

固体素子による量子技術: シリコン

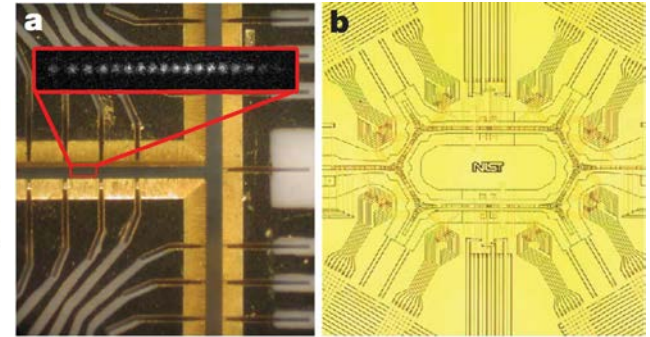
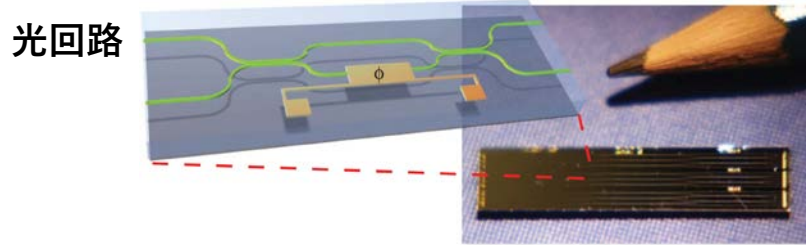
阿部 英介

理化学研究所 創発物性科学研究センター

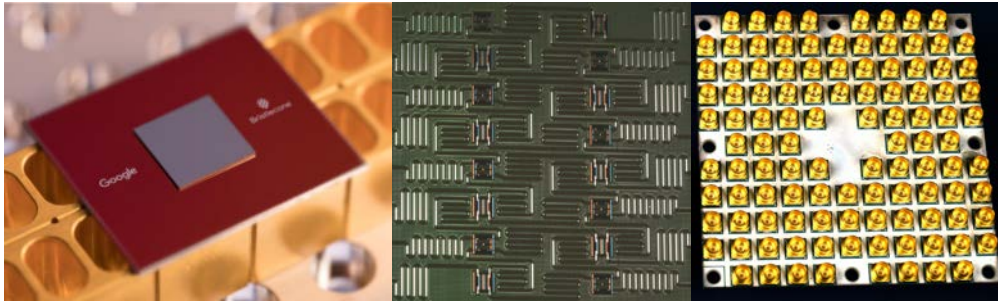
応用物理特別講義A

2020年度春学期後半 金曜4限@~~14-202~~オンライン講義

量子技術のプラットフォーム



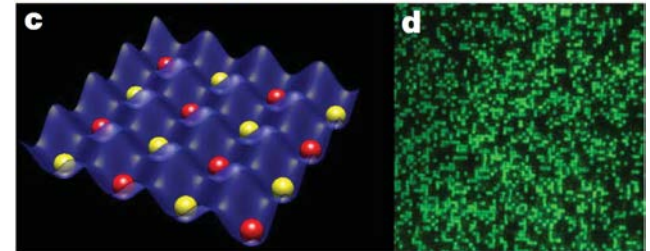
超伝導回路



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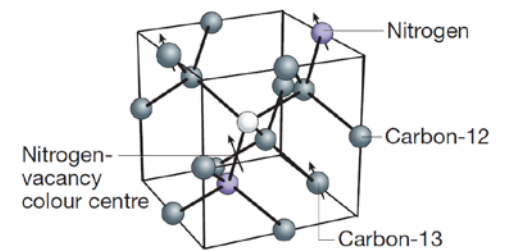
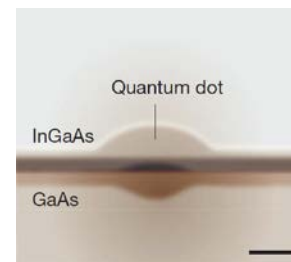
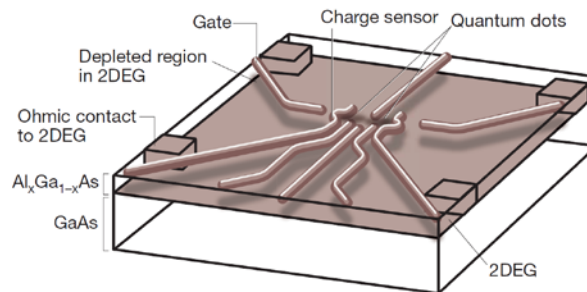
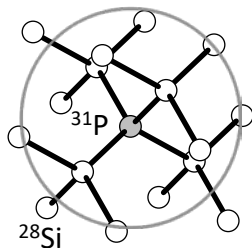
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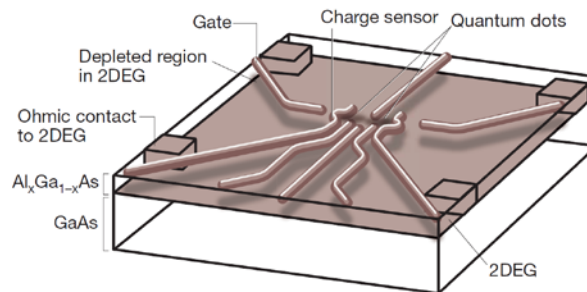
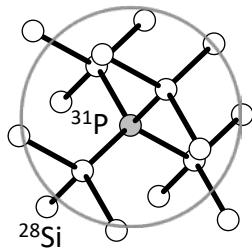
捕捉イオン/冷却原子

半導体スピン

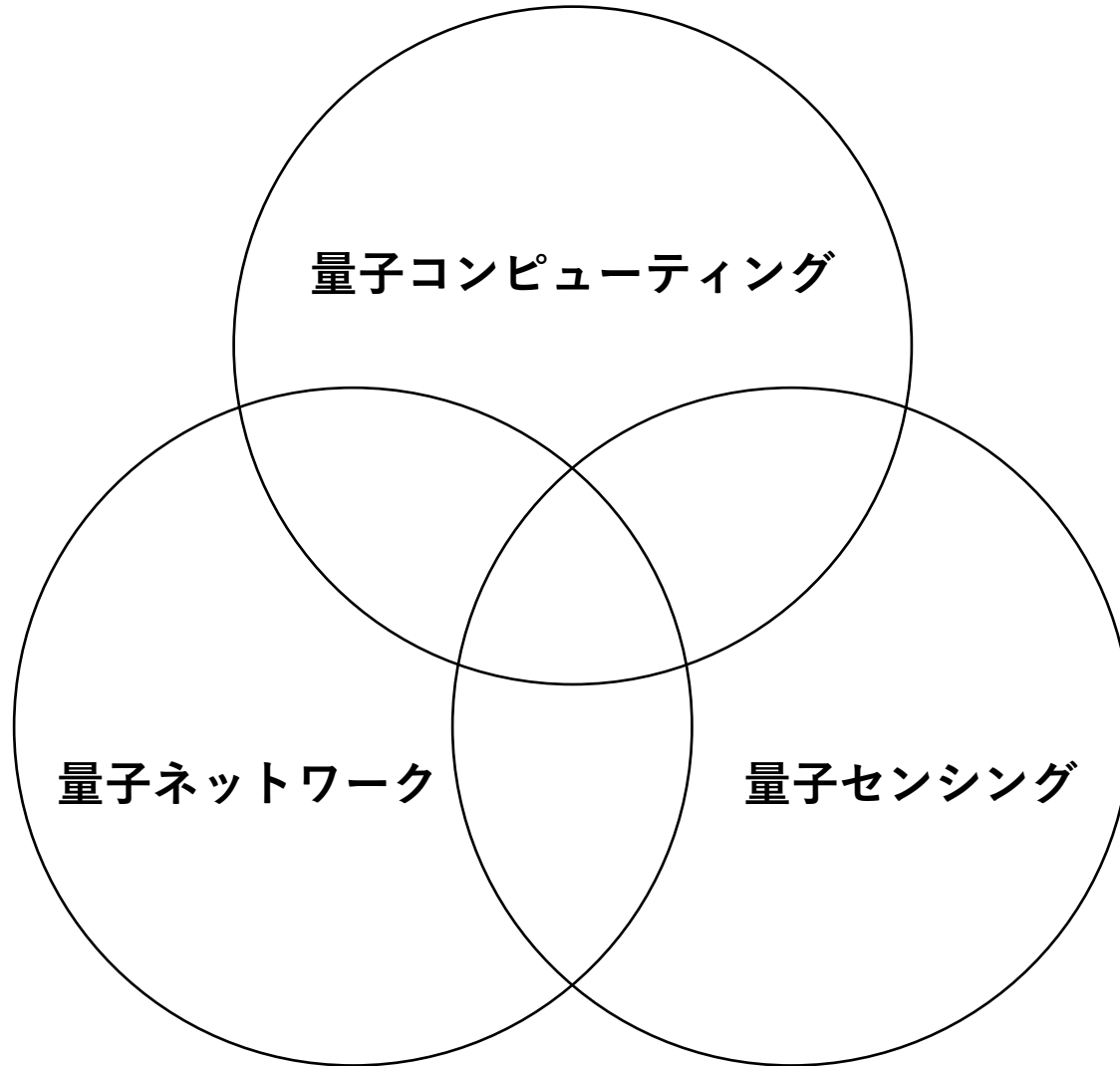


量子技術のプラットフォーム

半導体スピン



量子技術

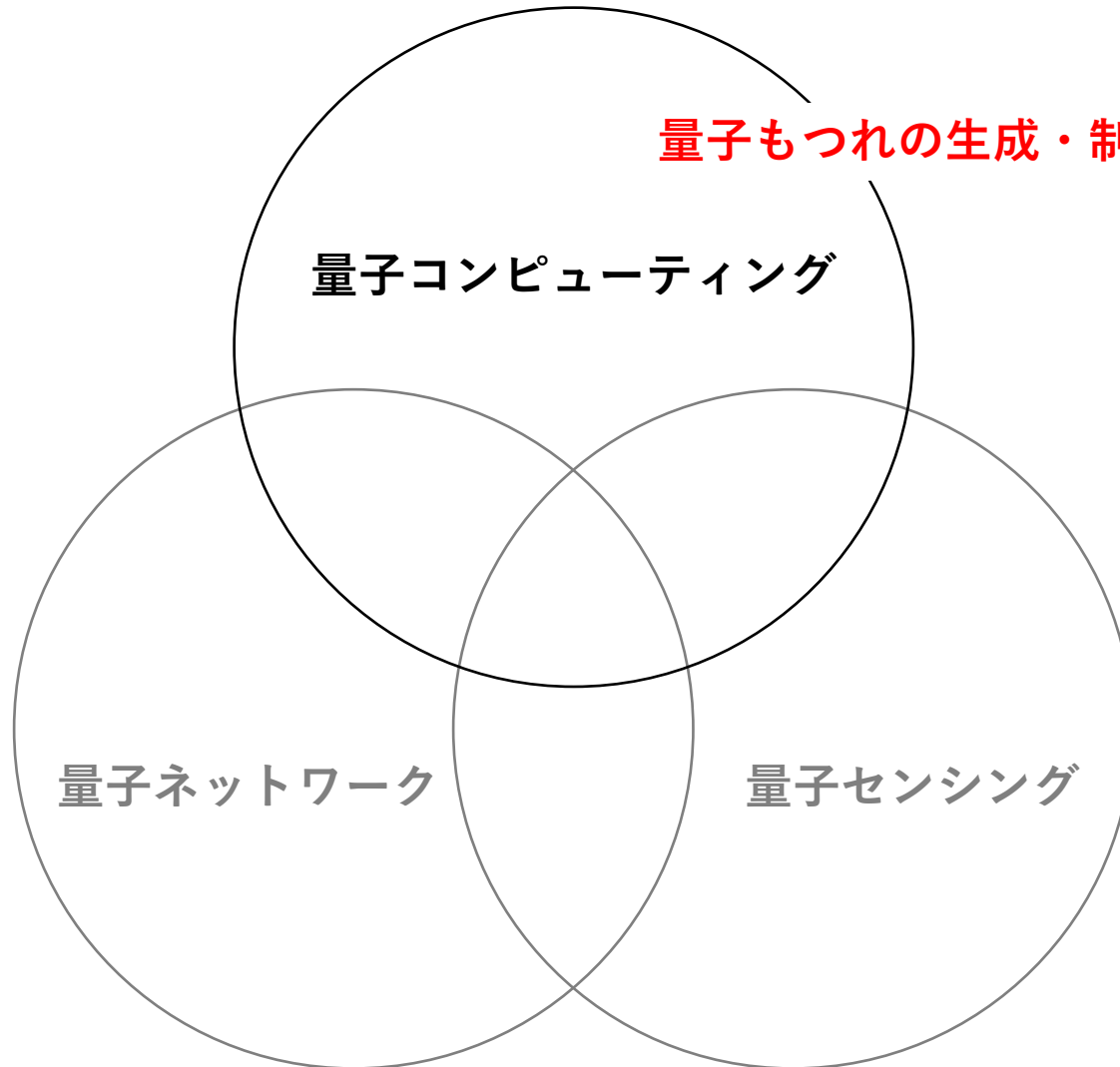


量子コンピューティング

量子ネットワーク

量子センシング

量子技術



量子もつれの生成・制御

量子コンピューティング

量子ネットワーク

量子センシング

講義内容

- **ドナースピン**
 - アンサンブル
 - 単一ドナー

- **量子ドットスピン**
 - MOS量子ドット
 - Si/SiGe量子ドット

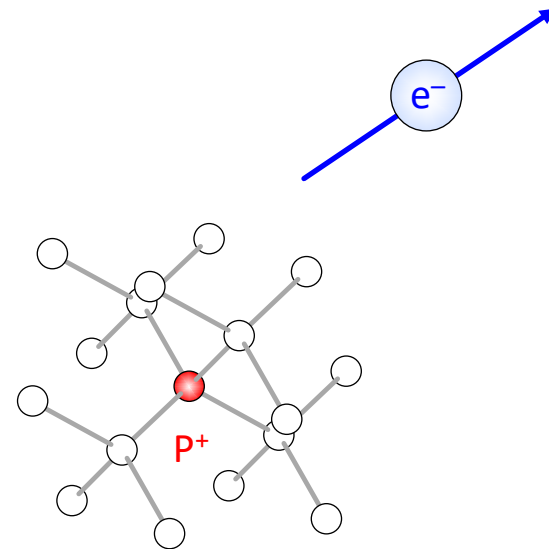
講義内容

- **ドナースピンの**
 - アンサンブル
 - 単一ドナー

- **量子ドットスピンの**
 - MOS量子ドット
 - Si/SiGe量子ドット

シリコン中のリンドナー

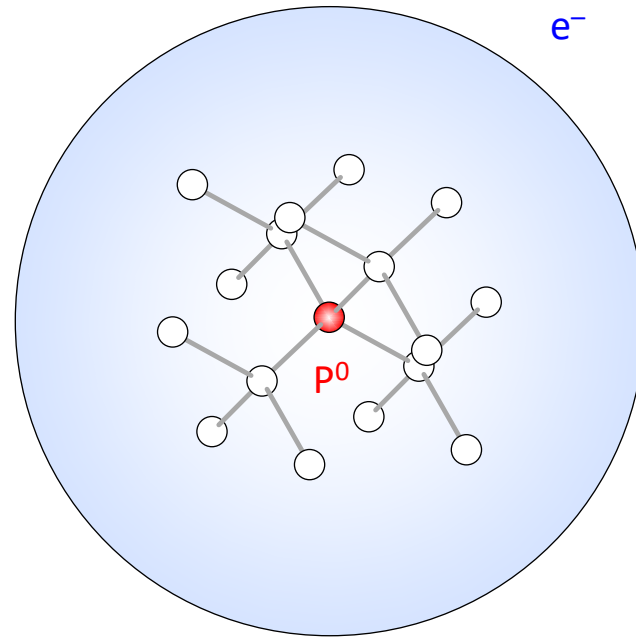
III (13)	IV (14)	V (15)
B	C	N
Al	Si	P
Ga	Ge	As



室温では100%イオン化
→ 自由電子として電気伝導に寄与

シリコン中のリンドナー

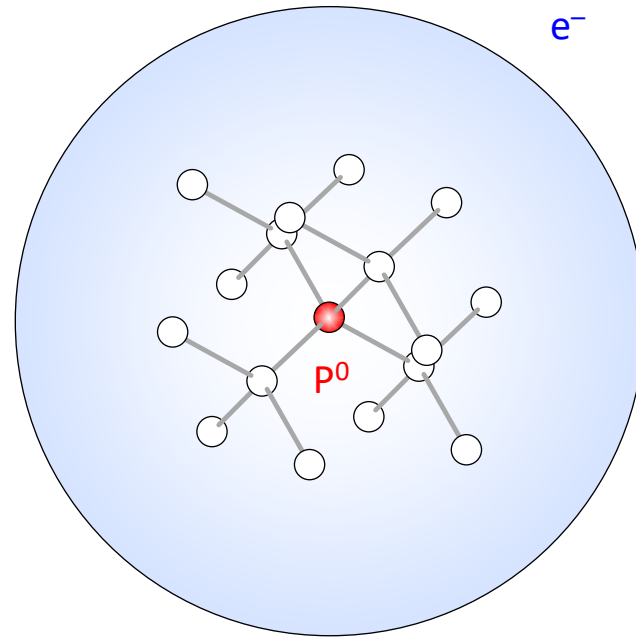
III (13)	IV (14)	V (15)
B	C	N
Al	Si	P
Ga	Ge	As



