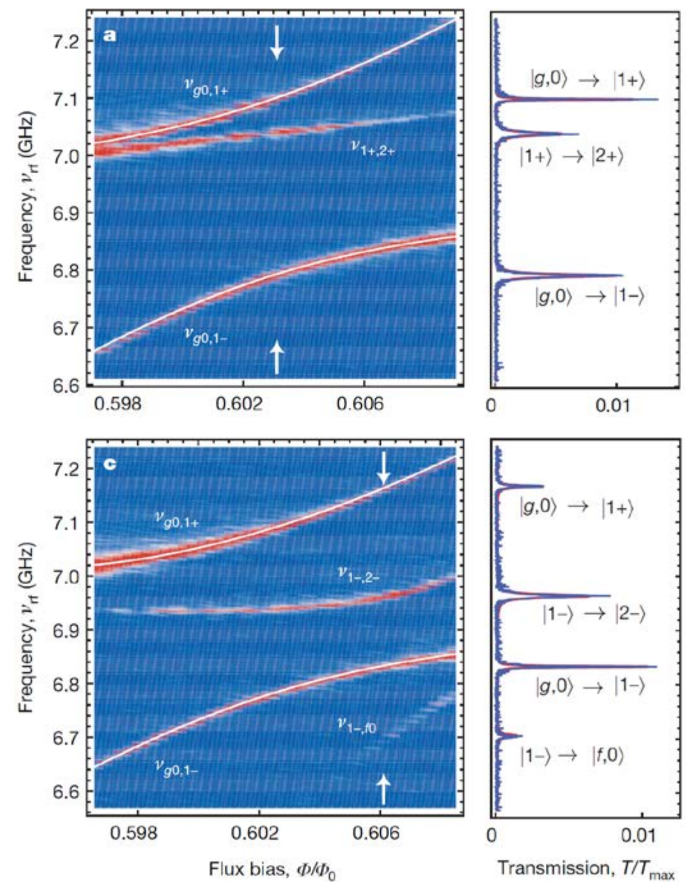
Climbing the Jaynes–Cummings ladder and observing its $\sqrt{n}$ nonlinearity in a cavity QED system

J. M. Fink[1], M. Göppl[1], M. Baur[1], R. Bianchetti[1], P. J. Leek[1], A. Blais[2] & A. Wallraff[1]

Nature **454**, 315 (2008) Fink *et al.*







分散領域

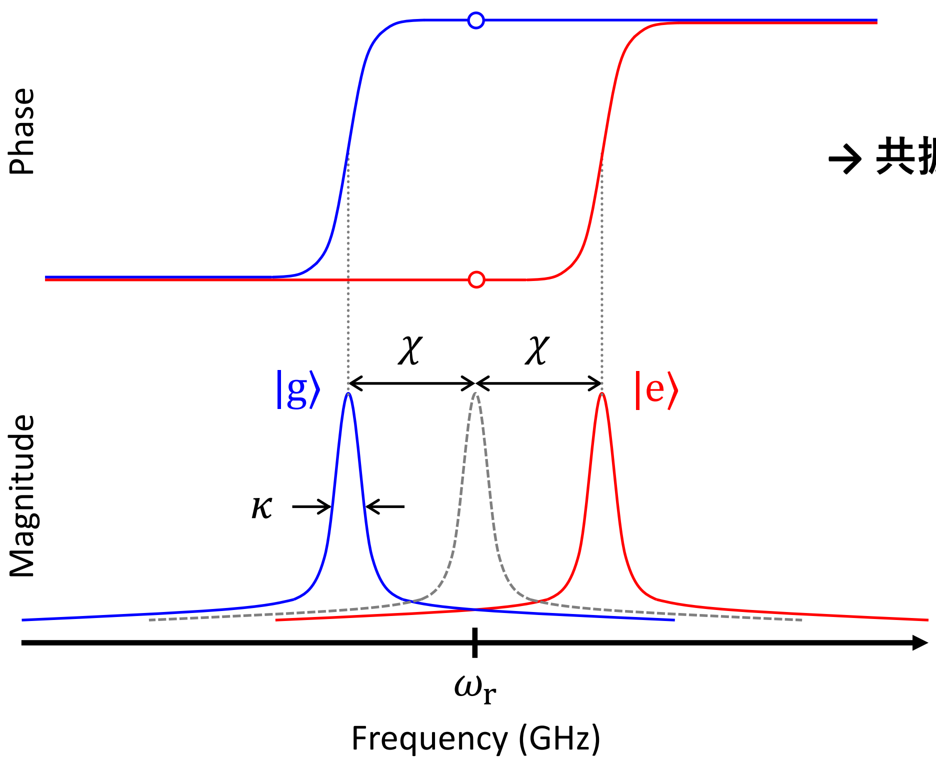
(Dispersive regime)