低温下(< 10 K): 電子はリンに束縛される
→ 水素原子様浅い不純物

シリコン中のリンドナー

III (13)	IV (14)	V (15)
B	C	N
Al	Si	P
Ga	Ge	As

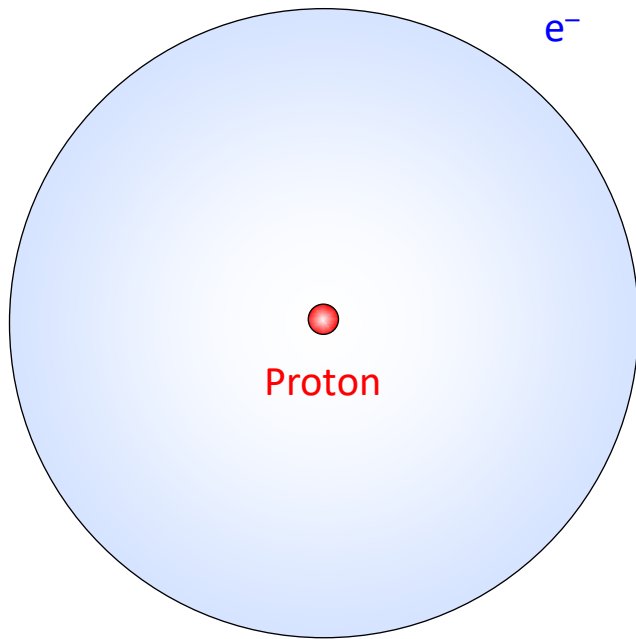


有効Bohr半径: $a_B^* = 3.2 \text{ nm}$

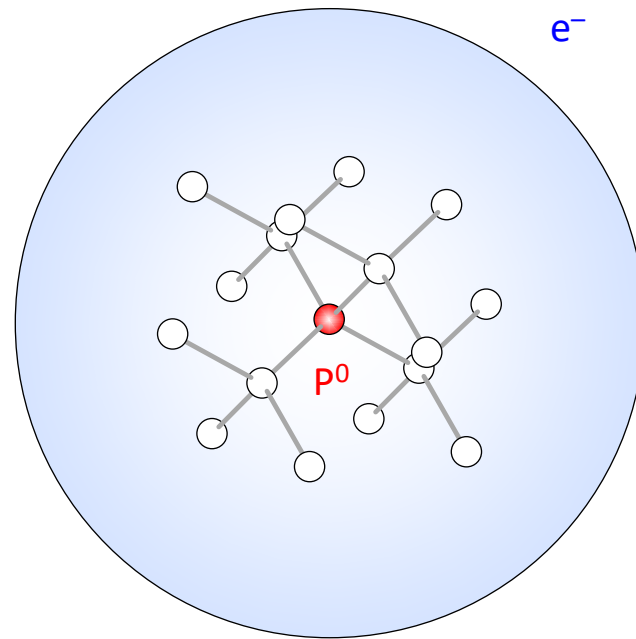
格子定数: $a_{\text{Si}} = 0.54 \text{ nm}$

シリコン中のリンドナー

水素原子



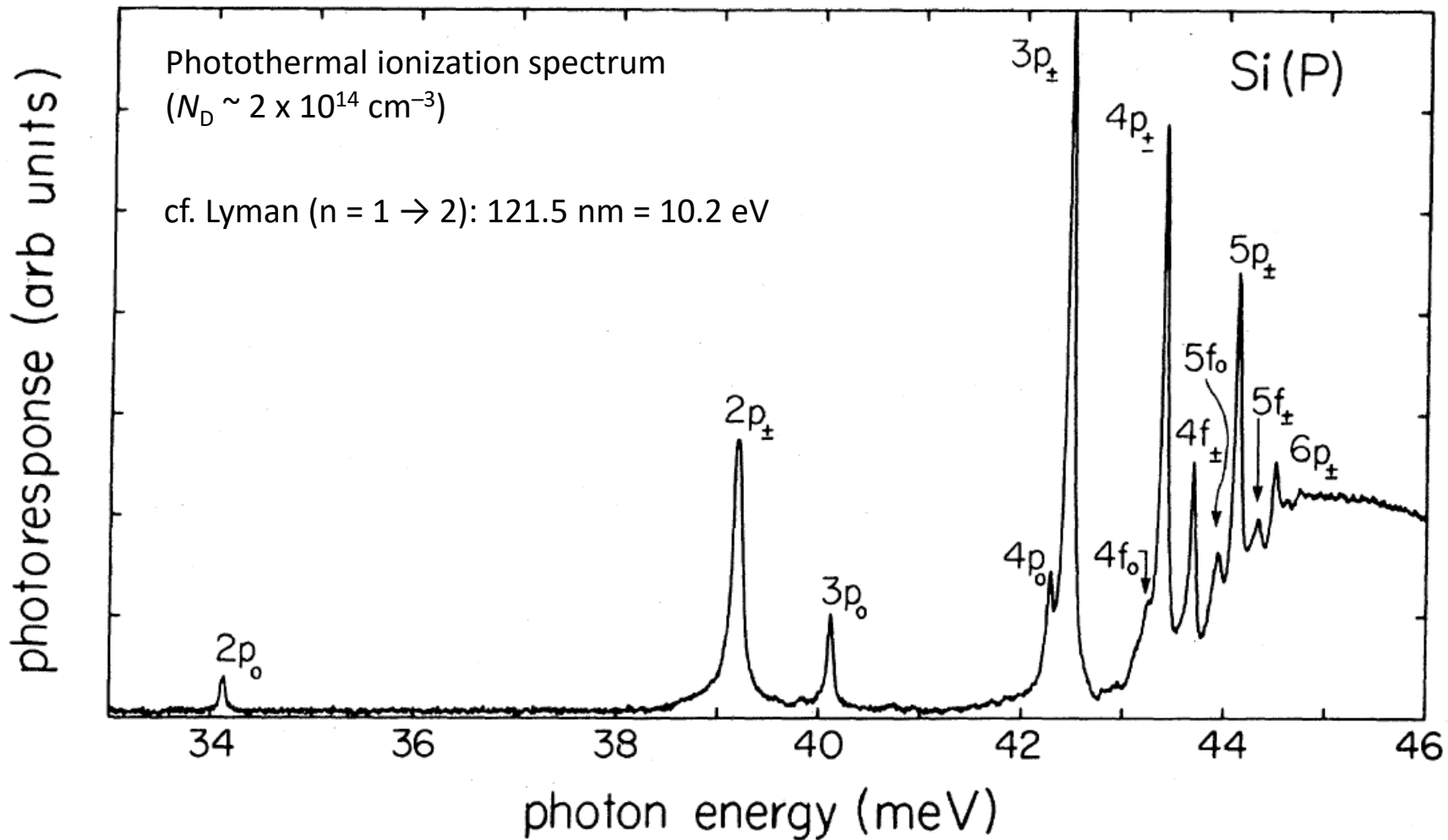
Bohr半径: $a_B = \epsilon_0 h^2 / \pi m e^2 = 0.053 \text{ nm}$



有効Bohr半径: $a_B^* = 3.2 \text{ nm}$

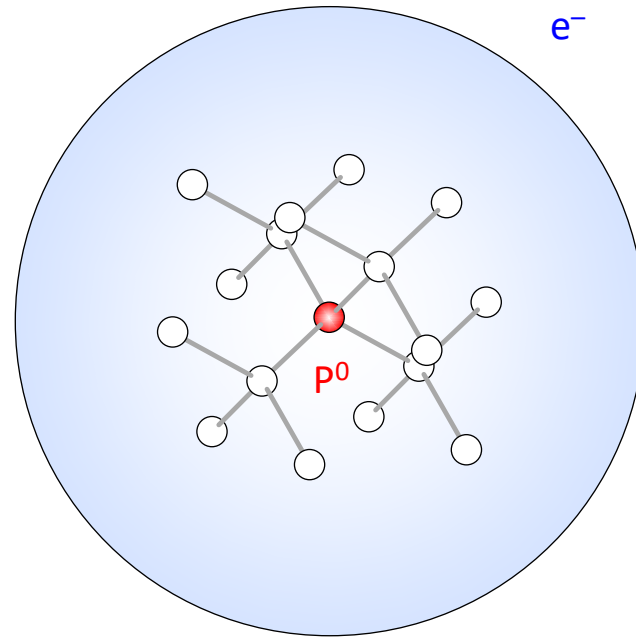
格子定数: $a_{Si} = 0.54 \text{ nm}$

シリコン中のリンドナー



シリコン中のリンドナー

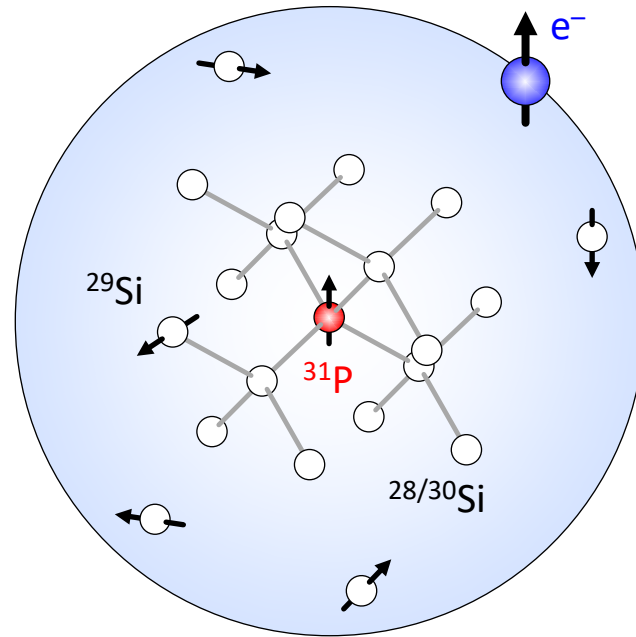
III (13)	IV (14)	V (15)
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シリコン中のリンドナー

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B	C	N
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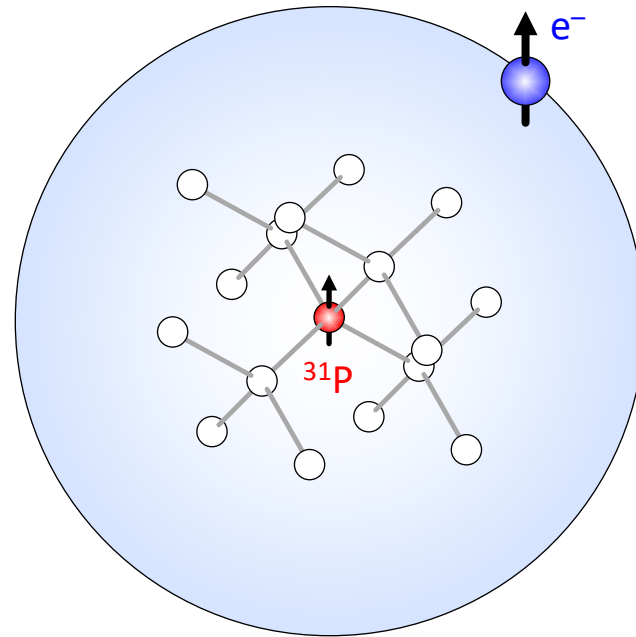
磁気環境

$^{28}\text{Si} : ^{29}\text{Si} (I = \frac{1}{2}) : ^{30}\text{Si} = 92.2\% : 4.7\% : 3.1\%$

$^{31}\text{P} (I = \frac{1}{2}) = 100\%$

シリコン中のリンドナー

III (13)	IV (14)	V (15)
B	C	N
Al	Si	P
Ga	Ge	As



磁気環境

同位体制御 $^{28}\text{Si} \rightarrow 99.995\%$

$^{31}\text{P} (I = 1/2) = 100\%$

シリコン中のリンドナー

スピンハミルトニアン

$$H_0 = \gamma_e B_0 S_z - \gamma_P B_0 I_z + a_0 S_z I_z$$

↑ ↑ ↙
電子スピン 核スピン 電子-核(超微細)
ゼーマン ゼーマン 相互作用

$$S_z = \frac{\sigma_z}{2} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$I_z = \frac{\sigma_z}{2} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$B_0 \approx 350 \text{ mT}$$

$$\gamma_e/2\pi = 27.97 \text{ GHz/T}$$

$$\gamma_P/2\pi = 17.23 \text{ MHz/T} \quad (\gamma_H/2\pi = 42.58 \text{ MHz/T})$$

$$a_0/2\pi = 117.53 \text{ MHz} \quad (a_H/2\pi = 1420.4 \text{ MHz, "21-cm line"})$$

シリコン中のリンドナー

スピンハミルトニアン

$$S_z = \frac{\sigma_z}{2} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$I_z = \frac{\sigma_z}{2} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$H_0 = \gamma_e B_0 S_z - \gamma_P B_0 I_z + a_0 S_z I_z$$

$$= \frac{\gamma_e B_0}{2} (\sigma_z \otimes I) - \frac{\gamma_P B_0}{2} (I \otimes \sigma_z) + \frac{a_0}{4} (\sigma_z \otimes \sigma_z)$$

$$= \frac{\gamma_e B_0}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{\gamma_P B_0}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{a_0}{4} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\gamma_e B_0}{2} - \frac{\gamma_P B_0}{2} + \frac{a_0}{4} & 0 & 0 & 0 \\ 0 & \frac{\gamma_e B_0}{2} + \frac{\gamma_P B_0}{2} - \frac{a_0}{4} & 0 & 0 \\ 0 & 0 & -\frac{\gamma_e B_0}{2} - \frac{\gamma_P B_0}{2} - \frac{a_0}{4} & 0 \\ 0 & 0 & 0 & -\frac{\gamma_e B_0}{2} + \frac{\gamma_P B_0}{2} + \frac{a_0}{4} \end{pmatrix}$$

$|\uparrow\uparrow\rangle$ $|\uparrow\downarrow\rangle$ $|\downarrow\uparrow\rangle$ $|\downarrow\downarrow\rangle$

シリコン中のリンドナー

スピンハミルトニアン

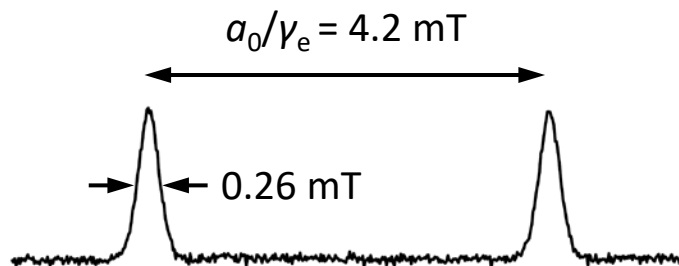
$$H_0 = \gamma_e B_0 S_z - \gamma_P B_0 I_z + a_0 S_z I_z$$

$$B_0 \approx 350 \text{ mT}$$

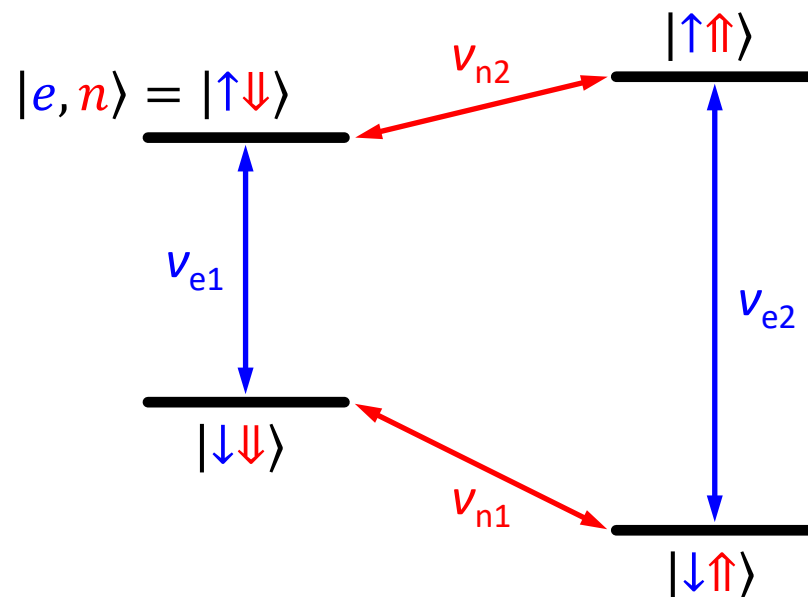
$$\gamma_e/2\pi = 27.97 \text{ GHz/T}$$

$$\gamma_P/2\pi = 17.23 \text{ MHz/T}$$

$$a_0/2\pi = 117.53 \text{ MHz}$$



アンサンブル電子スピン共鳴(natSi、磁場掃引)



$$\nu_{e1} = \gamma_e B_0 - a_0/2$$

$$\nu_{e2} = \gamma_e B_0 + a_0/2$$

$$\nu_{n1} = a_0/2 + \gamma_P B_0$$

$$\nu_{n2} = a_0/2 - \gamma_P B_0$$

ディビンチェンゾの要請

1. スケーラブルな量子ビット列
2. 初期化
3. 長いコヒーレンス時間
4. ユニバーサル量子ゲート
5. 射影測定



D. DiVincenzo

©RWTH Aachen U.

ディビンチェンゾの要請

1. スケーラブルな量子ビット列

2. 初期化

3. 長いコヒーレンス時間

→ $T_{2e} = 10 \text{ s}$ (*1), $T_{2n} = 180 \text{ min}$ (*2)

4. ユニバーサル量子ゲート

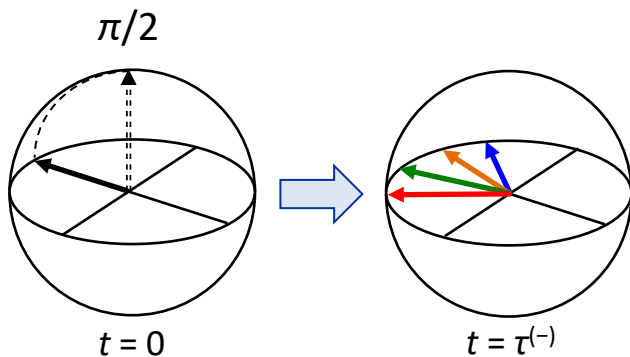
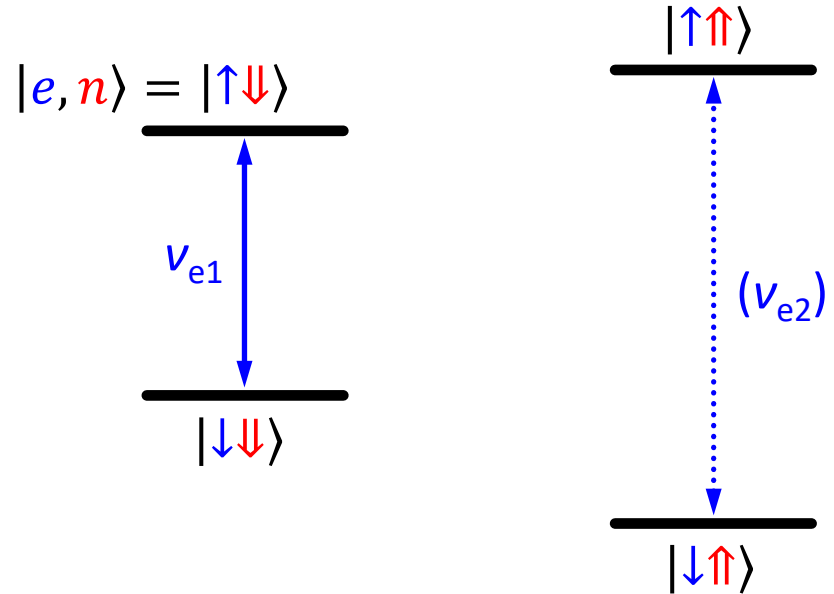
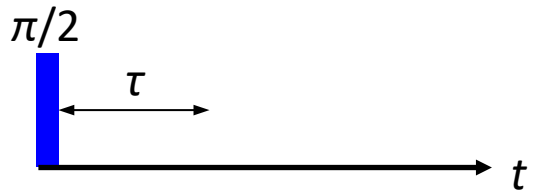
5. 射影測定

*1: Nature Mat. **11**, 143 (2012) Tyryshkin *et al.*

*2: Science **342**, 830 (2013) Saedi *et al.*

Fortschr. Phys. **48**, 771 (2000) DiVincenzo

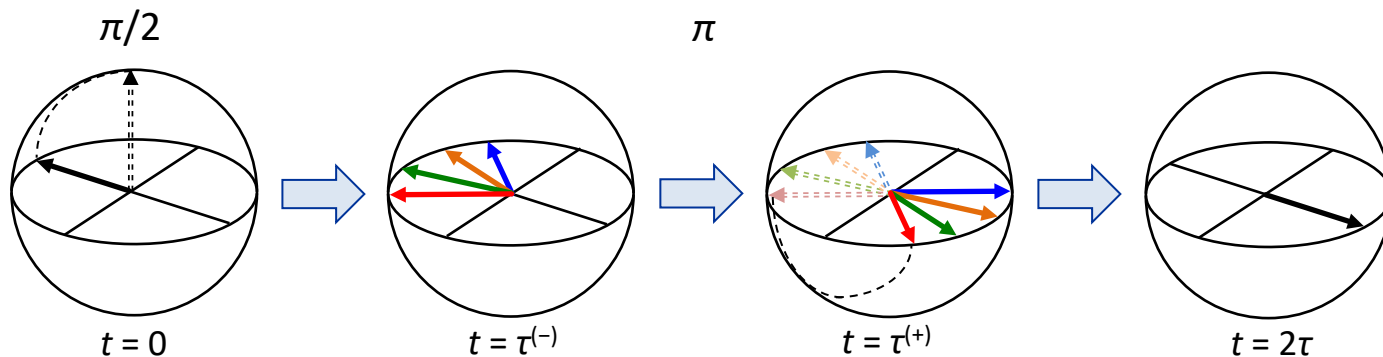
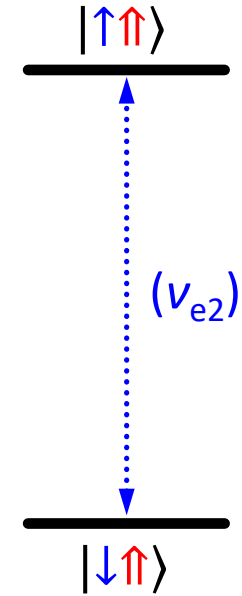
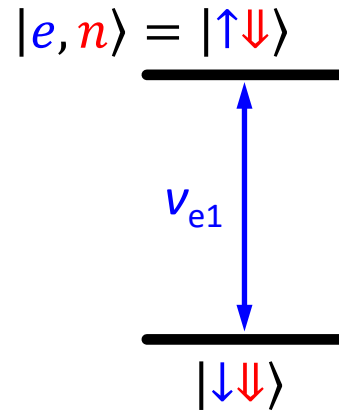
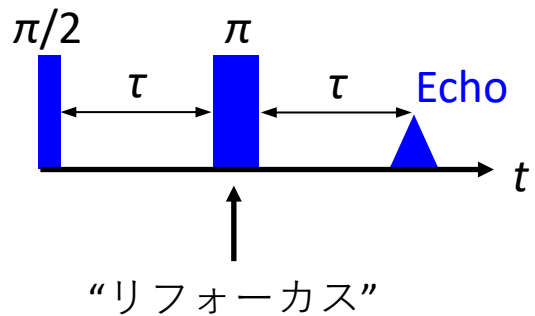
電子スピンのコヒーレンス: T_{2e}



不均一性(^{29}Si 由来)による見かけの減衰

$$T_{2e}^* \approx (\gamma_e 0.26 \text{ mT})^{-1} \approx 130 \text{ ns} \ll T_{2e} \approx 500 \mu\text{s}$$

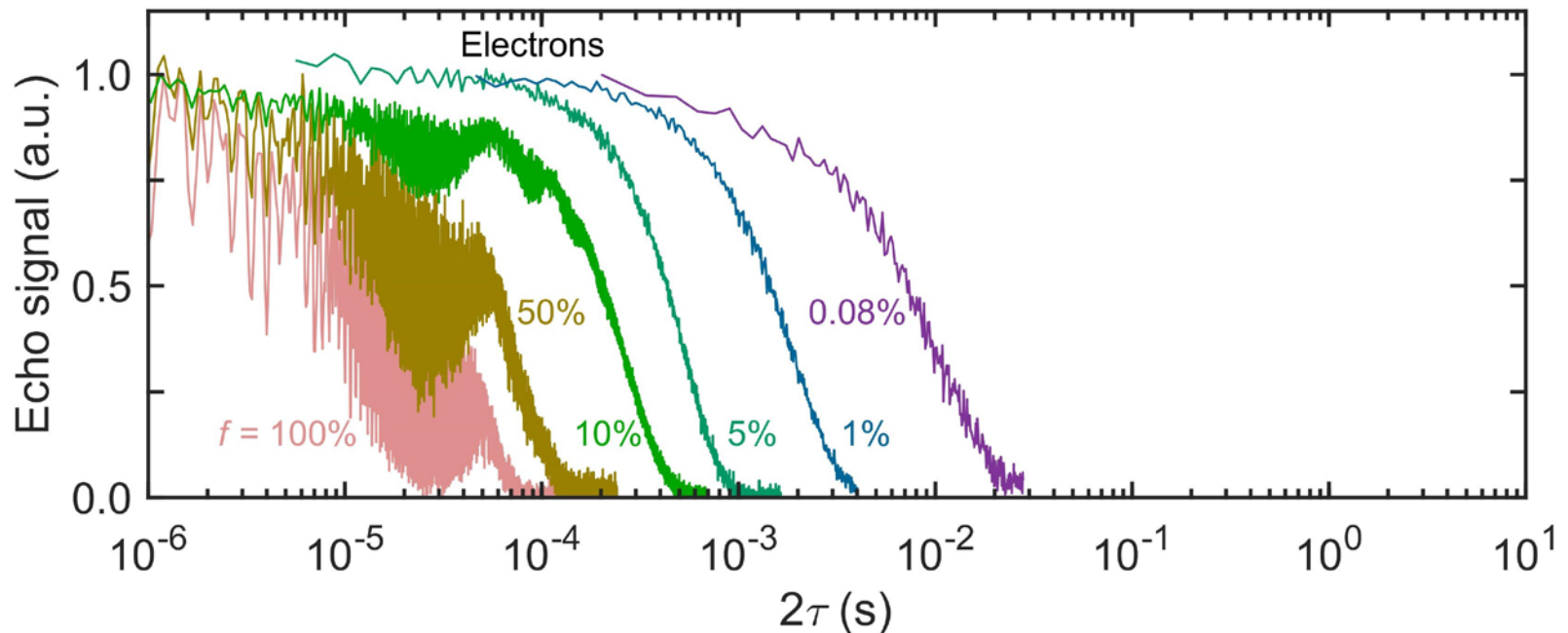
電子スピンのコヒーレンス: T_{2e}



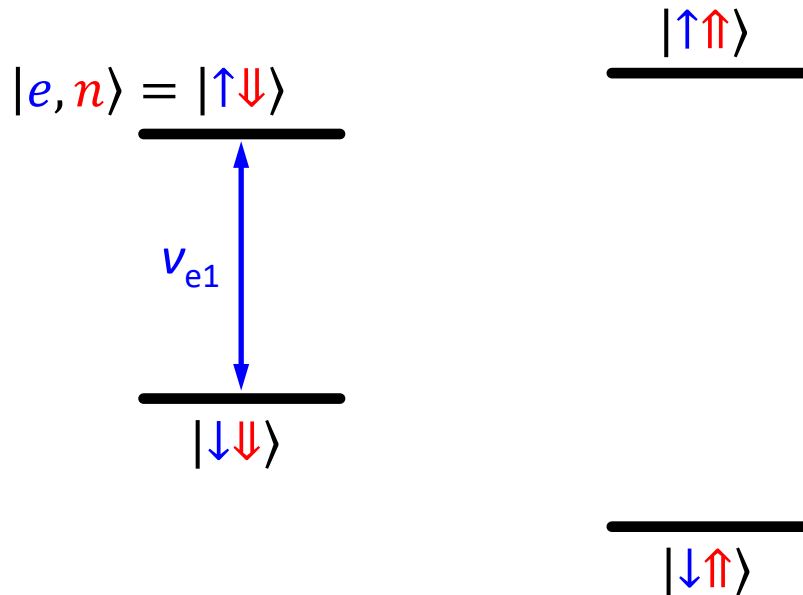
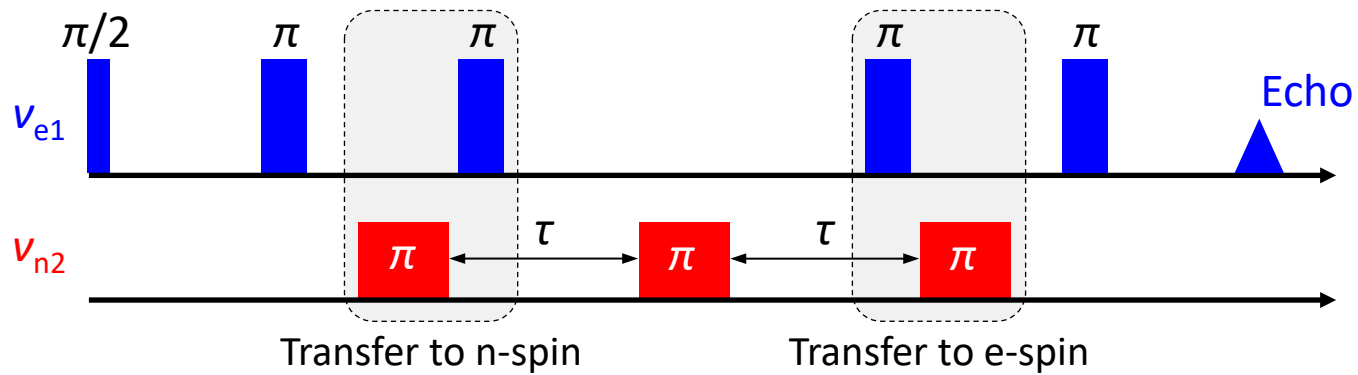
電子スピンコヒーレンス: T_{2e}

Electron spin coherence of phosphorus donors in silicon: Effect of environmental nuclei

Eisuke Abe,^{1,2,*} Alexei M. Tyryshkin,³ Shinichi Tojo,² John J. L. Morton,^{1,4} Wayne M. Witzel,⁵ Akira Fujimoto,² Joel W. Ager,⁶ Eugene E. Haller,^{6,7} Junichi Isoya,⁸ Stephen A. Lyon,³ Mike L. W. Thewalt,⁹ and Kohei M. Itoh²

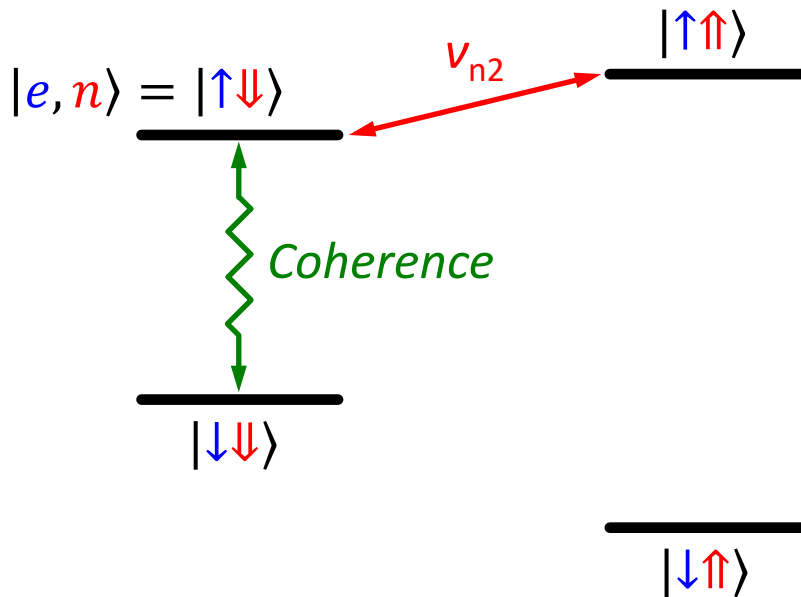
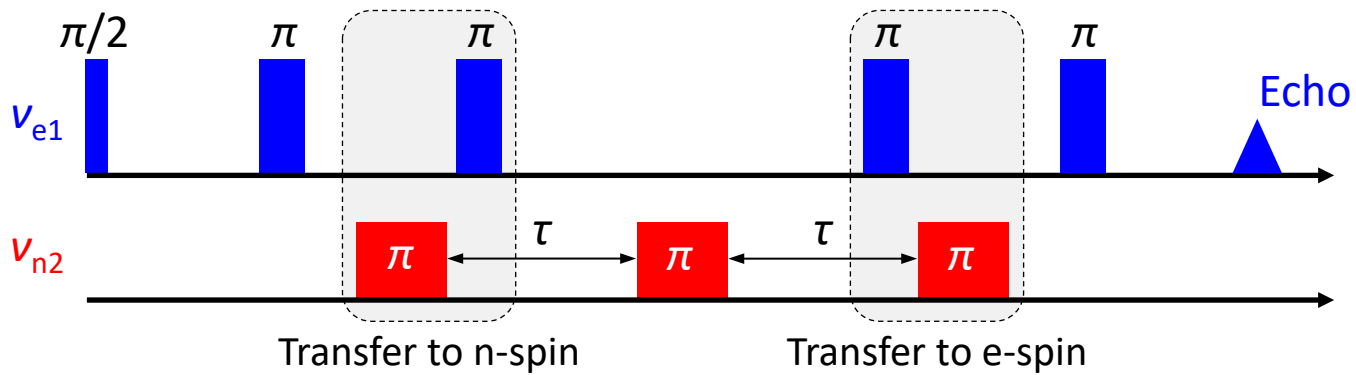


核スピンコヒーレンス: T_{2n0}



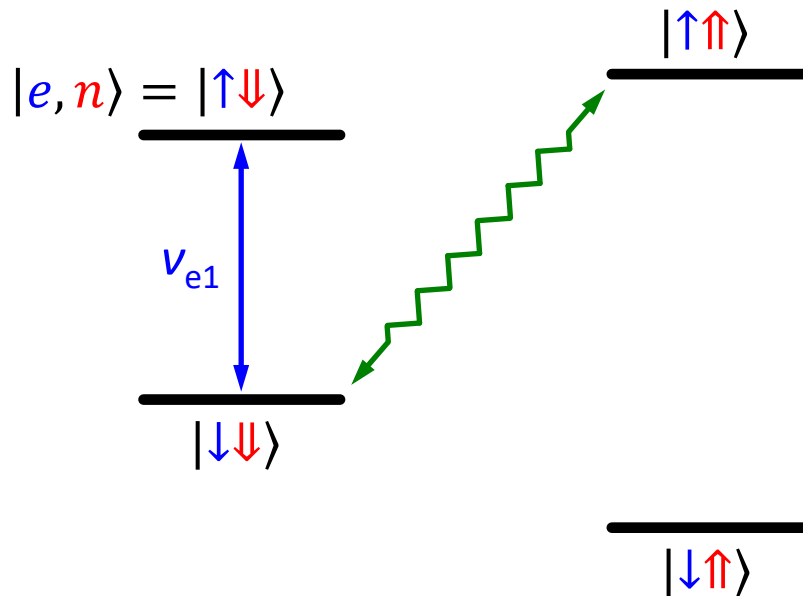
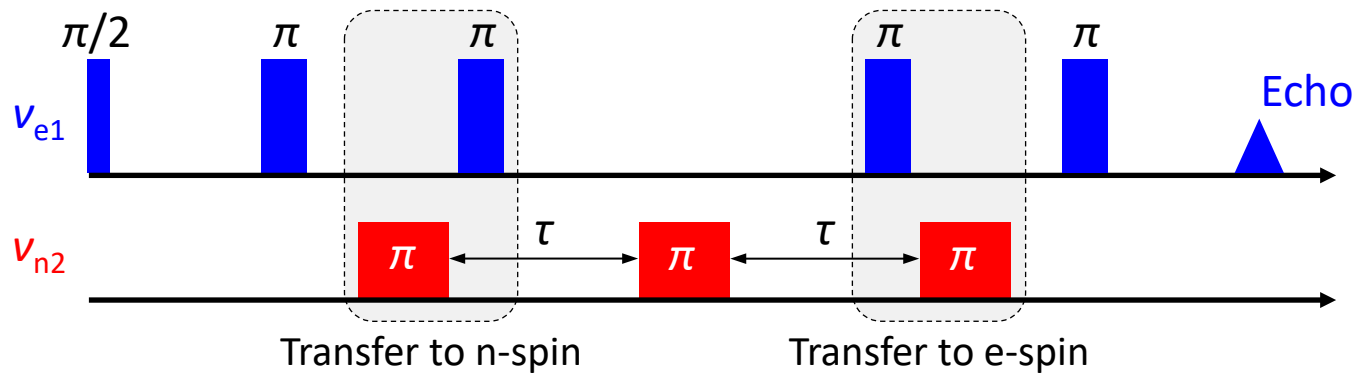
Electron Nuclear Double Resonance

核スピンコヒーレンス: T_{2n0}



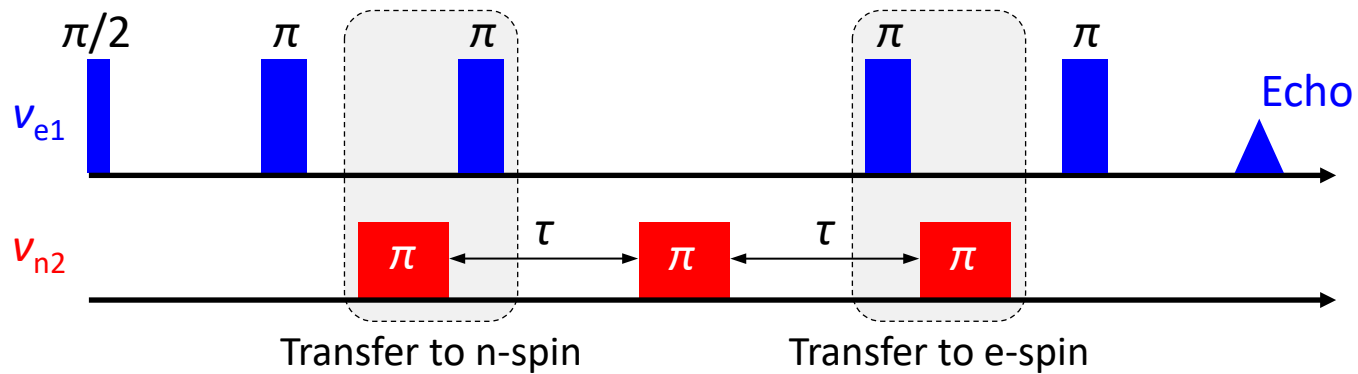
Electron Nuclear Double Resonance

核スピンコヒーレンス: T_{2n0}



Electron Nuclear Double Resonance

核スピンコヒーレンス: T_{2n0}



Electron Nuclear Double Resonance



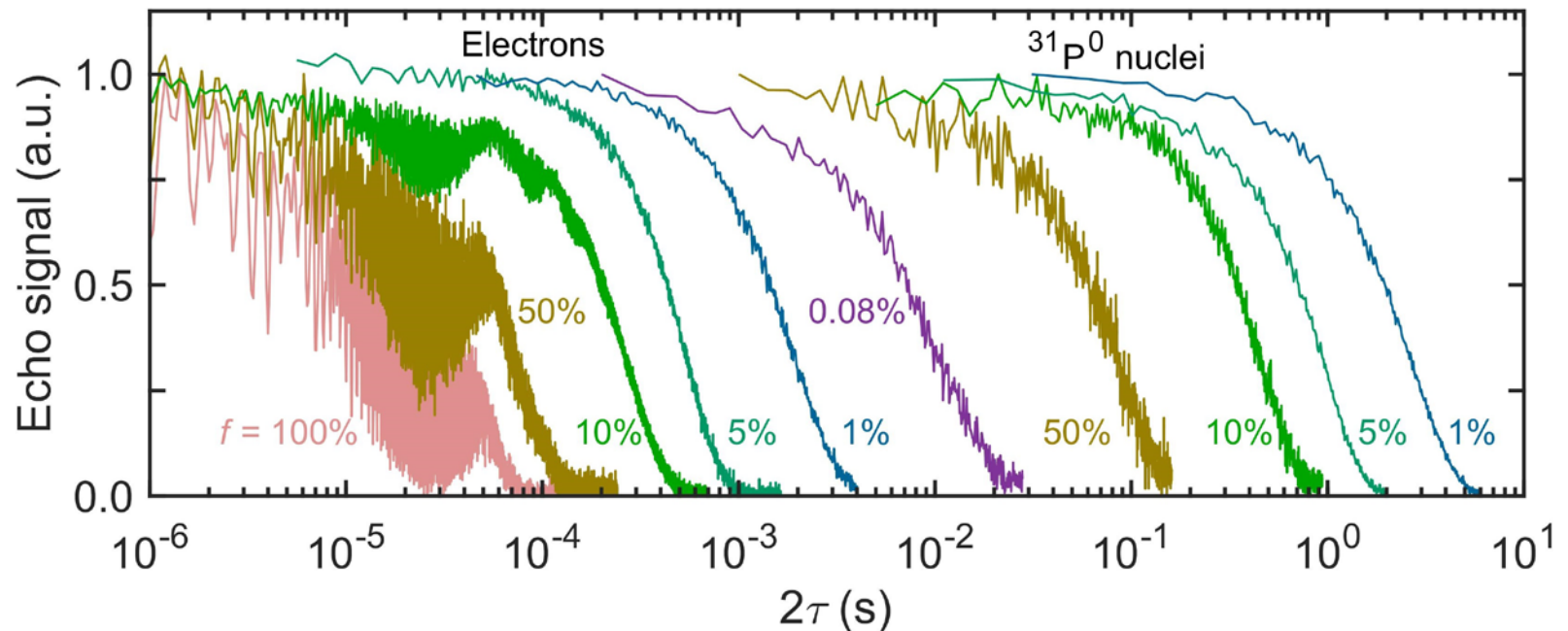
核スピンコヒーレンス: T_{2n0}

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Eisuke Abe,^{1,2,*} Alexei M. Tyryshkin,³ Shinichi Tojo,² John J. L. Morton,^{1,4} Wayne M. Witzel,⁵ Akira Fujimoto,² Joel W. Ager,⁶ Eugene E. Haller,^{6,7} Junichi Isoya,⁸ Stephen A. Lyon,³ Mike L. W. Thewalt,⁹ and Kohei M. Itoh²

Nuclear spin decoherence of neutral ^{31}P donors in silicon: Effect of environmental ^{29}Si nuclei

Evan S. Petersen,¹ A. M. Tyryshkin,¹ J. J. L. Morton,² E. Abe,³ S. Tojo,³ K. M. Itoh,³ M. L. W. Thewalt,⁴ and S. A. Lyon¹



Method of Polarizing Nuclei in Paramagnetic Substances

G. FEHER

Bell Telephone Laboratories, Murray Hill, New Jersey

(Received May 31, 1956)

OVERHAUSER¹ has shown that a saturation of the electron spin resonance leads to a large enhancement of the nuclear polarization. A necessary condition for this enhancement is that the nuclei relax via the electrons whose resonance is being saturated.

Polarization of Phosphorus Nuclei in Silicon

G. FEHER AND E. A. GERE

Bell Telephone Laboratories, Murray Hill, New Jersey

(Received May 31, 1956)

IN the preceding Letter a scheme for polarizing nuclei was described. This Letter deals with the experimental verification of the scheme.