$$H_{\mathrm{JC}} = \omega_{\mathrm{q}} \frac{\sigma_z}{2} + \omega_{\mathrm{r}} a^\dagger a + g(\sigma_+ a + \sigma_- a^\dagger)$$

$|\Delta| = |\omega_{\mathrm{q}} - \omega_{\mathrm{r}}| \gg g, \kappa$

$$\longrightarrow H_{\mathrm{JC}}^{\mathrm{disp}} = (\omega_{\mathrm{q}} + \chi)\frac{\sigma_z}{2} + (\omega_{\mathrm{r}} + \chi\sigma_z) a^\dagger a$$

$$\chi = \frac{g^2}{\Delta}$$



→ 共振器を介した量子ビットの測定

量子非破壊

e.g.
f_{q} = 8 GHz
f_{r} = 10 GHz
κ = 10 MHz (Q = 1000)
g = 100 MHz
2χ = 10 MHz

シュリーファー–ウォルフ変換

(Schrieffer-Wolff transformation)

対角行列H_0に対する摂動V(非対角成分のみ)をユニタリ変換により近似的に対角化

$$H = H_0 + V$$

に対して

$$S^\dagger = -S,\ \ [S, H_0] = -V$$

となるSを見つける

$$\longrightarrow\ UHU^\dagger = e^S H e^{-S}$$

$$= H_0 + \underbrace{V + [S, H_0]}_{0} + [S, V] + \underbrace{\frac{1}{2!}\left[S, [S, H_0]\right]}_{-\frac{1}{2}[S,V]} + \underbrace{\frac{1}{2!}\left[S, [S, V]\right] + \cdots}_{O(V^3)}$$

$$\approx H_0 + \frac{1}{2}[S, V]$$

分散領域

方針

$$S=\frac{g}{\Delta}(\sigma_+a-\sigma_-a^\dagger)$$

$$\Delta=\omega_\mathrm{q}-\omega_\mathrm{r}$$

とすると$S^\dagger=-S,[S,H_0]=-H_\mathrm{qr}$を満たす

$$\longrightarrow H_\mathrm{JC}^\mathrm{disp}=e^SH_\mathrm{JC}e^{-S}\approx H_0+\frac{1}{2}[S,H_\mathrm{qr}]$$

$$H_\mathrm{JC}=\underbrace{\omega_\mathrm{q}\frac{\sigma_z}{2}+\omega_\mathrm{r}a^\dagger a}_{H_0}+\underbrace{g(\sigma_+a+\sigma_-a^\dagger)}_{H_\mathrm{qr}}$$