Wolf Prize in Chemistry (2006)

1950年代に固体物理で業績を挙げたのち
生物物理(主に光合成)に転向

“...delighted to hear that EPR in Si is still alive and doing well and has branched out into new and exciting areas” (email to E.A. Dec. 2016)



G. Feher
(1924–2017)

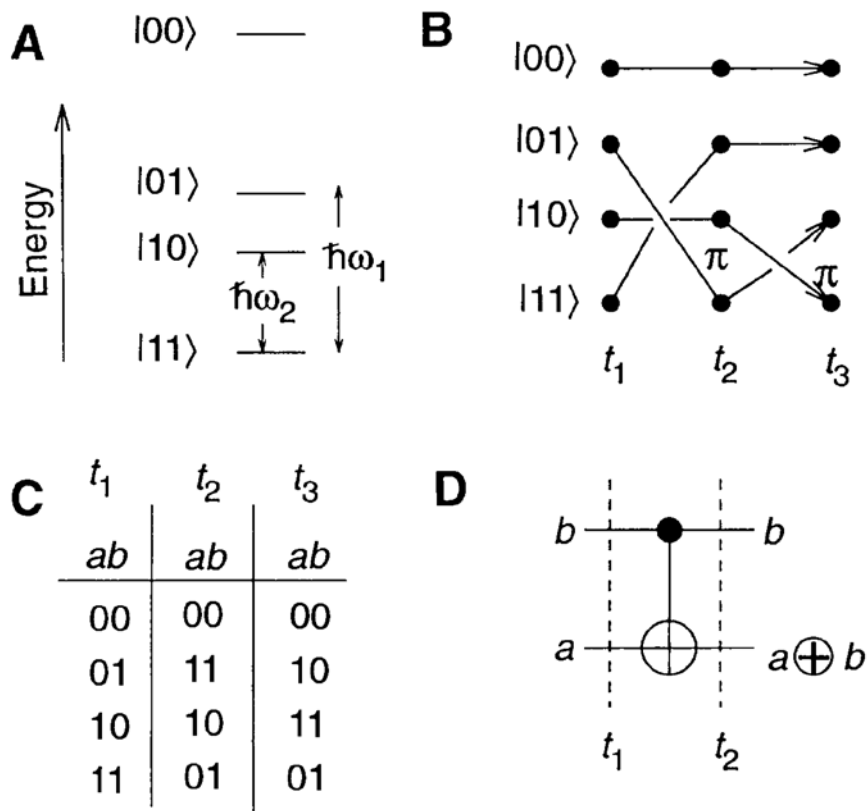
©R.A. Icaacson

Phys. Rev. **103**, 500 (1956) Feher

Phys. Rev. **103**, 501 (1956) Feher & Gere

Quantum Computation

David P. DiVincenzo



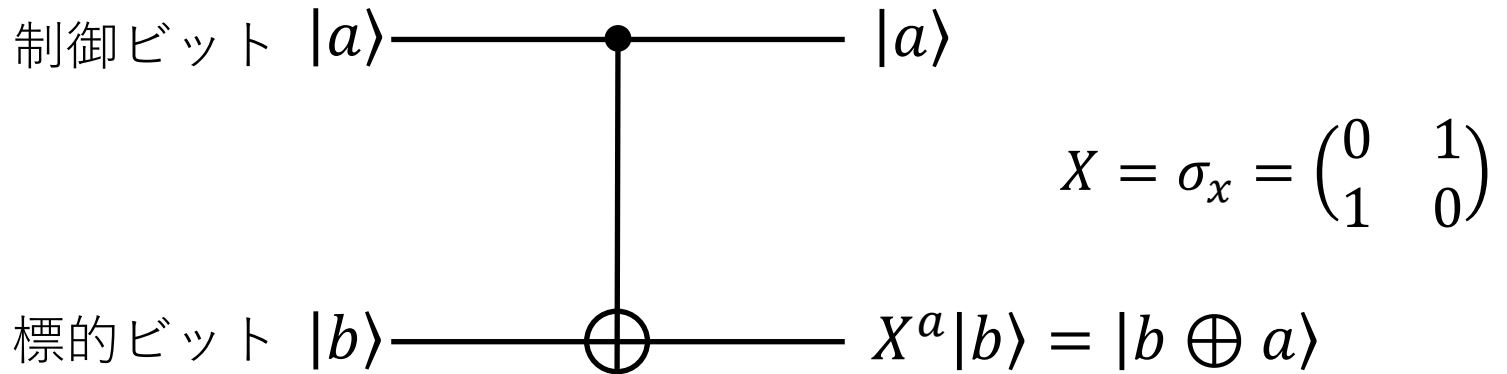
The XOR protocol is very closely related to procedures invented long ago in the field of resonance spectroscopies (13). In 1956, Feher introduced a procedure for polarization transfer in electron-nucleus double resonance (ENDOR), which contains the XOR protocol just discussed.

for many purposes in physics, chemistry, and biology, it is highly desirable to move the spin state of an electron onto a nearby nucleus. The fact that this procedure also performs an interesting logical function, XOR, was not previously noted by ENDOR spectroscopists.

Fig. 2. The action of the two-qubit XOR gate.

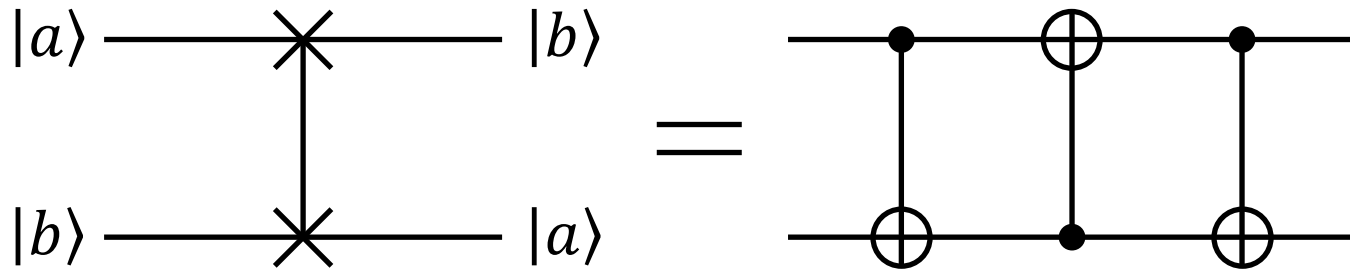
制御NOTゲート

制御ビットが1のとき標的ビットを反転



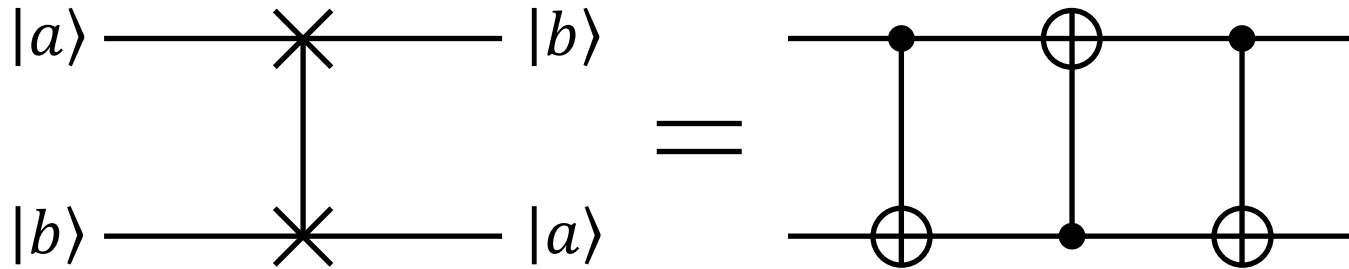
$$\begin{aligned} \text{CNOT}|00\rangle &= |00\rangle \\ \text{CNOT}|01\rangle &= |01\rangle \\ \text{CNOT}|10\rangle &= |11\rangle \\ \text{CNOT}|11\rangle &= |10\rangle \end{aligned} \iff \text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

SWAPゲート



$$\begin{aligned} \text{SWAP}|00\rangle &= |00\rangle \\ \text{SWAP}|01\rangle &= |10\rangle \\ \text{SWAP}|10\rangle &= |01\rangle \\ \text{SWAP}|11\rangle &= |11\rangle \end{aligned} \iff \text{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

SWAPゲート

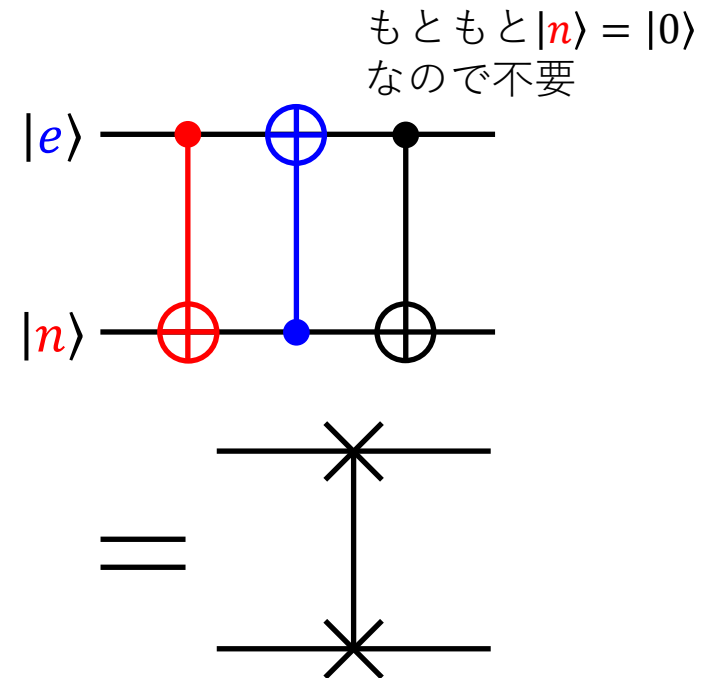
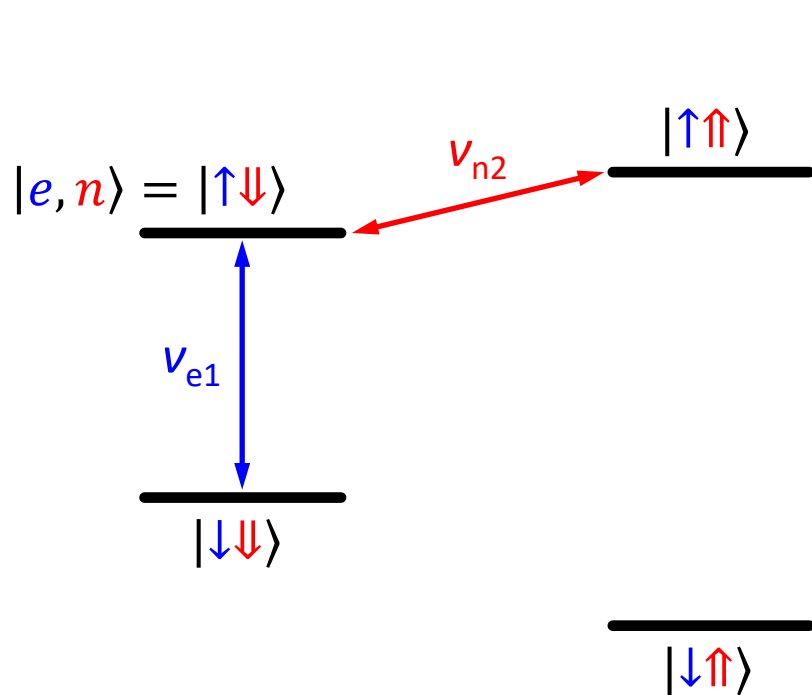
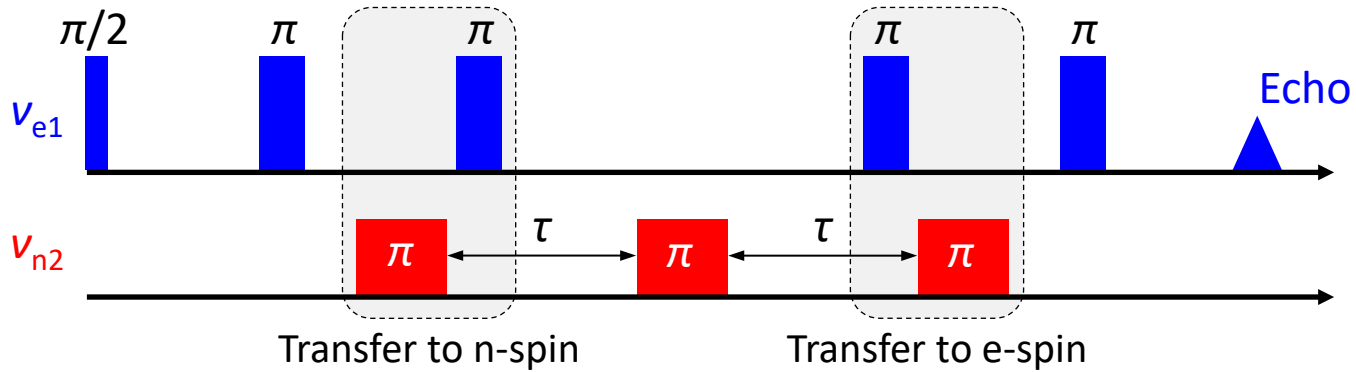


SWAPに必要な手続き

- 第1ビットの情報を第2ビットへ書き込む
- 第2ビットの情報を第1ビットへ書き込む
- 第1ビットの情報を第1ビットから消去する
- 第2ビットの情報を第2ビットから消去する

$$\begin{aligned} |a\rangle|b\rangle &\xrightarrow{C_{12}} |a\rangle|b \oplus a\rangle \\ &\xrightarrow{C_{21}} |a \oplus (b \oplus a)\rangle|b \oplus a\rangle = |b\rangle|b \oplus a\rangle \\ &\xrightarrow{C_{12}} |b\rangle|(b \oplus a) \oplus b\rangle = |b\rangle|a\rangle \end{aligned}$$

ENDOR と SWAP

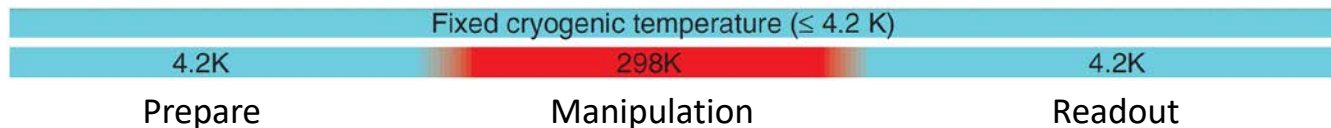
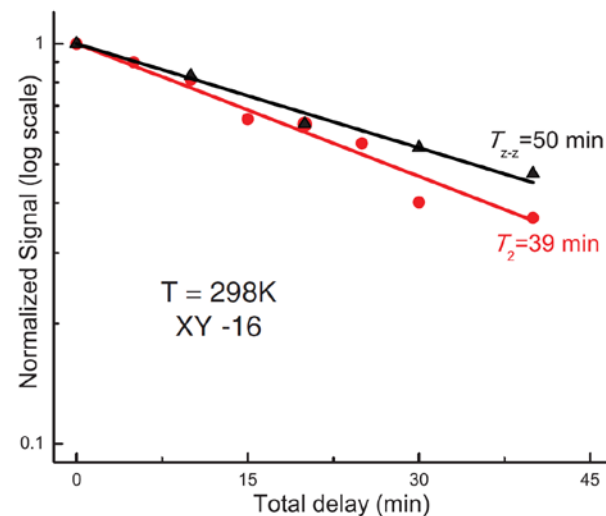
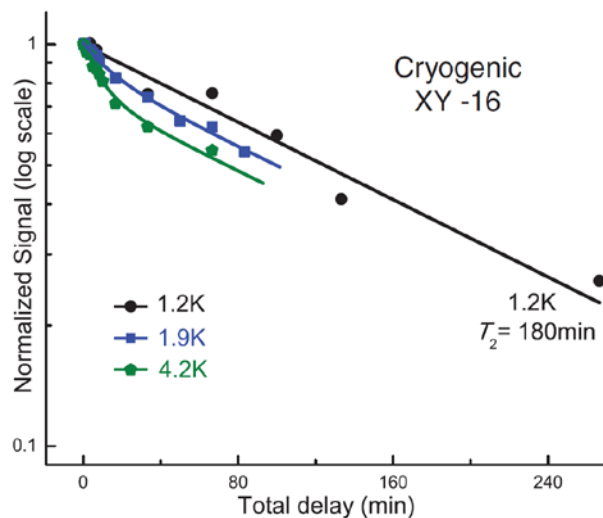
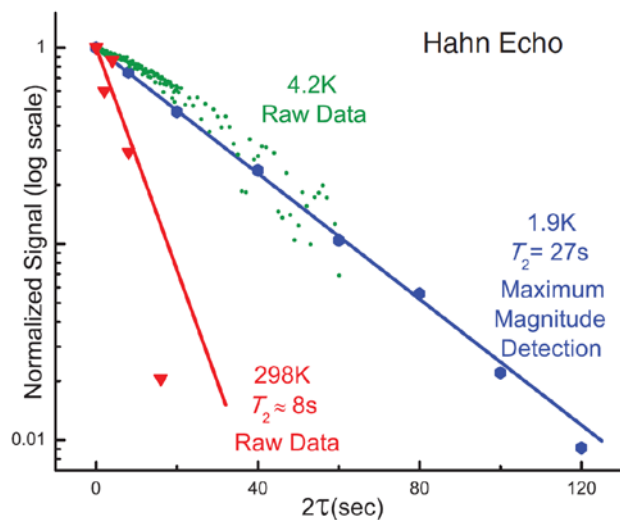


核スピンコヒーレンス: T_{2n+}

Room-Temperature Quantum Bit Storage Exceeding 39 Minutes Using Ionized Donors in Silicon-28

Auger-electron-detected NMR

Kamyar Saeedi,¹ Stephanie Simmons,² Jeff Z. Salvail,¹ Phillip Dluhy,¹ Helge Riemann,³ Nikolai V. Abrosimov,³ Peter Becker,⁴ Hans-Joachim Pohl,⁵ John J. L. Morton,⁶ Mike L. W. Thewalt^{1*}



レポート課題 2 (20点)

低磁場におけるSi:Pのスピンハミルトニアンは

$$H_1 = \gamma_e B_0 S_z - \gamma_P B_0 I_z + a_0 \mathbf{S} \cdot \mathbf{I}$$

と書かれる。ただし

$$\mathbf{S} \cdot \mathbf{I} = \frac{1}{2} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix} = \frac{1}{4} (\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z)$$

である。

(1) H_1 を4x4行列で表せ。基底は $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$ の順とする。

(2) H_1 を対角化し、4つ固有値を B_0 の関数として求めよ。

ヒント:(1)より $|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle$ は対角的であることが示されるので、 $|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle$ に対する2x2行列Aについて永年方程式 $|A - \lambda I| = 0$ を解けばよい。

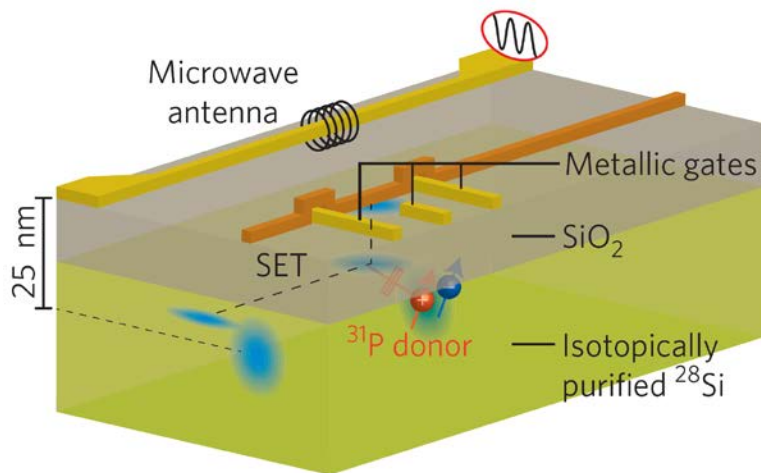
(3) 十分大きな磁場($B_0 \gg a_0/\gamma_e$)では、 H_1 と H_0 の固有値は一致することを確認せよ。

講義内容

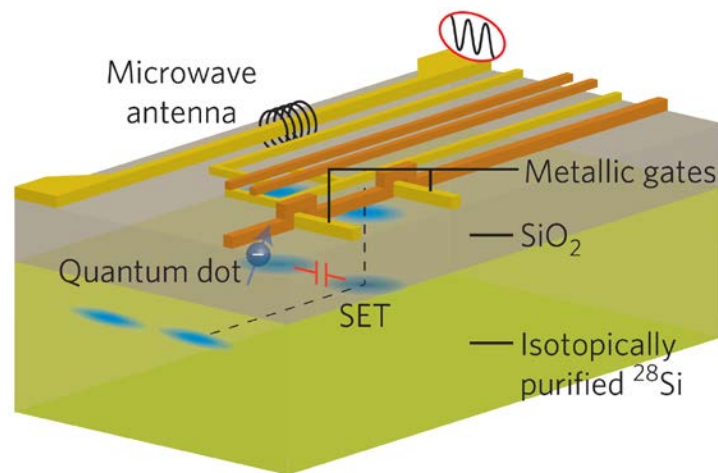
- **ドナースピン**
 - アンサンブル
 - 単一ドナー

- **量子ドットスピン**
 - MOS量子ドット
 - Si/SiGe量子ドット

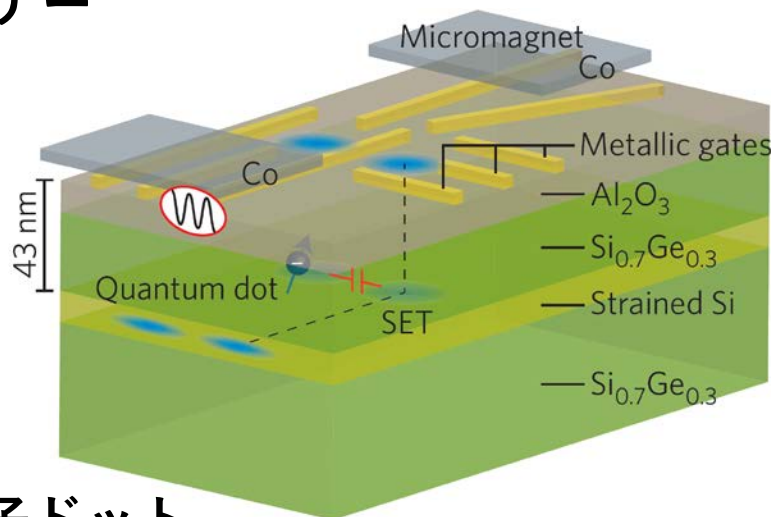
Si単一スピンの量子ビット



単一リンドナー

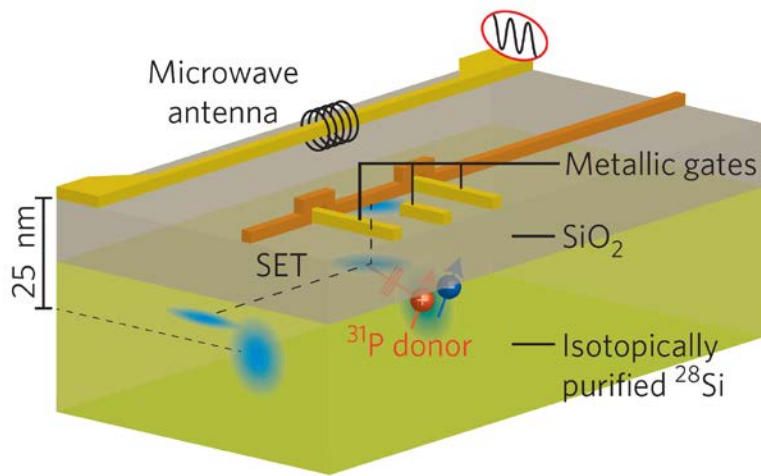


MOS量子ドット

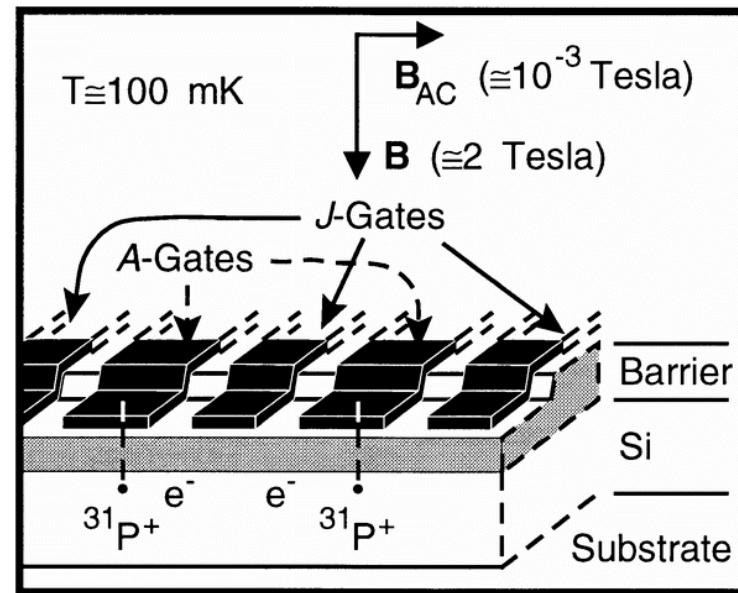


Si/SiGe量子ドット

Si単一スピン量子ビット



単一リンドナー



98年の提案から10年以上を掛けて実験が進展@豪国

Nature **393**, 133 (1998) Kane

Nature Nano. **9**, 966 (2014) Schreiber & Bluhm

ディビンチェンゾの要請

1. スケーラブルな量子ビット列
2. 初期化
3. 長いコヒーレンス時間
4. ユニバーサル量子ゲート
5. 射影測定

ディビンチェンゾの要請

1. スケーラブルな量子ビット列

→ スピン系における最大の課題

2. 初期化

→ スピン緩和(T_1), スピン依存トンネル etc

3. 長いコヒーレンス時間

→ $T_{2e} = 10$ s, $T_{2n} = 180$ min

4. ユニバーサル量子ゲート

→ 1量子ビット制御 + CNOT

5. 射影測定

→ スピン-電荷変換

ディビンチェンゾの要請

フィデリティ(忠実度, 精度) > 99%

1. スケーラブルな量子ビット列

→ スピン系における最大の課題

2. 初期化

→ スピン緩和(T_1), スピン依存トンネル etc

3. 長いコヒーレンス時間

→ 量子誤り訂正($T_2 \rightarrow \infty$)

4. ユニバーサル量子ゲート

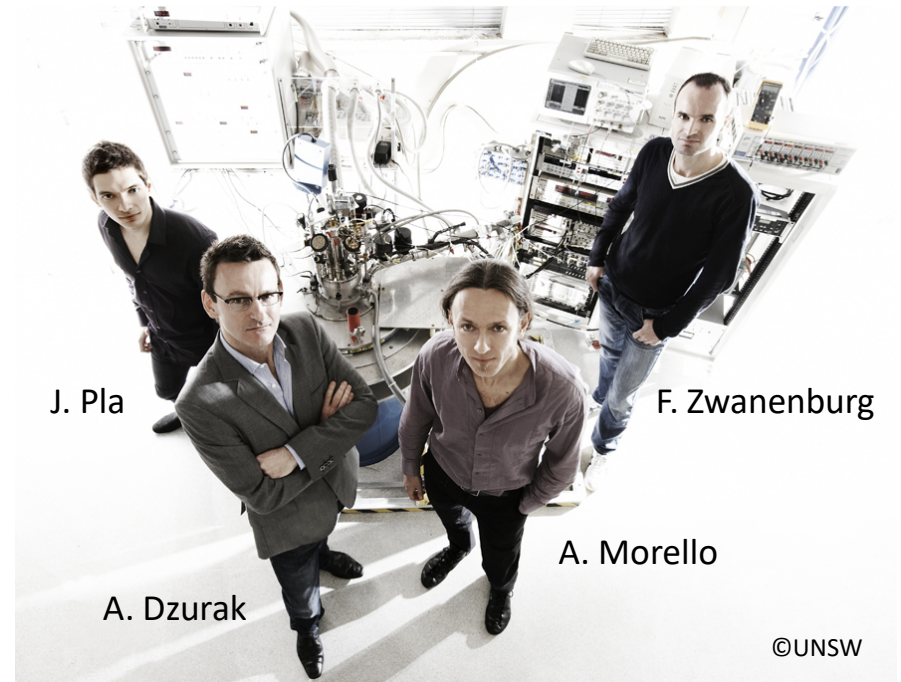
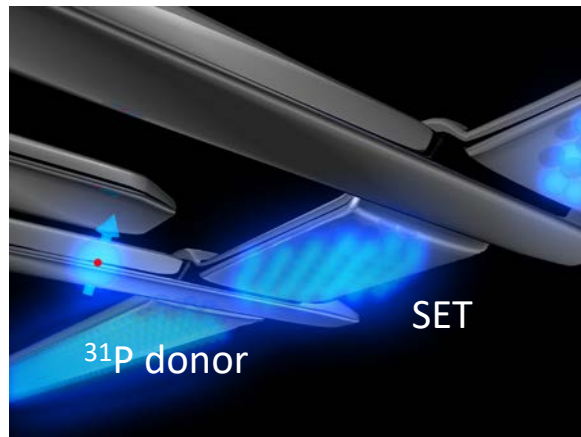
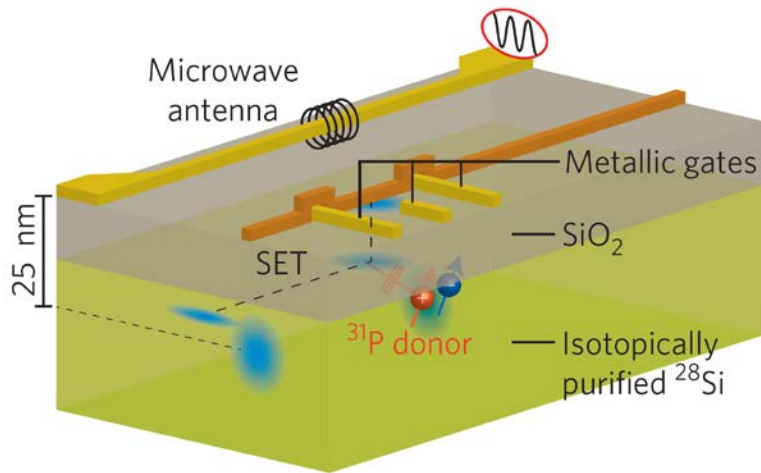
→ 1量子ビット制御 + CNOT

5. 射影測定

→ スピン-電荷変換

Single-shot readout of an electron spin in silicon

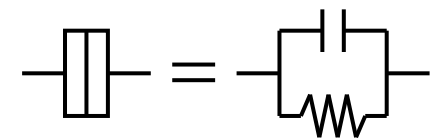
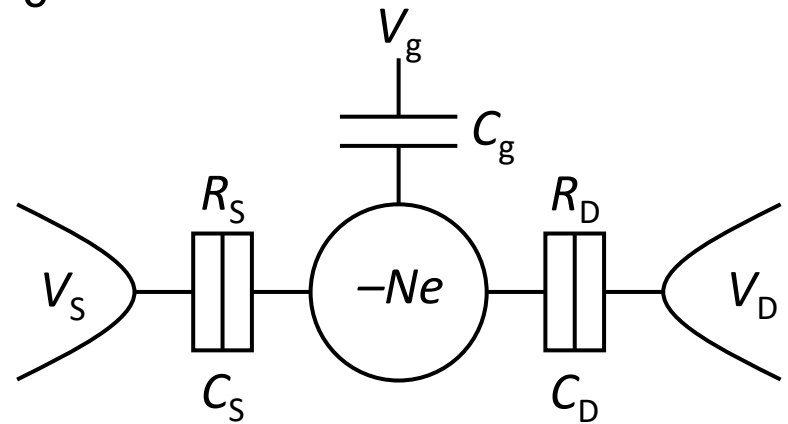
Andrea Morello¹, Jarryd J. Pla¹, Floris A. Zwanenburg¹, Kok W. Chan¹, Kuan Y. Tan¹, Hans Huebl^{1†}, Mikko Möttönen^{1,3,4}, Christopher D. Nugroho^{1†}, Changyi Yang², Jessica A. van Donkelaar², Andrew D. C. Alves², David N. Jamieson², Christopher C. Escott¹, Lloyd C. L. Hollenberg², Robert G. Clark^{1†} & Andrew S. Dzurak¹



©UNSW

単電子トランジスタ

(aka 量子ドット)



定相互作用モデル(Constant interaction model)

- SETをキャパシタ回路でモデル化
- SET内には N 個の電子
- $V_{RB, LB}$ はドットが形成されたら固定($C_{S/D}$, $R_{S/D}$ に寄与)
- V_g はQD準位の操作のみで電流は流れない

定相互作用モデル

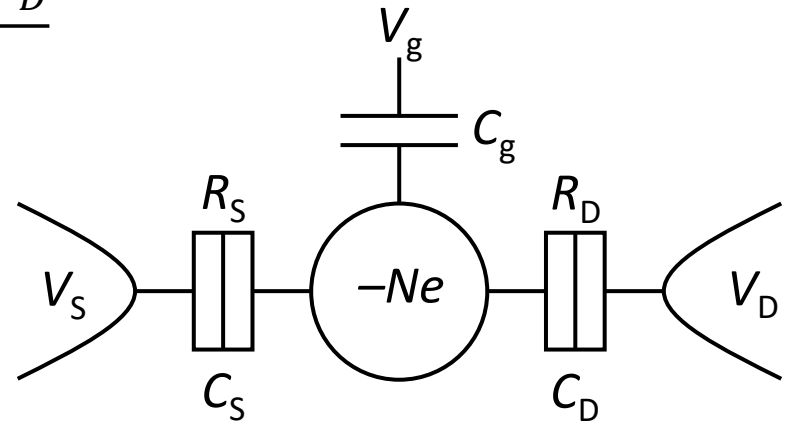
SETの電荷 Q と電圧 V

$$Q = -(N - N_0)e = C_S(V - V_S) + C_g(V - V_g) + C_D(V - V_D)$$

➔
$$V = \frac{-(N - N_0)e + C_S V_S + C_g V_g + C_D V_D}{C_\Sigma}$$

$$C_\Sigma = C_S + C_g + C_D$$

N_0 : 背景正電荷の補償分



全エネルギー

$$U(N) = \frac{1}{2C_\Sigma} \left[-(N - N_0)e + C_S V_S + C_g V_g + C_D V_D \right]^2 + \sum_{n=1}^N E_n$$
$$= \frac{e^2}{2C_\Sigma} N^2 - \frac{e^2}{C_\Sigma} N N_0 - \frac{e}{C_\Sigma} N (C_S V_S + C_g V_g + C_D V_D) + E_N + \dots$$

定相互作用モデル

電気化学ポテンシャル

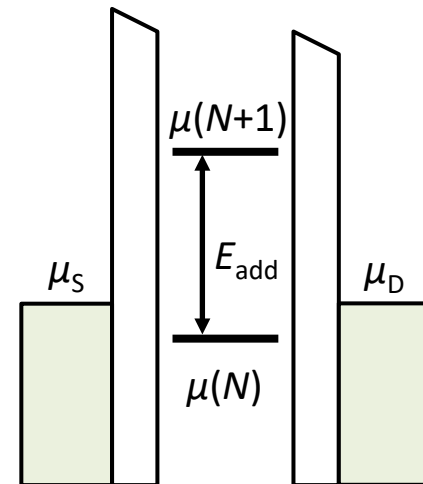
$$\mu(N) \equiv U(N) - U(N - 1)$$

$$= \frac{e^2}{C_{\Sigma}} \left(N - N_0 - \frac{1}{2} \right) - \frac{e}{C_{\Sigma}} (C_S V_S + C_g V_g + C_D V_D) + E_N$$

付加(addition)エネルギー

$$E_{\text{add}}(N) = \mu(N + 1) - \mu(N) = E_C + \Delta E$$

$$E_C = \frac{e^2}{C_{\Sigma}} : \text{帯電エネルギー}$$

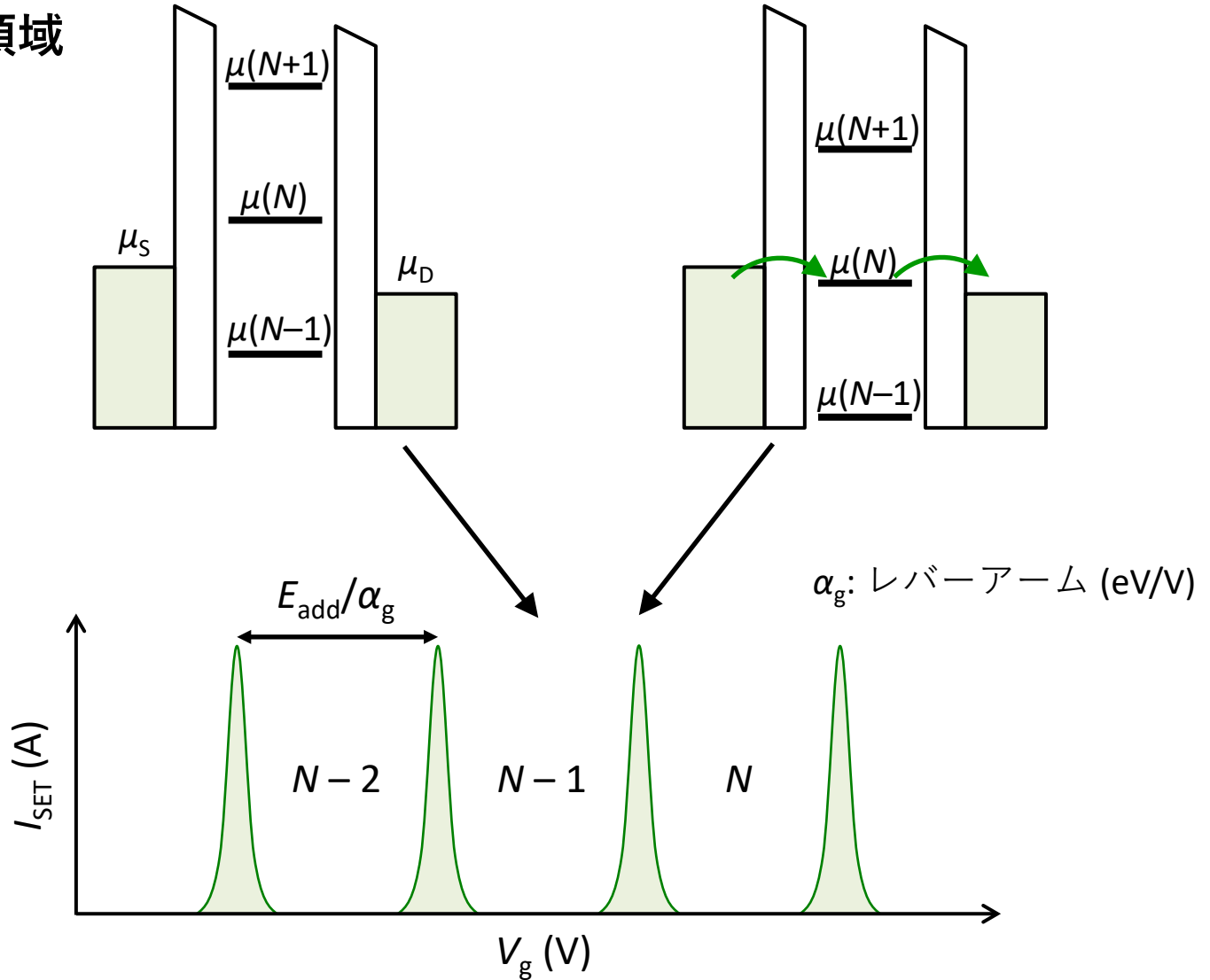


SET準位、ソース、ドレインの μ の
相対位置で伝導を理解する

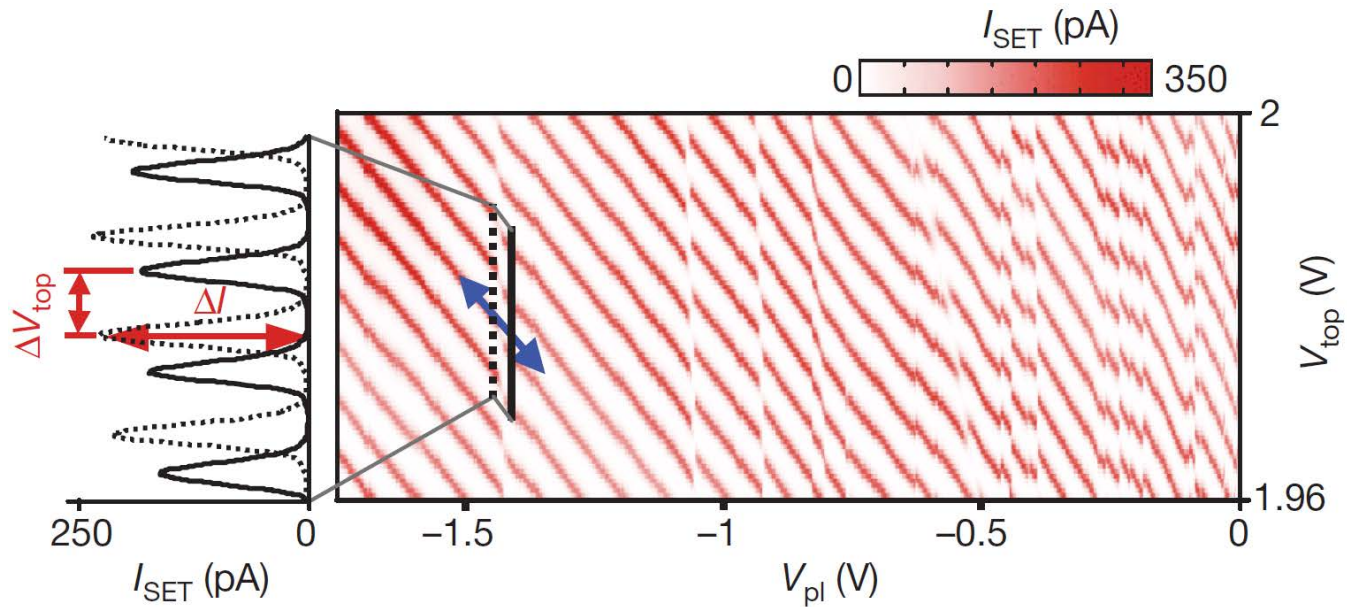
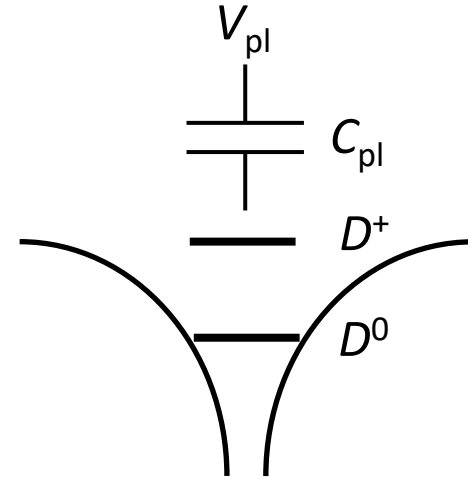
クーロン振動