***S*の確認**

$$[S,H_0]=\frac{g}{\Delta}[\sigma_+a-\sigma_-a^\dagger,\omega_\mathrm{q}\frac{\sigma_z}{2}+\omega_\mathrm{r}a^\dagger a]$$

$$=\frac{g\omega_\mathrm{q}}{2\Delta}[\sigma_+,\sigma_z]a+\frac{g\omega_\mathrm{r}}{\Delta}\sigma_+[a,a^\dagger a]-\frac{g\omega_\mathrm{q}}{2\Delta}[\sigma_-,\sigma_z]a^\dagger-\frac{g\omega_\mathrm{r}}{\Delta}\sigma_-[a^\dagger,a^\dagger a]$$

$$=\frac{g\omega_\mathrm{q}}{2\Delta}(-2\sigma_+)a+\frac{g\omega_\mathrm{r}}{\Delta}\sigma_+a-\frac{g\omega_\mathrm{q}}{2\Delta}(2\sigma_-)a^\dagger-\frac{g\omega_\mathrm{r}}{\Delta}\sigma_-(-a^\dagger)$$

$$=-g\frac{(\omega_\mathrm{q}-\omega_\mathrm{r})}{\Delta}\sigma_+a-g\frac{(\omega_\mathrm{q}-\omega_\mathrm{r})}{\Delta}\sigma_-a^\dagger=-H_\mathrm{qr}$$

$$[\sigma_z,\sigma_\pm]=\pm2\sigma_\pm$$
$$[a^\dagger a,a^\dagger]=a^\dagger$$
$$[a^\dagger a,a]=-a$$

分散領域

計算

$$\chi = \frac{g^2}{\Delta}$$

$$[S, H_{\mathrm{qr}}] = \frac{g^2}{\Delta}[\sigma_+ a - \sigma_- a^\dagger, \sigma_+ a + \sigma_- a^\dagger]$$

$$= \chi\big([\sigma_+ a, \sigma_+ a] + [\sigma_+ a, \sigma_- a^\dagger] - [\sigma_- a^\dagger, \sigma_+ a] - [\sigma_- a^\dagger, \sigma_- a^\dagger]\big)$$

$$= 2\chi[\sigma_+ a, \sigma_- a^\dagger]$$

$$= 2\chi\sigma_+\sigma_- a a^\dagger - 2\chi\sigma_-\sigma_+ a^\dagger a$$

$$= 2\chi\frac{1}{2}(1+\sigma_z)(1+a^\dagger a) - 2\chi\frac{1}{2}(1-\sigma_z)\, a^\dagger a$$

$$= \chi(1 + a^\dagger a + \sigma_z + \sigma_z a^\dagger a) - \chi\,(a^\dagger a - \sigma_z a^\dagger a)$$

$$= \chi + 2\chi\sigma_z a^\dagger a + \chi\sigma_z$$

$$\sigma_+\sigma_- = \frac{1}{2}(1+\sigma_z)$$

$$\sigma_-\sigma_+ = \frac{1}{2}(1-\sigma_z)$$

$$[a, a^\dagger] = 1$$

$$\longrightarrow\ H_{\mathrm{JC}}^{\mathrm{disp}} \approx \omega_{\mathrm{q}}\frac{\sigma_z}{2} + \omega_{\mathrm{r}} a^\dagger a + \frac{1}{2}[S, H_{\mathrm{qr}}] = (\omega_{\mathrm{q}} + \chi)\frac{\sigma_z}{2} + (\omega_{\mathrm{r}} + \chi\sigma_z) a^\dagger a$$

1Qゲート

外場(マイクロ波パルス)ハミルトニアン

$$H_{\mathrm{d}}(t) = E_d(t)(a^\dagger e^{-i\omega_{\mathrm{d}}t} + ae^{i\omega_{\mathrm{d}}t})$$

分散領域

$$S = \frac{g}{\Delta}(\sigma_+ a - \sigma_- a^\dagger)$$

$$H_{\mathrm{d}}^{\mathrm{disp}} = e^S H_{\mathrm{d}}(t) e^{-S}$$

$$= H_{\mathrm{d}} + \overbrace{[S, H_{\mathrm{d}}] + \frac{1}{2!}[S,[S,H_{\mathrm{d}}]] + \frac{1}{3!}\left[S,\left[S,[S,H_{\mathrm{d}}]\right]\right]}^{O\left(\frac{g^2}{\Delta^2}\right)} + \cdots$$