低バイアス領域

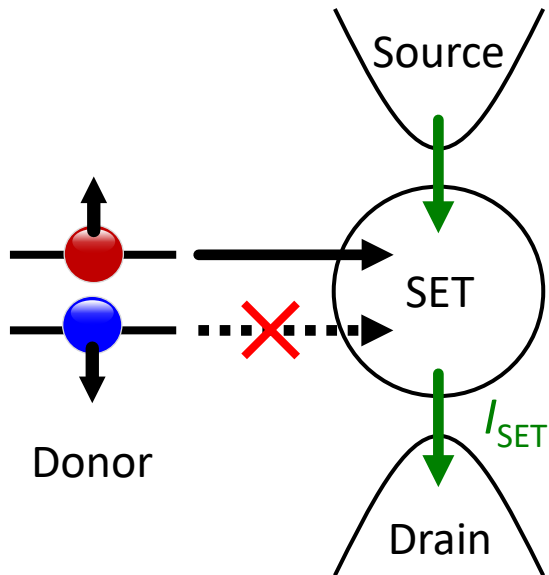
$$|\mu_S - \mu_D| \approx 0$$



ドナー-SETハイブリッド



スピン-電荷変換



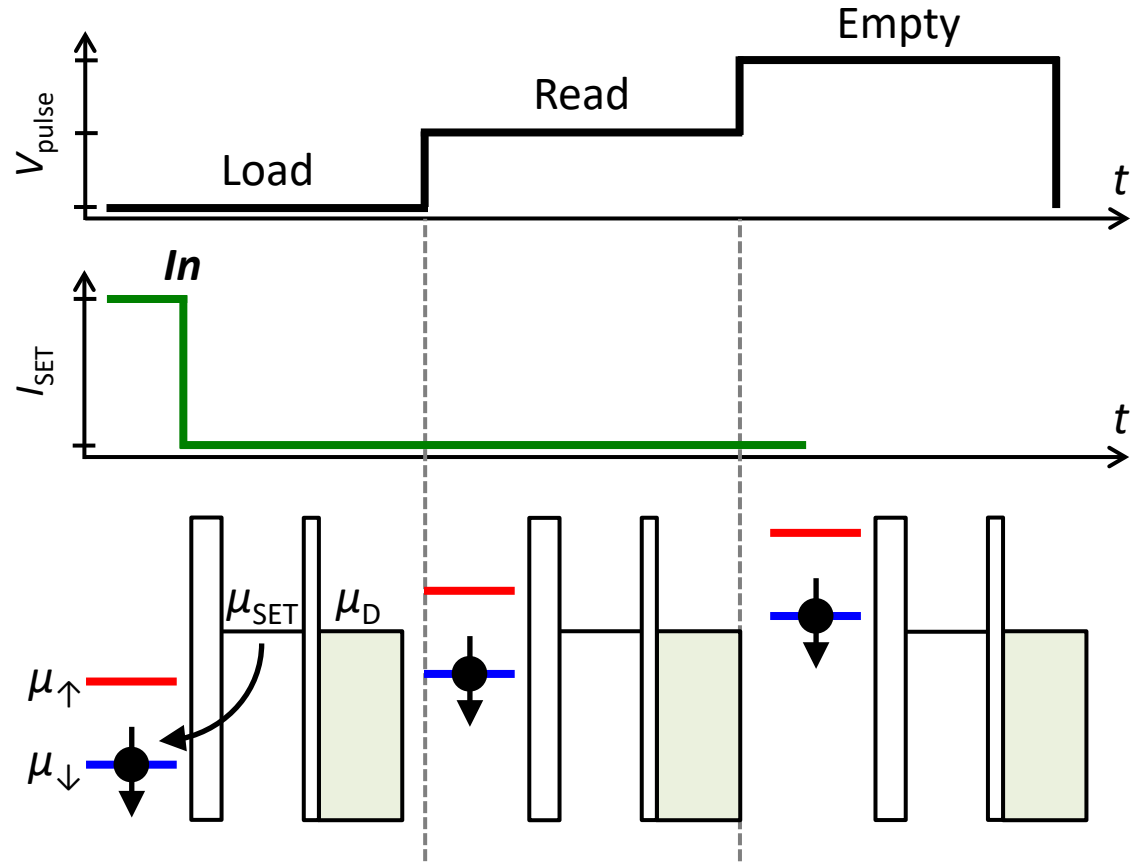
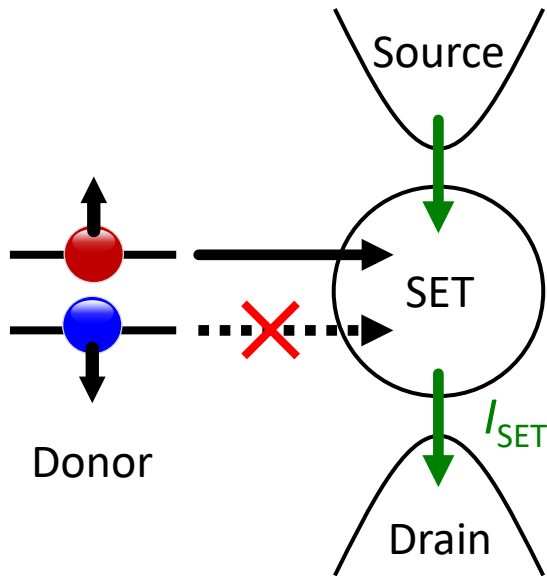
$$E(D^+) - E(D^0) = 45 \text{ meV}$$

$$E_C = 1.5 \text{ meV}$$

$$E_z = 28 \text{ GHz} = 116 \text{ } \mu\text{eV} @ B_0 = 1 \text{ T}$$

$$T_{\text{elec}} = 200 \text{ mK} = 17 \text{ } \mu\text{eV}$$

スピン測定(↓)



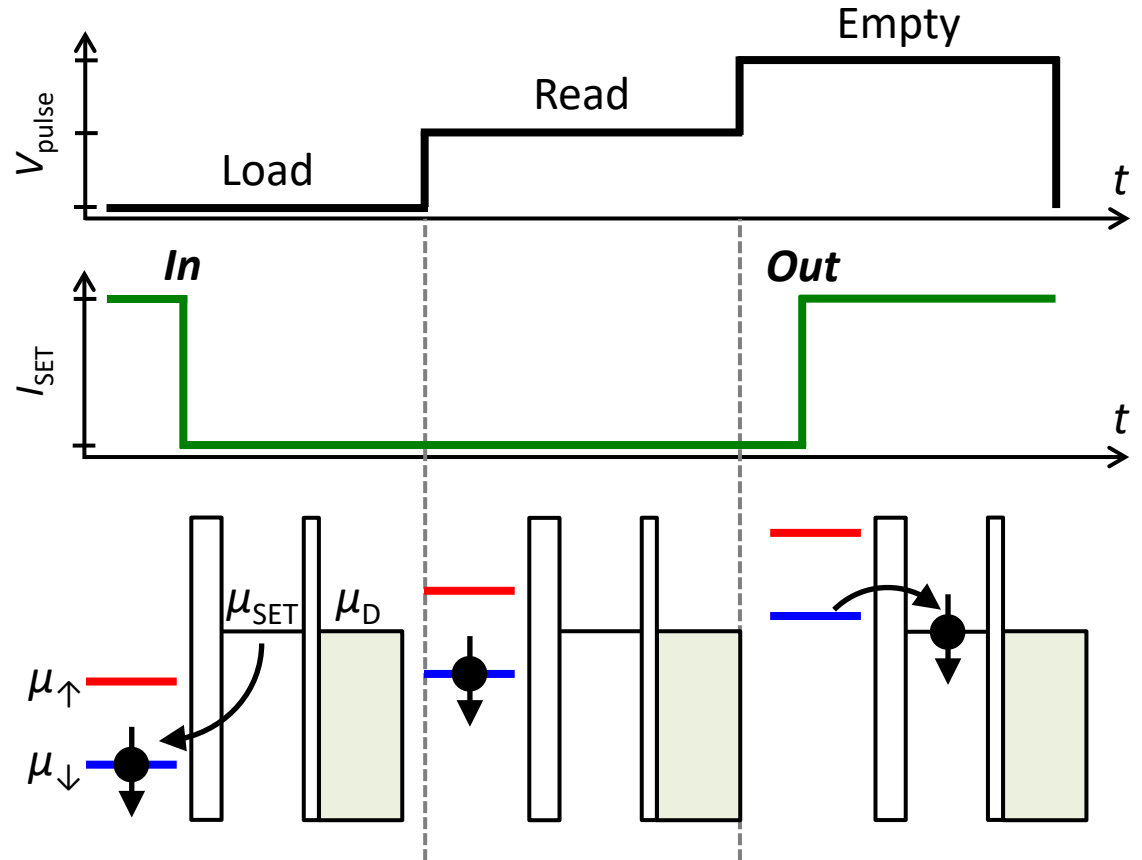
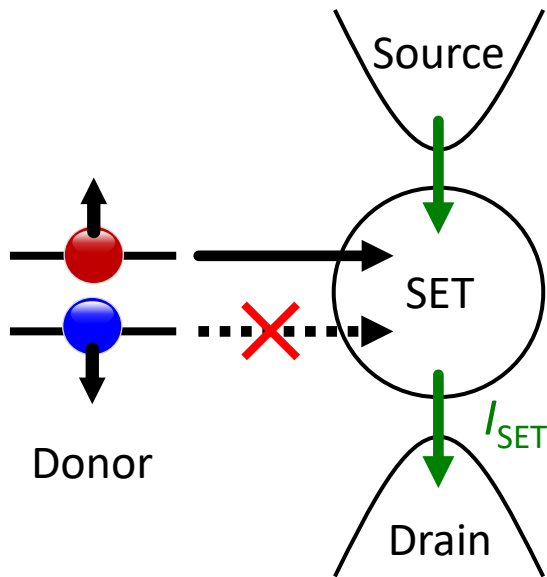
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スピン測定(↓)



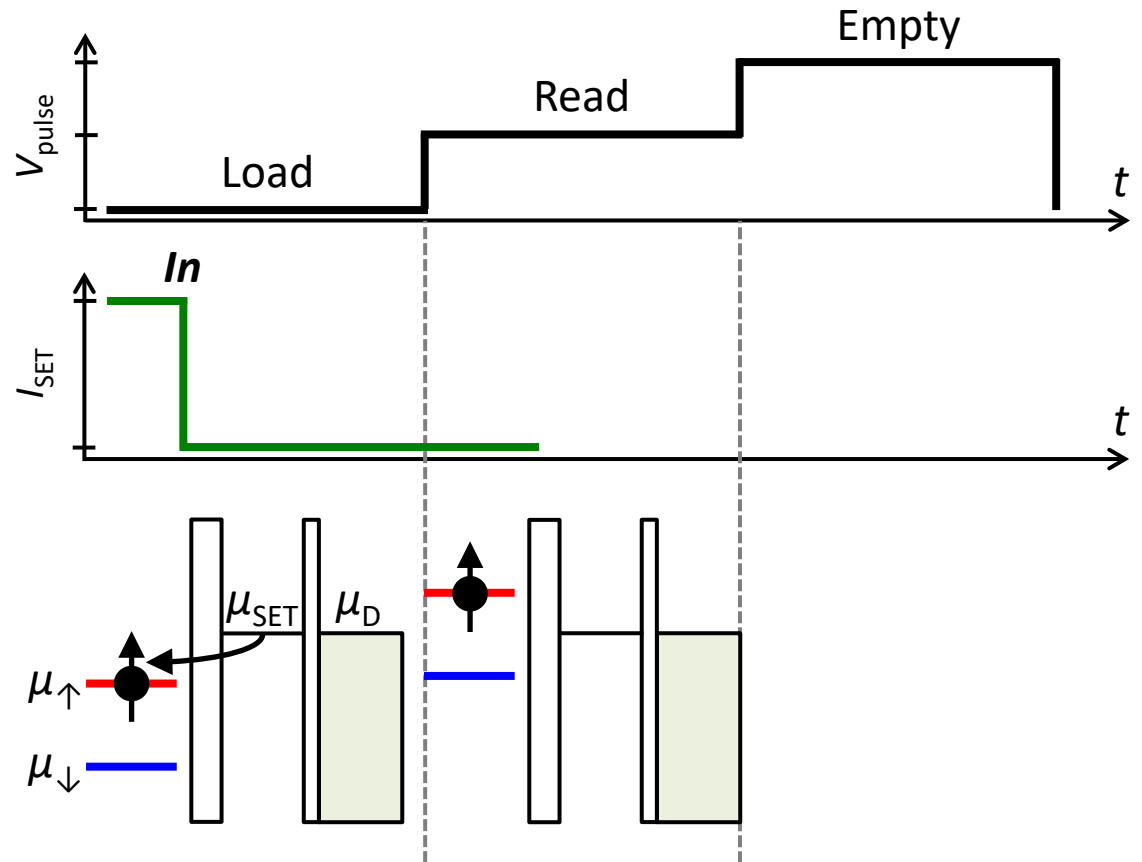
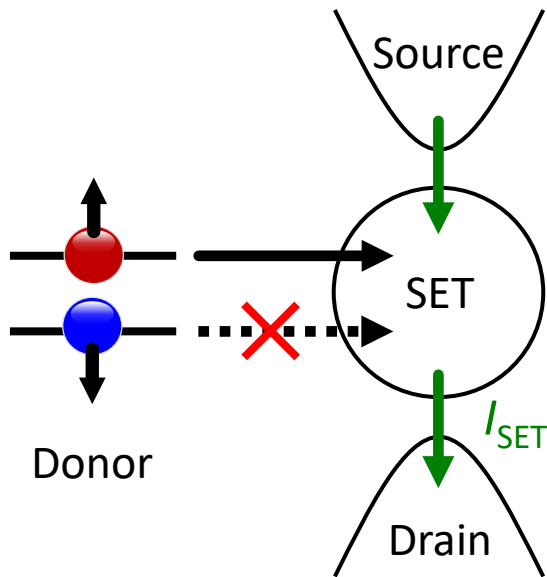
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スピン測定(↑)



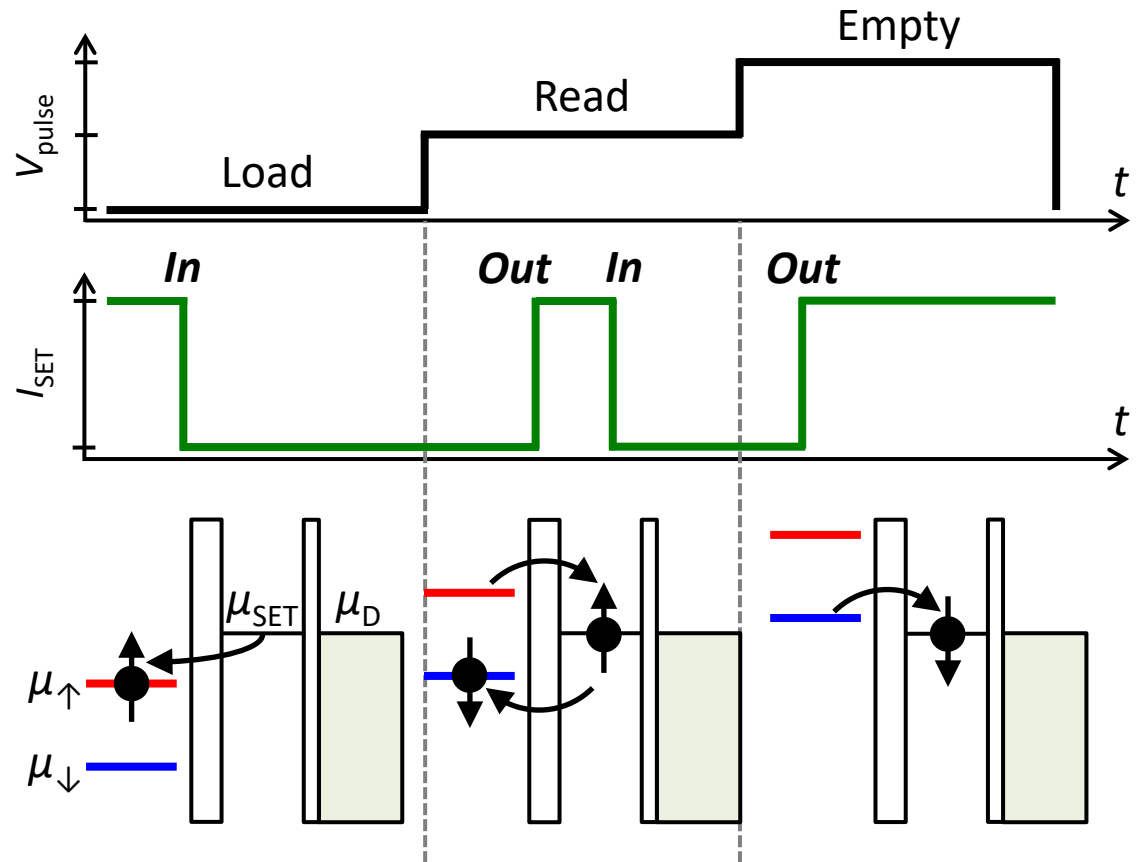
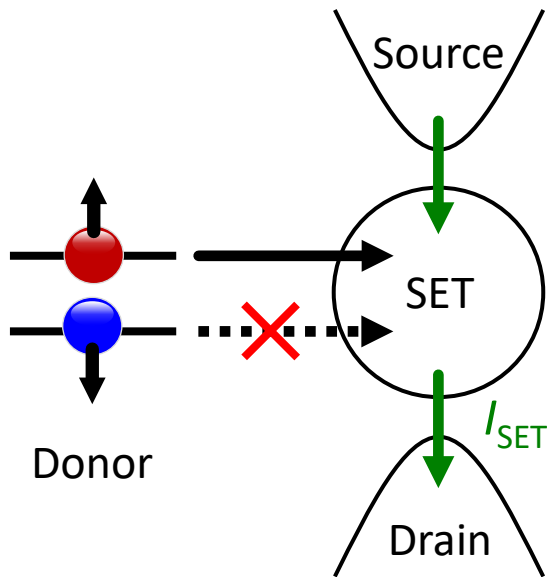
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スピン測定(↑)



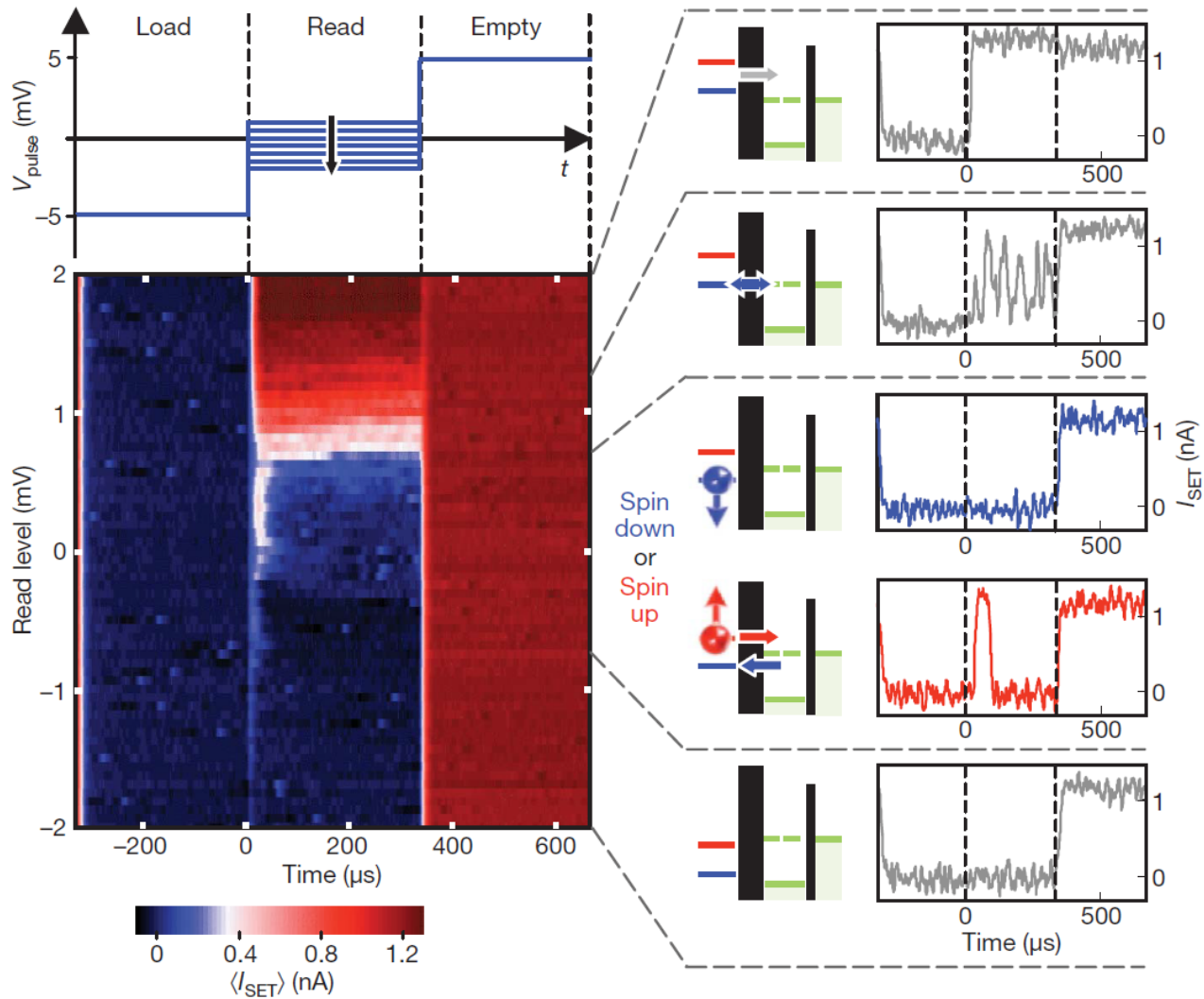
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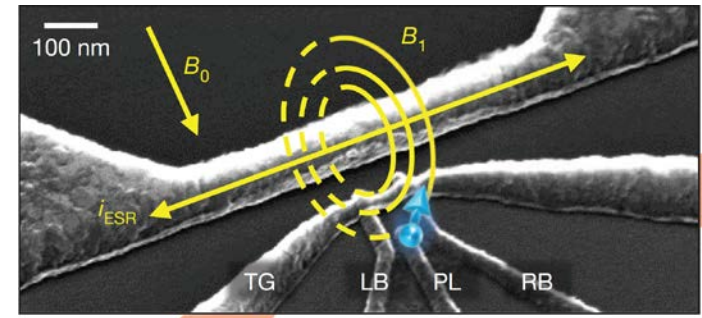
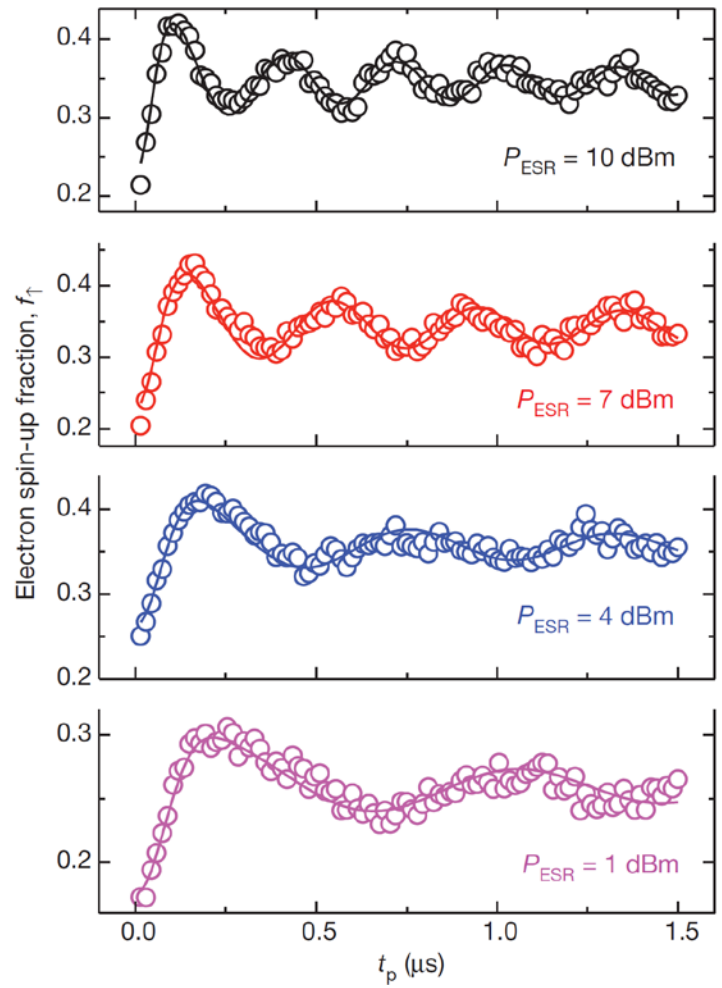
$$T_{\text{elec}} = 200 \text{ mK} = 17 \text{ } \mu\text{eV}$$

電子スピン単発読み出し



A single-atom electron spin qubit in silicon

Jarryd J. Pla¹, Kuan Y. Tan^{1†}, Juan P. Dehollain¹, Wee H. Lim¹, John J. L. Morton^{2†}, David N. Jamieson³, Andrew S. Dzurak¹ & Andrea Morello¹



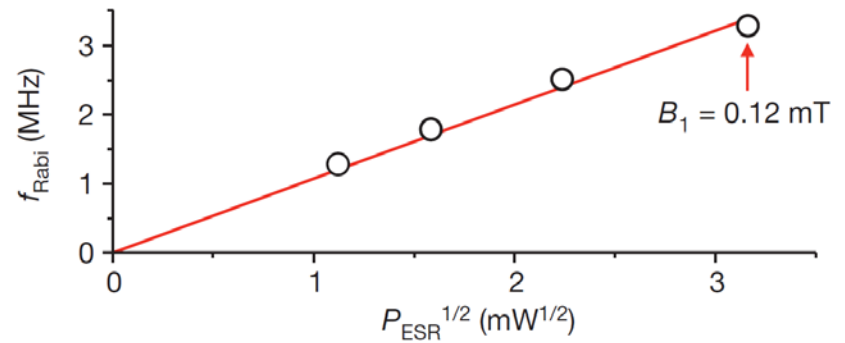
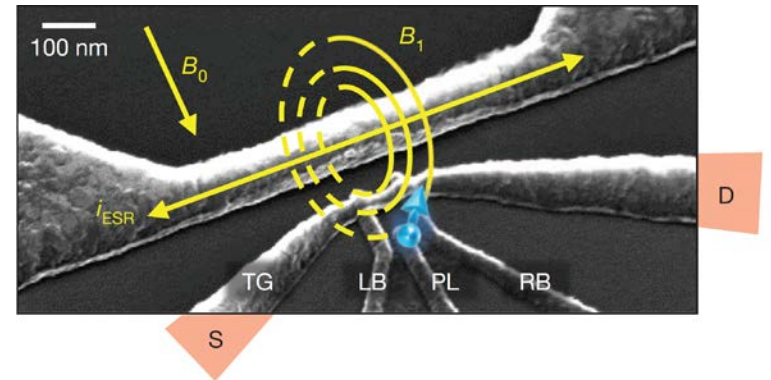
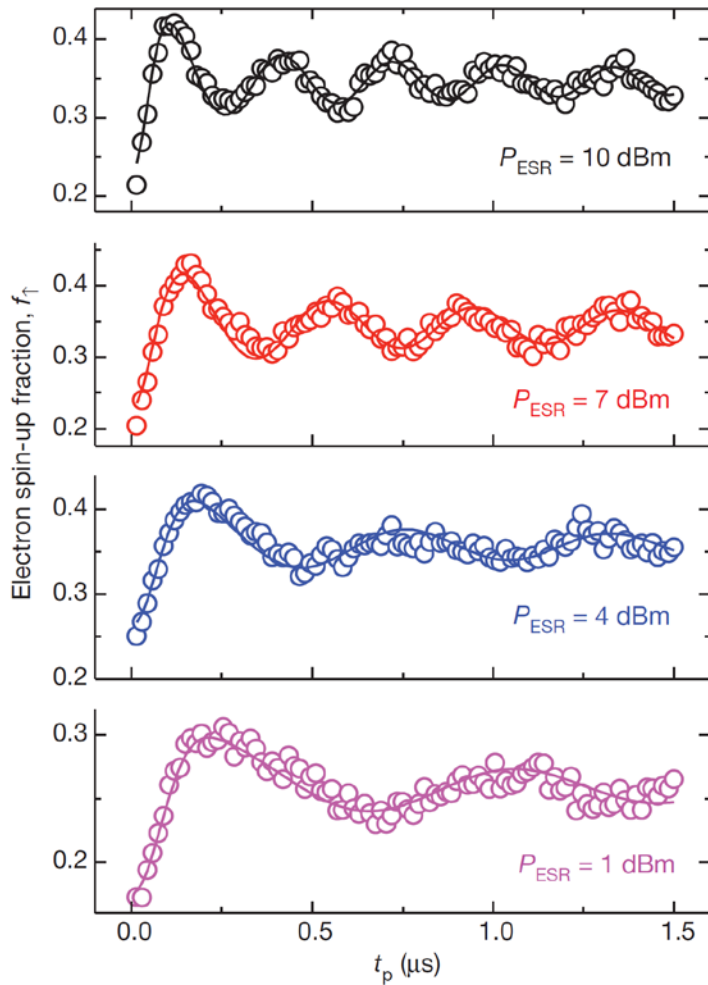
Isidor Rabi
(1898–1988)

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ラビ振動: natSiデバイス

A single-atom electron spin qubit in silicon

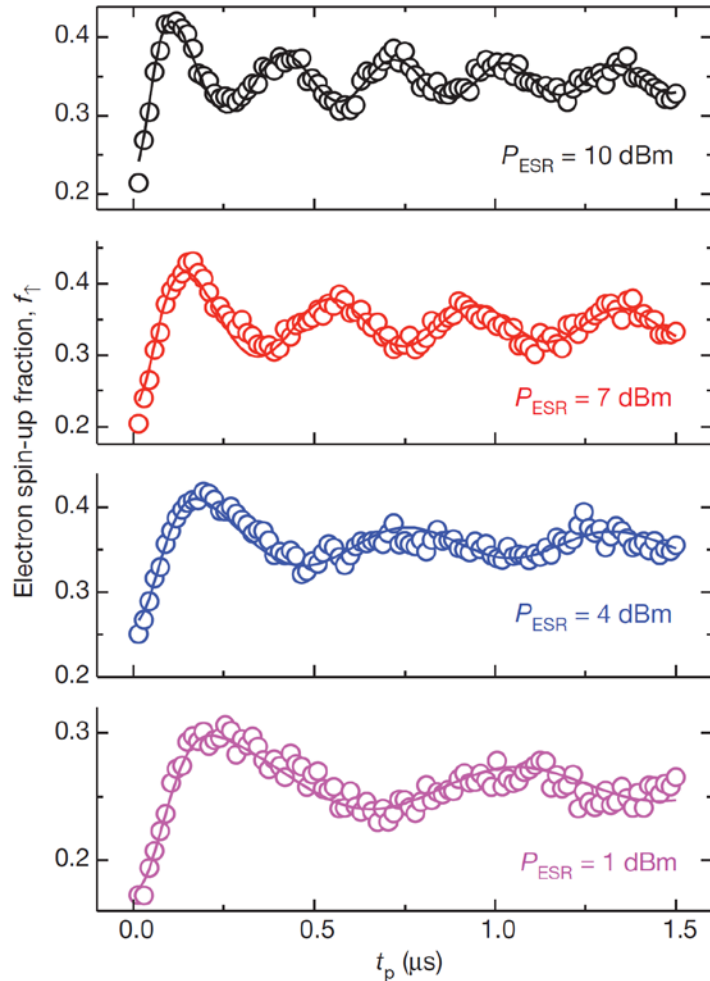
Jarryd J. Pla¹, Kuan Y. Tan^{1†}, Juan P. Dehollain¹, Wee H. Lim¹, John J. L. Morton^{2†}, David N. Jamieson³, Andrew S. Dzurak¹ & Andrea Morello¹



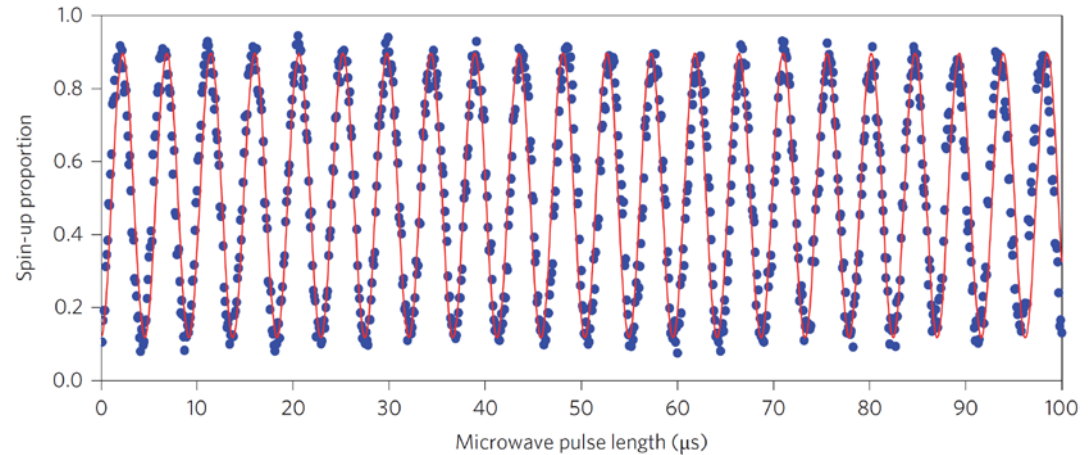
ラビ振動: natSiデバイス

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& Andrea Morello¹



同位体²⁸Si(99.92%)デバイス

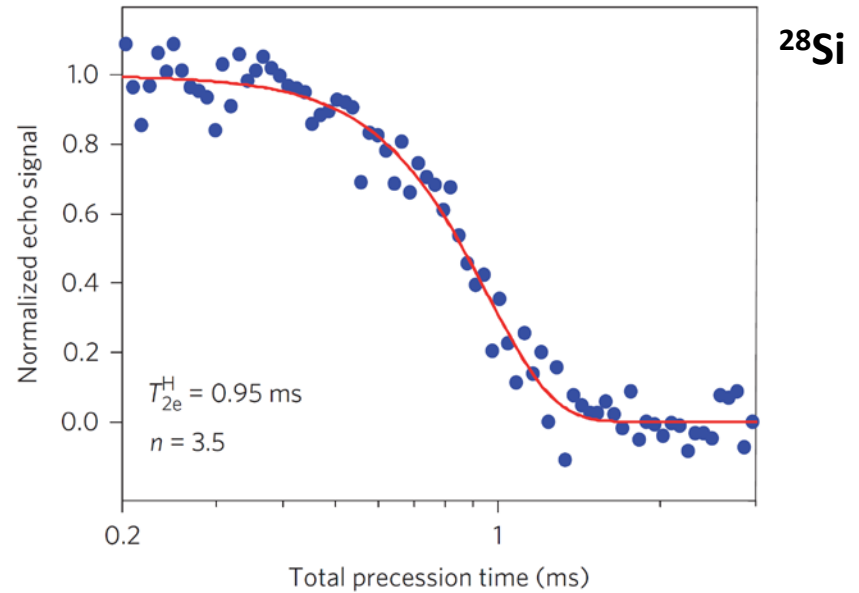
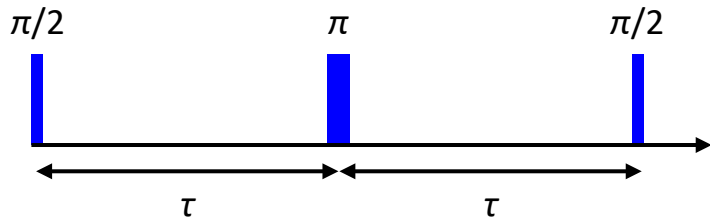


Nature Nano. **9**, 986 (2014) Muhonen *et al.*

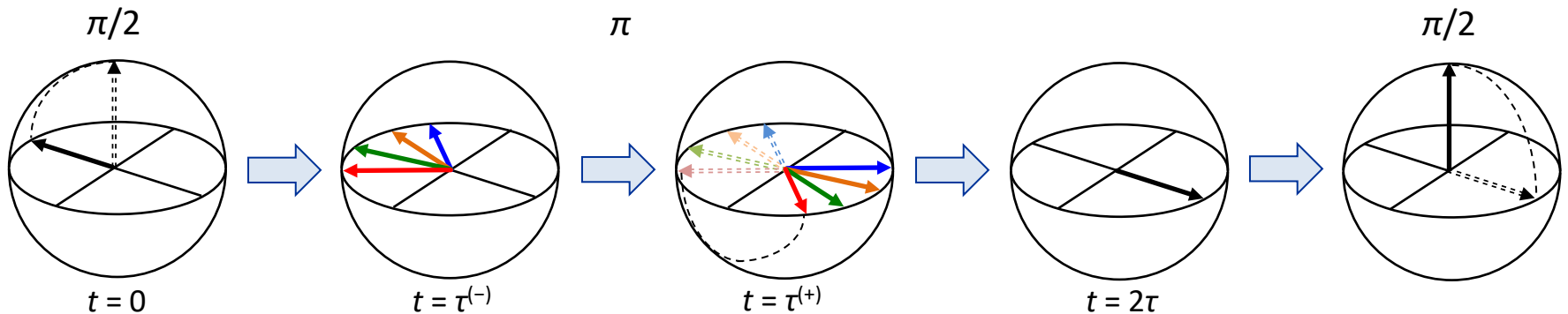
ラビ振動: natSiデバイス

Nature **489**, 541 (2012) Pla *et al.*

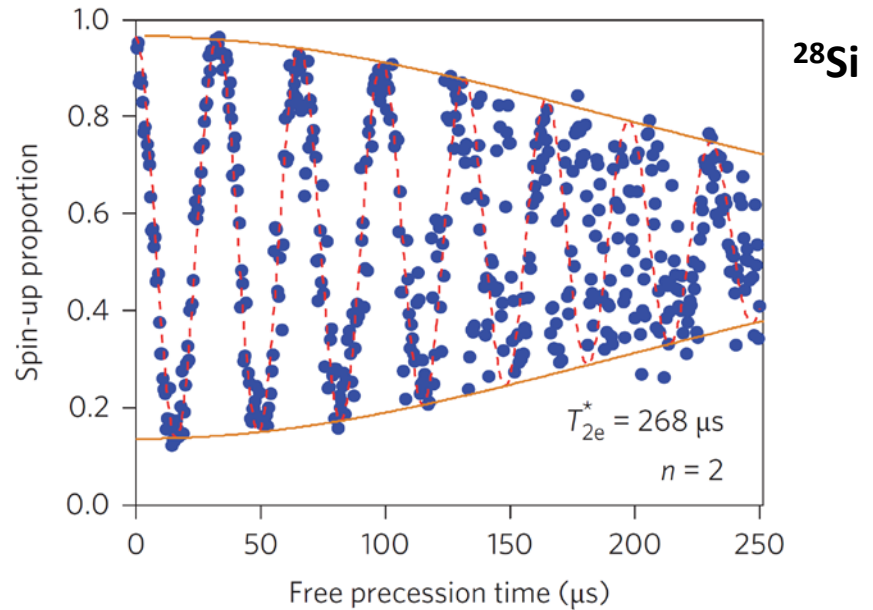
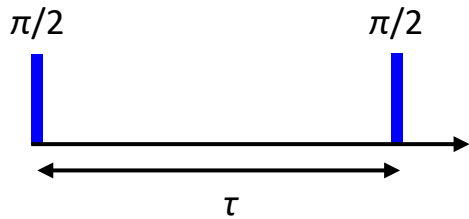
スピンエコー: T_{2e}



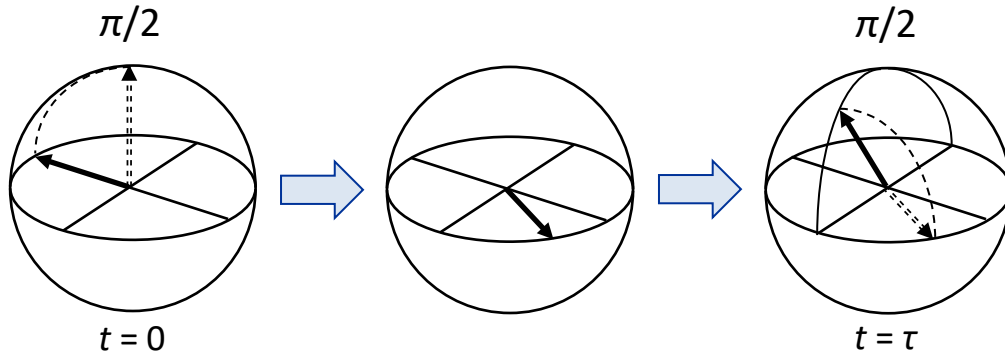
(共鳴周波数の回転系)



ラムゼー干渉: T_{2e}^*



(共鳴から少し外れた回転系)



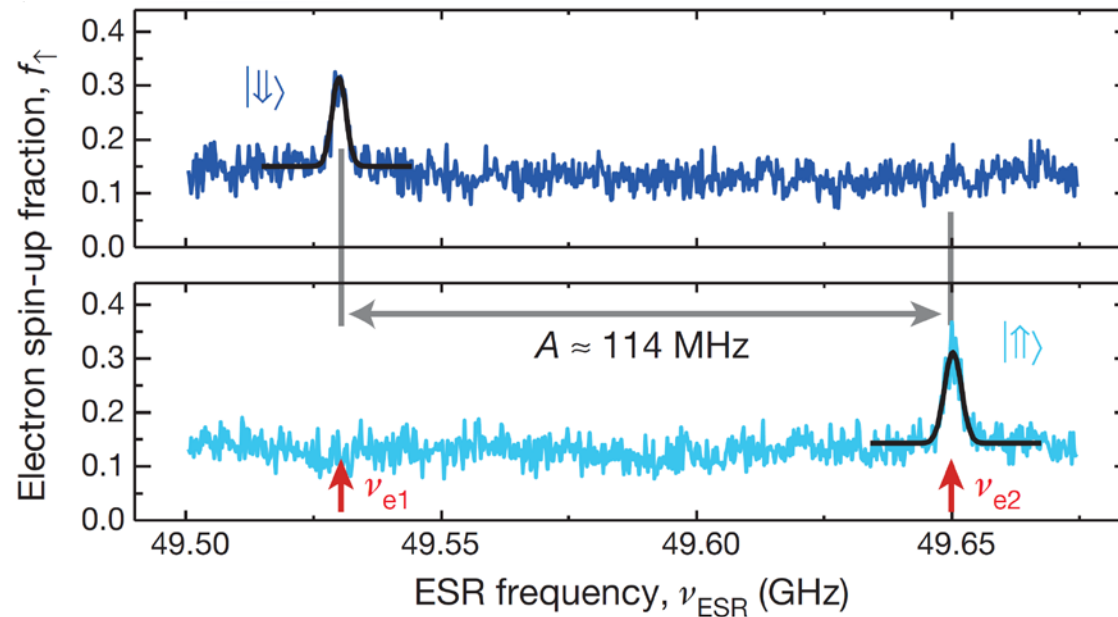
Norman Ramsey
(1915–2011)