$$\approx E_d(t)\left(a^\dagger e^{-i\omega_{\mathrm{d}}t} + ae^{i\omega_{\mathrm{d}}t}\right) + \frac{E_d(t)g}{\Delta}(\sigma_+ e^{-i\omega_{\mathrm{d}}t} + \sigma_- e^{i\omega_{\mathrm{d}}t})$$

$$\left[\sigma_+ a - \sigma_- a^\dagger, a^\dagger\right] = \sigma_+$$

$$\left[\sigma_+ a - \sigma_- a^\dagger, a\right] = \sigma_-$$

$$\left[a, a^\dagger\right] = 1$$

1Qゲート

回転座標系($H_{\mathrm{d}}^{\mathrm{rot}}$)

$$H_{\mathrm{d}}^{\mathrm{rot}} = \omega_{\mathrm{d}}\left(\frac{\sigma_z}{2} + a^\dagger a\right)$$

$$\Omega_{\mathrm{R}} = \frac{2E_d(t)g}{\Delta}$$

$$\longrightarrow H_{1\mathrm{q}}^{\mathrm{rot}} = e^{iH_{\mathrm{d}}^{\mathrm{rot}}t}(H_{\mathrm{JC}}^{\mathrm{disp}} + H_{\mathrm{d}}^{\mathrm{disp}})e^{-iH_{\mathrm{d}}^{\mathrm{rot}}t}$$

$$= (\omega_{\mathrm{r}} - \omega_{\mathrm{d}} + \chi\sigma_z)a^\dagger a + \left(\omega_{\mathrm{q}} - \omega_{\mathrm{d}} + \chi\right)\frac{\sigma_z}{2} + E_d(t)\left(a^\dagger + a\right) + \Omega_{\mathrm{R}}\frac{\sigma_x}{2}$$

CBH公式

$$A = i\omega_{\mathrm{d}}\frac{\sigma_z}{2}t \qquad B_+ = \sigma_+ \qquad B_- = \sigma_-$$

$$[\sigma_z, \sigma_\pm] = \pm 2\sigma_\pm$$

$$[A, B_+] = i\frac{\omega_{\mathrm{d}}}{2}t[\sigma_z, \sigma_+] = i\omega_{\mathrm{d}}t\sigma_+ \qquad [A, B_-] = i\frac{\omega_{\mathrm{d}}}{2}t[\sigma_z, \sigma_-] = -i\omega_{\mathrm{d}}t\sigma_-$$

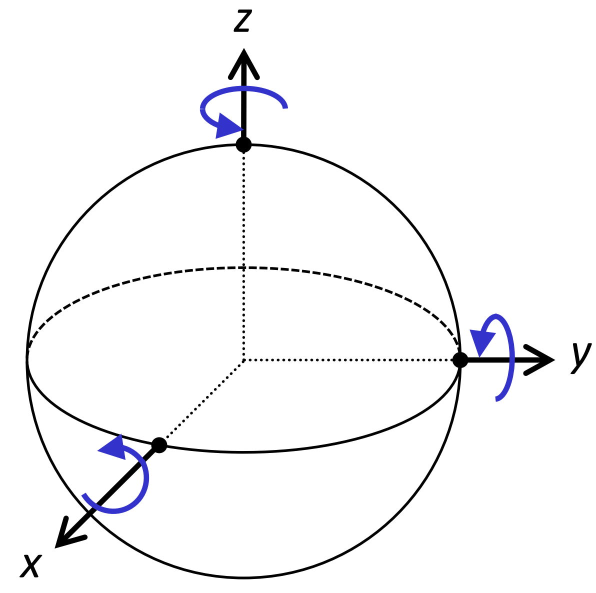
$$\longrightarrow e^A\left(\sigma_+ e^{-i\omega_{\mathrm{d}}t} + \sigma_- e^{i\omega_{\mathrm{d}}t}\right)e^{-A} = \sum_{n=0}^{\infty}\frac{(i\omega_{\mathrm{d}}t)^n}{n!}\sigma_+ e^{-i\omega_{\mathrm{d}}t} + \sum_{n=0}^{\infty}\frac{(-i\omega_{\mathrm{d}}t)^n}{n!}\sigma_- e^{i\omega_{\mathrm{d}}t}$$

$$= \sigma_+ + \sigma_+ = \sigma_x$$

1Qゲート

$$R_x\left(\varphi=\frac{\Omega_\mathrm{R}\tau}{2}\right)=e^{-i\Omega_\mathrm{R}\frac{\sigma_x}{2}\tau}$$

$$=\begin{pmatrix}\cos\left(\frac{\Omega_\mathrm{R}\tau}{2}\right) & -i\sin\left(\frac{\Omega_\mathrm{R}\tau}{2}\right)\\ -i\sin\left(\frac{\Omega_\mathrm{R}\tau}{2}\right) & \cos\left(\frac{\Omega_\mathrm{R}\tau}{2}\right)\end{pmatrix}$$



指数演算子

$$A^2=I$$

$$e^{iAx}=\sum_{n=0}^{\infty}\frac{(iAx)^n}{n!}=\cos x\cdot I+i\sin x\cdot A$$

$$R_y(\varphi)=e^{-i\varphi\sigma_y}=\begin{pmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{pmatrix}$$

$$R_z(\varphi)=e^{-i\varphi\sigma_z}=\begin{pmatrix}e^{-i\varphi} & 0\\ 0 & e^{i\varphi}\end{pmatrix}$$

実験では回転軸は装置系の基準位相で決まる

z軸周りの回転は装置系の基準位相を変えるだけで“仮想的”に実行できる

ZY分解

任意のユニタリ行列(1Qゲート)は*z*軸と*y*軸の回転の組み合わせで実現

$$U = \begin{pmatrix} e^{i(\alpha-\beta/2-\delta/2)}\cos\frac{\gamma}{2} & -e^{i(\alpha-\beta/2+\delta/2)}\sin\frac{\gamma}{2} \\ e^{i(\alpha+\beta/2-\delta/2)}\sin\frac{\gamma}{2} & e^{i(\alpha+\beta/2+\delta/2)}\cos\frac{\gamma}{2} \end{pmatrix}$$

$$= e^{i\alpha}\begin{pmatrix} e^{-i\beta/2} & 0 \\ 0 & e^{i\beta/2} \end{pmatrix}\begin{pmatrix} \cos\frac{\gamma}{2} & -\sin\frac{\gamma}{2} \\ \sin\frac{\gamma}{2} & \cos\frac{\gamma}{2} \end{pmatrix}\begin{pmatrix} e^{-i\delta/2} & 0 \\ 0 & e^{i\delta/2} \end{pmatrix}$$

→ 分解の仕方は一意ではない

交差共鳴ゲート

(Cross-resonance, CR)

結合2量子ビットハミルトニアン

$$H_{2\mathrm{q}} = \underbrace{\omega_{\mathrm{q}1}\frac{\sigma_z^1}{2} + \omega_{\mathrm{q}2}\frac{\sigma_z^2}{2}}_{\mathrm{H_{qq}}} + \underbrace{J(\sigma_+^1\sigma_-^2 + \sigma_-^1\sigma_+^2)}_{H_J}$$

$$H_{\mathrm{JC}} = \omega_{\mathrm{q}}\frac{\sigma_z}{2} + \omega_{\mathrm{r}}a^\dagger a + g(\sigma_+ a + \sigma_- a^\dagger)$$

SW変換

$$e^S H_{2\mathrm{q}} e^{-S} \approx H_{\mathrm{qq}} + \frac{1}{2}[S, H_J] \qquad S = \frac{J}{\Delta_{\mathrm{qq}}}(\sigma_+^1\sigma_-^2 - \sigma_-^1\sigma_+^2)$$

$$\Delta_{\mathrm{qq}} \equiv \omega_{\mathrm{q}1} - \omega_{\mathrm{q}2}$$

$$\Delta_{\mathrm{qq}} \gg J$$

$$S^\dagger = -S$$

$$[S, H_{\mathrm{qq}}] = -H_J$$

$$\longrightarrow H_{2\mathrm{q}}^{\mathrm{disp}} = \left(\omega_{\mathrm{q}1} + \frac{J^2}{\Delta_{\mathrm{qq}}}\right)\frac{\sigma_z^1}{2} + \left(\omega_{\mathrm{q}2} - \frac{J^2}{\Delta_{\mathrm{qq}}}\right)\frac{\sigma_z^2}{2}$$

交差共鳴ゲート

Sの確認

$$[S, H_{\mathrm{qq}}] = \frac{\omega_{\mathrm{q1}}J}{2\Delta_{\mathrm{qq}}}[\sigma_+^1\sigma_-^2 - \sigma_-^1\sigma_+^2, \sigma_z^1] + \frac{\omega_{\mathrm{q2}}J}{2\Delta_{\mathrm{qq}}}[\sigma_+^1\sigma_-^2 - \sigma_-^1\sigma_+^2, \sigma_z^2]$$

$$[\sigma_z, \sigma_\pm] = \pm 2\sigma_\pm$$

$$= \frac{\omega_{\mathrm{q1}}J}{2\Delta_{\mathrm{qq}}}\{(-2\sigma_+^1)\sigma_-^2 - (2\sigma_-^1)\sigma_+^2\} + \frac{\omega_{\mathrm{q2}}J}{2\Delta_{\mathrm{qq}}}\{\sigma_+^1(2\sigma_-^2) - \sigma_-^1(-2\sigma_+^2)\}$$

$$= \frac{\omega_{\mathrm{q1}}J}{\Delta_{\mathrm{qq}}}(-\sigma_+^1\sigma_-^2 - \sigma_-^1\sigma_+^2) + \frac{\omega_{\mathrm{q2}}J}{\Delta_{\mathrm{qq}}}(\sigma_+^1\sigma_-^2 + \sigma_-^1\sigma_+^2)$$

$$= -\frac{(\omega_{\mathrm{q1}} - \omega_{\mathrm{q2}})}{\Delta_{\mathrm{qq}}}J(\sigma_+^1\sigma_-^2 + \sigma_-^1\sigma_+^2) = -H_J$$

交差共鳴ゲート

計算

$$\left[S, H_J\right] = \frac{J^2}{\Delta_{\mathrm{qq}}}\left[\sigma_+^1\sigma_-^2 - \sigma_-^1\sigma_+^2, \sigma_+^1\sigma_-^2 + \sigma_-^1\sigma_+^2\right]$$

$$= \frac{J^2}{\Delta_{\mathrm{qq}}}\left(\left[\sigma_+^1\sigma_-^2, \sigma_+^1\sigma_-^2\right] + \left[\sigma_+^1\sigma_-^2, \sigma_-^1\sigma_+^2\right] - \left[\sigma_-^1\sigma_+^2, \sigma_+^1\sigma_-^2\right] - \left[\sigma_-^1\sigma_+^2, \sigma_-^1\sigma_+^2\right]\right)$$

$$= \frac{2J^2}{\Delta_{\mathrm{qq}}}\left[\sigma_+^1\sigma_-^2, \sigma_-^1\sigma_+^2\right]$$

$$= \frac{2J^2}{\Delta_{\mathrm{qq}}}\left(\sigma_+^1\sigma_-^1\sigma_-^2\sigma_+^2 - \sigma_-^1\sigma_+^1\sigma_+^2\sigma_-^2\right)$$

$$= \frac{J^2}{2\Delta_{\mathrm{qq}}}\left\{\left(1 + \sigma_z^1\right)\left(1 - \sigma_z^2\right) - \left(1 - \sigma_z^1\right)\left(1 + \sigma_z^2\right)\right\}$$

$$= \frac{J^2}{\Delta_{\mathrm{qq}}}\left(\sigma_z^1 - \sigma_z^2\right)$$

$$\sigma_+\sigma_- = \frac{1}{2}\left(1 + \sigma_z\right)$$

$$\sigma_-\sigma_+ = \frac{1}{2}\left(1 - \sigma_z\right)$$

$$\longrightarrow H_{2\mathrm{q}}^{\mathrm{disp}} = H_{\mathrm{qq}} + \frac{1}{2}\left[S, H_J\right] = \left(\omega_{\mathrm{q1}} + \frac{J^2}{\Delta_{12}}\right)\frac{\sigma_z^1}{2} + \left(\omega_{\mathrm{q2}} - \frac{J^2}{\Delta_{12}}\right)\frac{\sigma_z^2}{2}$$

交差共鳴ゲート

外場(Q1に印加)ハミルトニアン

$$H_\mathrm{d}^\mathrm{CR}(t) = E_\mathrm{q}(t)(\sigma_+^1 e^{-i\omega_\mathrm{d} t} + \sigma_-^1 e^{-i\omega_\mathrm{d} t})$$

分散領域

$$H_\mathrm{d}^\mathrm{disp,CR} = e^S H_\mathrm{d}^\mathrm{CR}(t) e^{-S}$$

$$= H_\mathrm{d}^\mathrm{CR} + [S, H_\mathrm{d}^\mathrm{CR}] + \overbrace{\frac{1}{2!}\left[S,[S,H_\mathrm{d}^\mathrm{CR}]\right] + \frac{1}{3!}\left[S,\left[S,[S,H_\mathrm{d}^\mathrm{CR}]\right]\right]}^{O\left(\frac{J^2}{\Delta_\mathrm{qq}^2}\right)} + \cdots$$

$$\approx E_\mathrm{q}(t)\left(\sigma_+^1 e^{-i\omega_\mathrm{d} t} + \sigma_-^1 e^{i\omega_\mathrm{d} t}\right) + \frac{E_\mathrm{q}(t)J}{\Delta_\mathrm{qq}} \sigma_z^1 (\sigma_+^2 e^{-i\omega_\mathrm{d} t} + \sigma_-^2 e^{i\omega_\mathrm{d} t})$$

$$S = \frac{J}{\Delta_\mathrm{qq}}(\sigma_+^1\sigma_-^2 - \sigma_-^1\sigma_+^2)$$

$$[\sigma_+^1\sigma_-^2 - \sigma_-^1\sigma_+^2, \sigma_+^1] = \sigma_z^1\sigma_+^2$$

$$[\sigma_+^1\sigma_-^2 - \sigma_-^1\sigma_+^2, \sigma_-^1] = \sigma_z^1\sigma_-^2$$

$$[\sigma_+, \sigma_-] = \sigma_z$$

交差共鳴ゲート

$$H_{2\mathrm{q}}^{\mathrm{disp}} = \left(\omega_{\mathrm{q}1} + \frac{J^2}{\Delta_{12}}\right)\frac{\sigma_z^1}{2} + \left(\omega_{\mathrm{q}2} - \frac{J^2}{\Delta_{12}}\right)\frac{\sigma_z^2}{2}$$

$$H_{\mathrm{d}}^{\mathrm{disp,CR}} = E_{\mathrm{q}}(t)\left(\sigma_+^1 e^{-i\omega_{\mathrm{d}}t} + \sigma_-^1 e^{i\omega_{\mathrm{d}}t}\right) + \frac{E_{\mathrm{q}}(t)J}{\Delta_{\mathrm{qq}}}\sigma_z^1(\sigma_+^2 e^{-i\omega_{\mathrm{d}}t} + \sigma_-^2 e^{i\omega_{\mathrm{d}}t})$$

回転座標系($H_{\mathrm{d}}^{\mathrm{rot,CR}}$)

$$H_{\mathrm{d}}^{\mathrm{rot,CR}} = \omega_{\mathrm{d}}\left(\frac{\sigma_z^1}{2} + \frac{\sigma_z^2}{2}\right)$$

$$\longrightarrow H_{2\mathrm{q}}^{\mathrm{rot}} = e^{iH_{\mathrm{d}}^{\mathrm{rot,CR}}t}(H_{2\mathrm{q}}^{\mathrm{disp}} + H_{\mathrm{d}}^{\mathrm{disp,CR}} - H_{\mathrm{d}}^{\mathrm{rot,CR}})e^{-iH_{\mathrm{d}}^{\mathrm{rot,CR}}t}$$

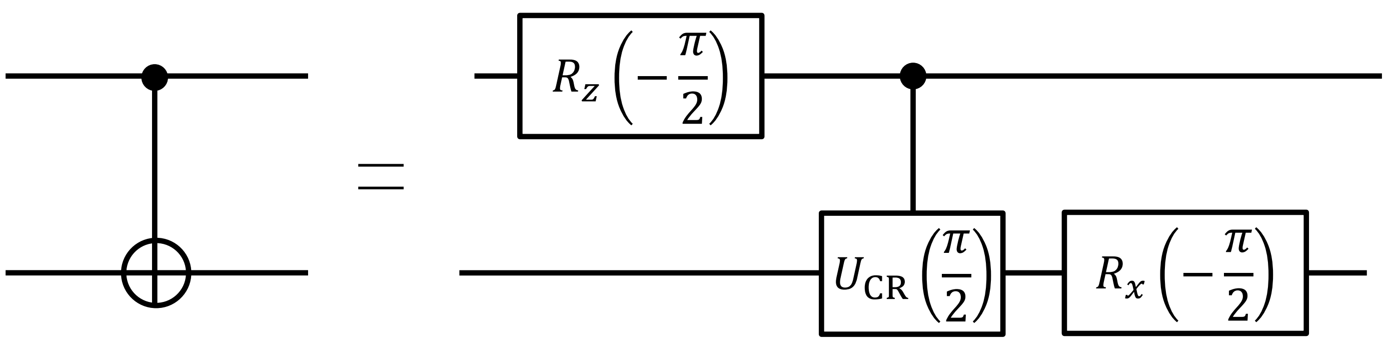
$$= \left(\omega_{\mathrm{q}1} - \omega_{\mathrm{d}} + \frac{J^2}{\Delta_{\mathrm{qq}}}\right)\frac{\sigma_z^1}{2} + \left(\omega_{\mathrm{q}2} - \omega_{\mathrm{d}} - \frac{J^2}{\Delta_{\mathrm{qq}}}\right)\frac{\sigma_z^2}{2} + \underbrace{E_{\mathrm{q}}(t)\sigma_x^1 + \Omega_{\mathrm{CR}}\sigma_z^1\sigma_x^2}_{\text{1Qゲートと同様の形}}$$

$$\Omega_{\mathrm{CR}} = \frac{E_{\mathrm{q}}(t)J}{\Delta_{\mathrm{qq}}}$$

CNOTゲート

$$\omega_{\mathrm{d}} = \omega_{\mathrm{q2}} - \frac{J^2}{\Delta_{\mathrm{qq}}} \longrightarrow H_{\mathrm{d}}^{\mathrm{rot}} = \Omega_{\mathrm{CR}} \sigma_z^1 \sigma_x^2$$

$$\longrightarrow U_{\mathrm{CR}}(\theta = \Omega_{\mathrm{CR}}\tau) = e^{-iH_{\mathrm{d}}^{\mathrm{rot}}\tau} = \begin{pmatrix} \cos\theta & -i\sin\theta & 0 & 0 \\ -i\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & i\sin\theta \\ 0 & 0 & i\sin\theta & \cos\theta \end{pmatrix}$$



$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$U_{\mathrm{CR}}\left(\frac{\pi}{2}\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i & 0 & 0 \\ -i & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & i & 1 \end{pmatrix}$$