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High-fidelity readout and control of a nuclear spin qubit in silicon

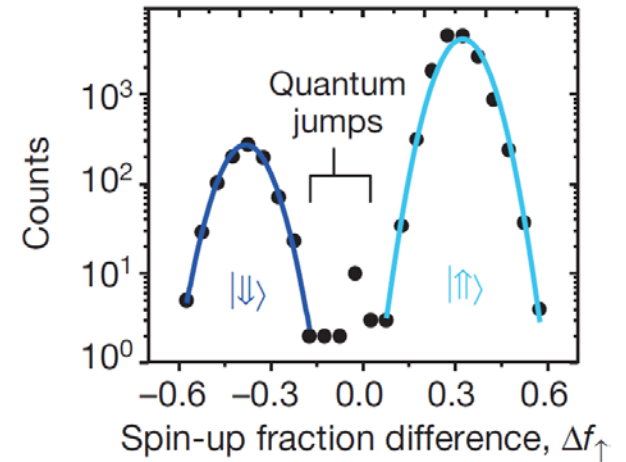
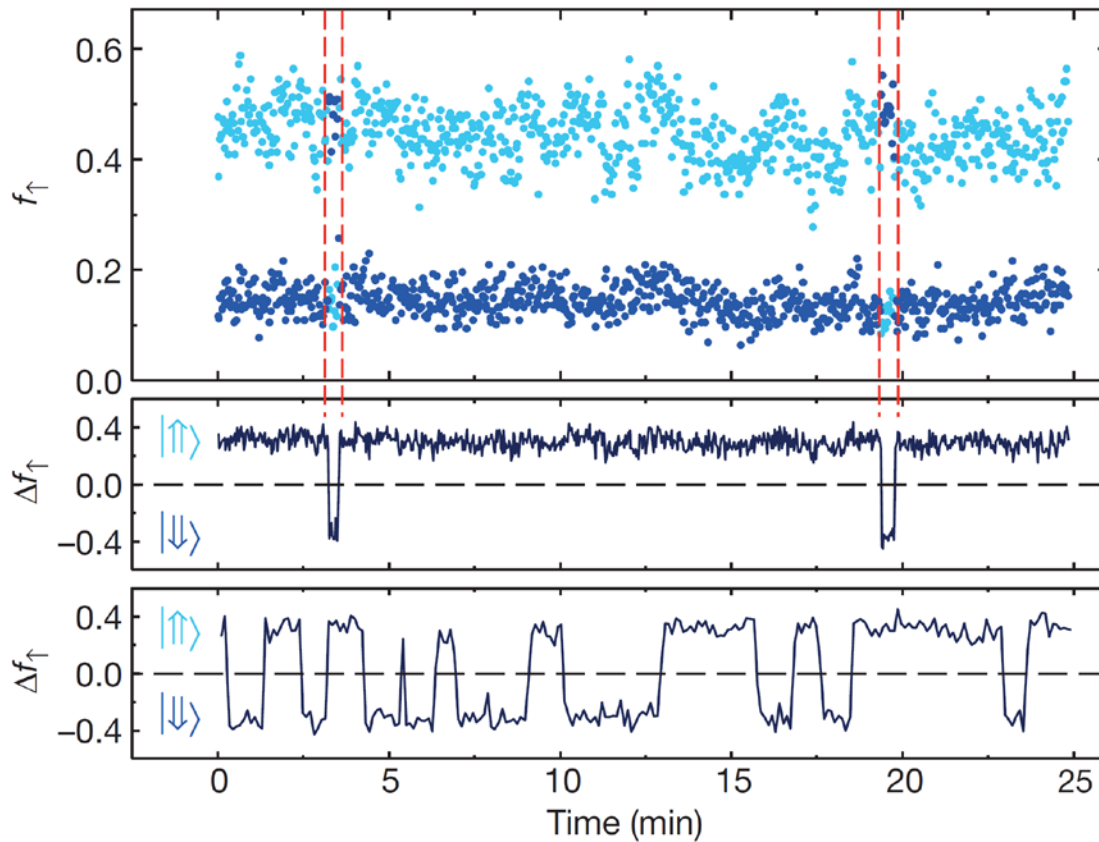
Jarryd J. Pla¹, Kuan Y. Tan^{1†}, Juan P. Dehollain¹, Wee H. Lim^{1†}, John J. L. Morton², Floris A. Zwanenburg^{1†}, David N. Jamieson³, Andrew S. Dzurak¹ & Andrea Morello¹



- 電子スピン遷移周波数 $\nu_{e1,2} = \gamma_e B_0 \mp a_0/2$ は核スピン状態に依存する
 - 電子スピン遷移によって核スピン状態は変わらない
- 量子非破壊(QND)測定

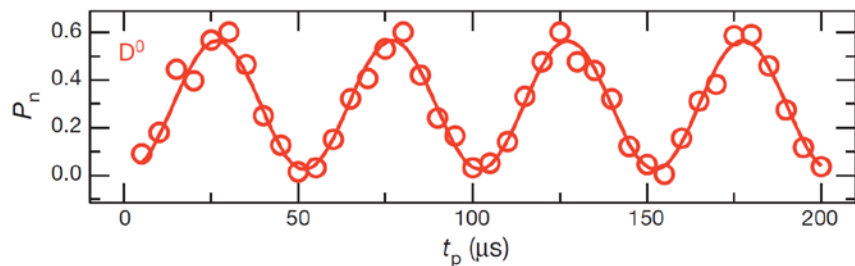
High-fidelity readout and control of a nuclear spin qubit in silicon

Jarryd J. Pla¹, Kuan Y. Tan^{1†}, Juan P. Dehollain¹, Wee H. Lim^{1†}, John J. L. Morton², Floris A. Zwanenburg^{1†}, David N. Jamieson³, Andrew S. Dzurak¹ & Andrea Morello¹

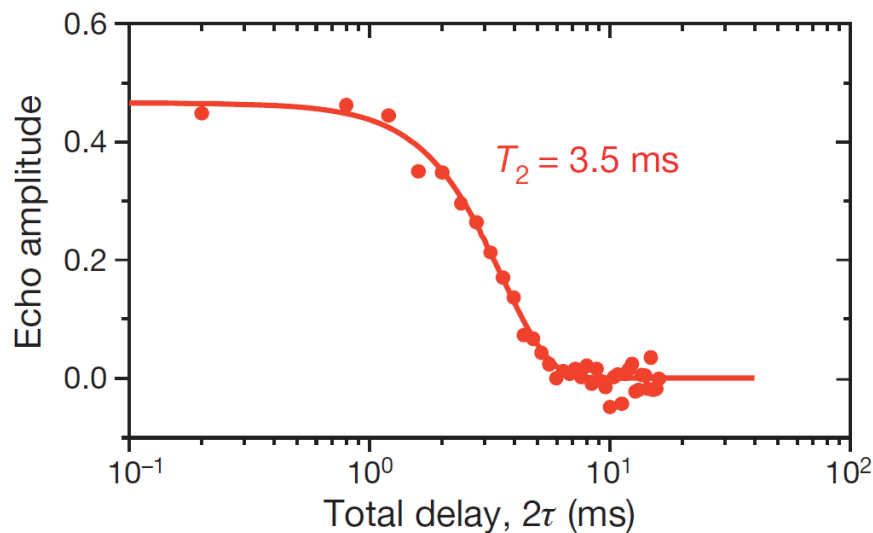
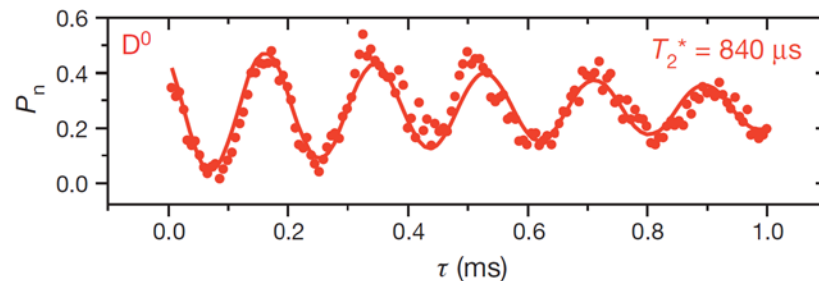


単一核スピンコヒーレント制御

ラビ振動(^{nat}Si)



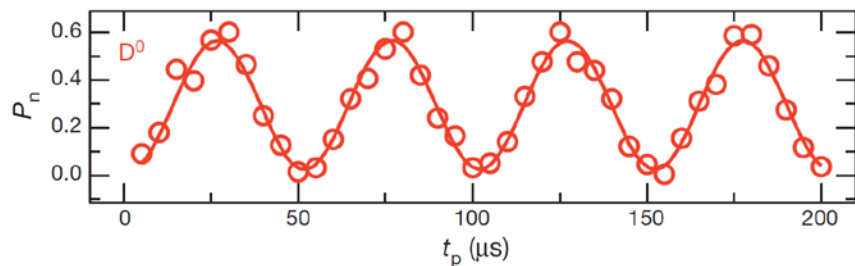
ラムゼー干渉(^{nat}Si)



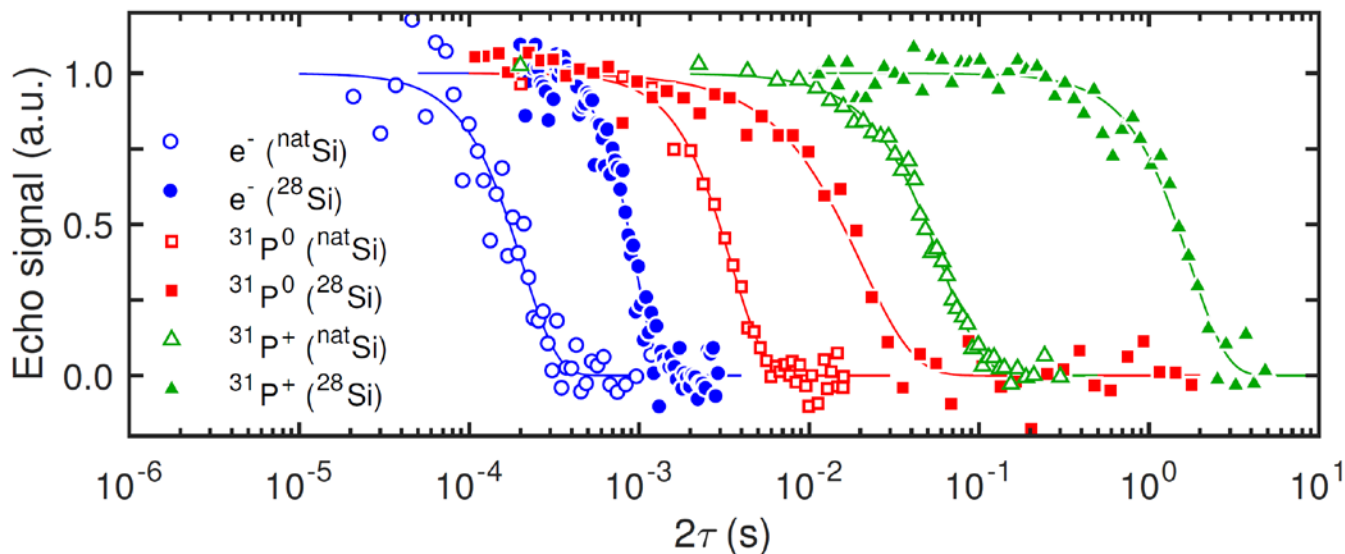
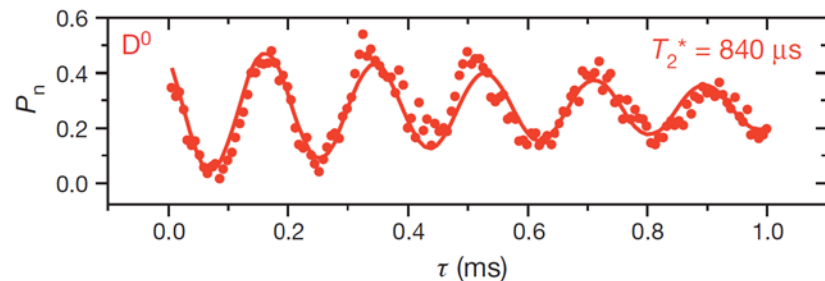
エコー(^{nat}Si)

単一核スピンコヒーレント制御

ラビ振動(^{nat}Si)



ラムゼー干渉(^{nat}Si)



Nature **489**, 541 (2012) Pla *et al.*

Nature **496**, 334 (2013) Pla *et al.*

Nature Nano. **9**, 986 (2014) Muhonen *et al.*

Coherent electrical control of a single high-spin nucleus in silicon

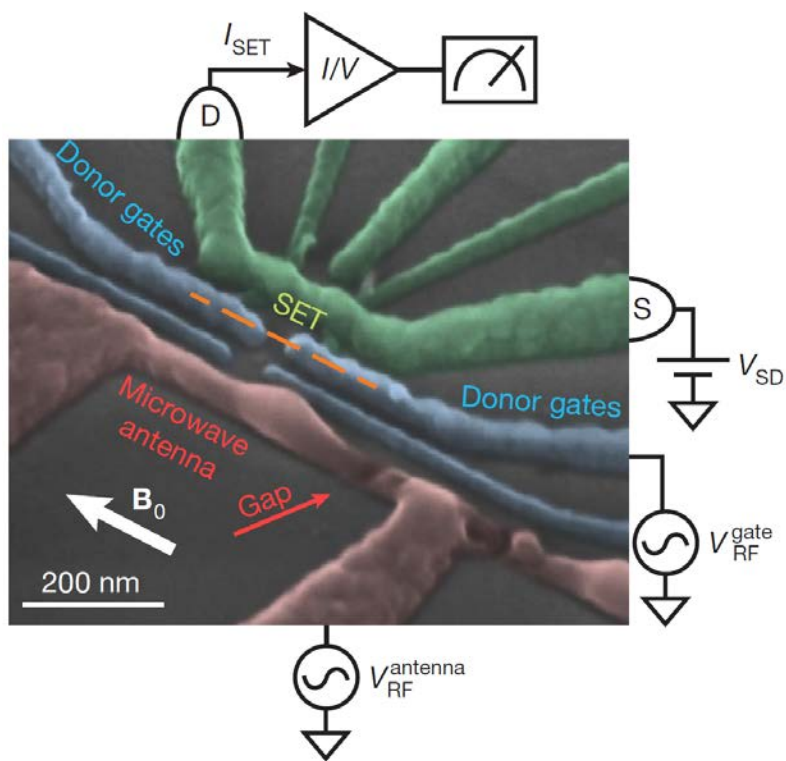
<https://doi.org/10.1038/s41586-020-2057-7>

Received: 10 June 2019

Accepted: 30 January 2020

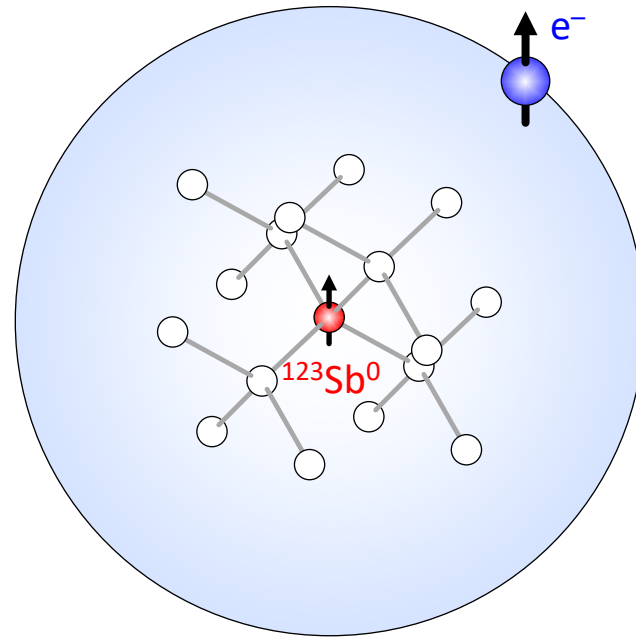
Published online: 11 March 2020

Serwan Asaad^{1,6}, Vincent Mourik^{1,6}, Benjamin Joecker¹, Mark A. I. Johnson¹, Andrew D. Baczewski², Hannes R. Firgau¹, Mateusz T. Mądzik¹, Vivien Schmitt¹, Jarryd J. Pla³, Fay E. Hudson¹, Kohei M. Itoh⁴, Jeffrey C. McCallum⁵, Andrew S. Dzurak¹, Arne Laucht¹ & Andrea Morello¹✉



シリコン中のアンチモンドナー

III (13)	IV (14)	V (15)
B	C	N
Al	Si	P
Ga	Ge	As
		Sb



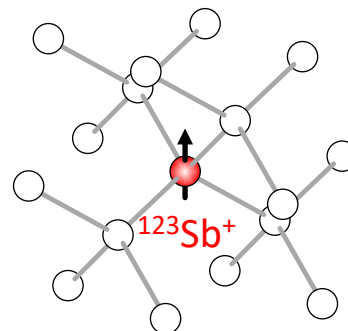
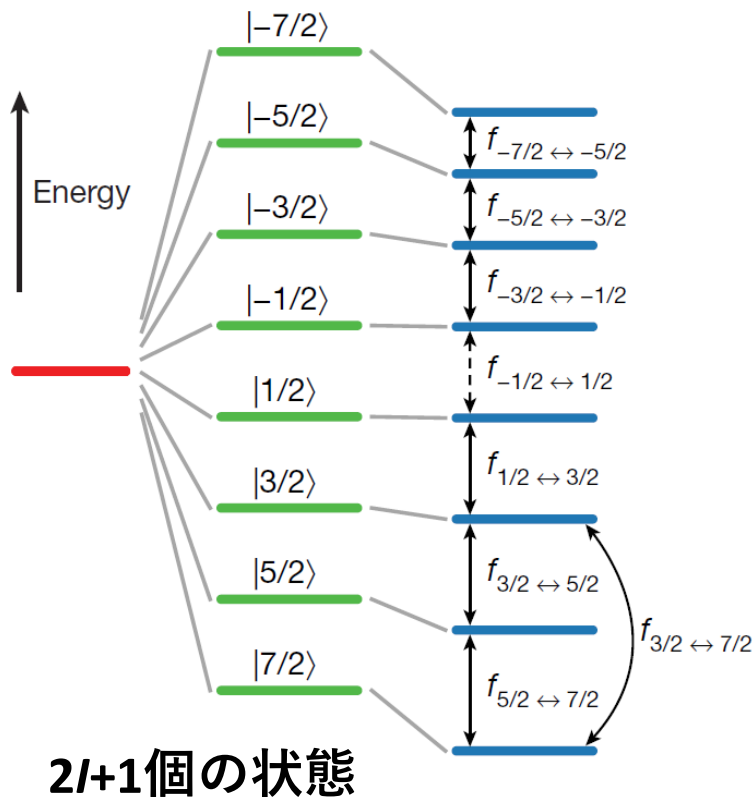
^{121}Sb ($I = 5/2$) : ^{123}Sb ($I = 7/2$) = 57.36% : 42.64%

シリコン中のアンチモンドナー

Zeeman
 $\gamma_n B_0 I_z$

Quadrupole
 $Q_{xx} I_x^2$

四重極子相互作用: 原子核位置の電場勾配に起因
($I \geq 1$ の原子核では電荷分布が球対象でない)



^{121}Sb ($I = 5/2$) : ^{123}Sb ($I = 7/2$) = 57.36% : 42.64%

Coherent electrical control of a single high-spin nucleus in silicon

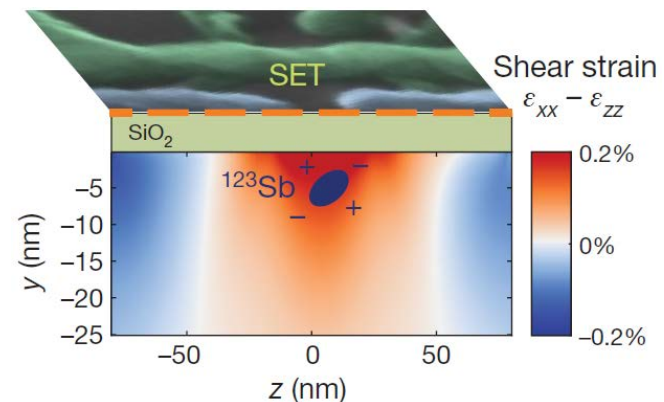
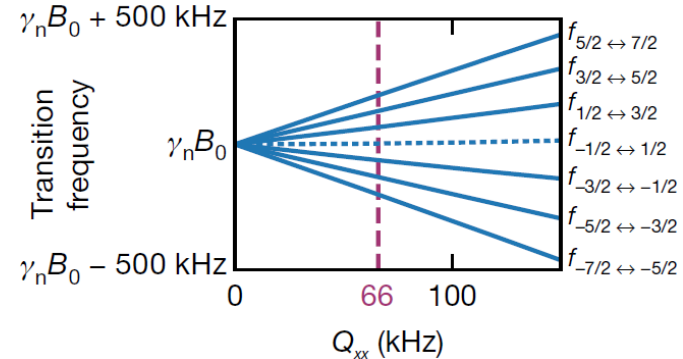
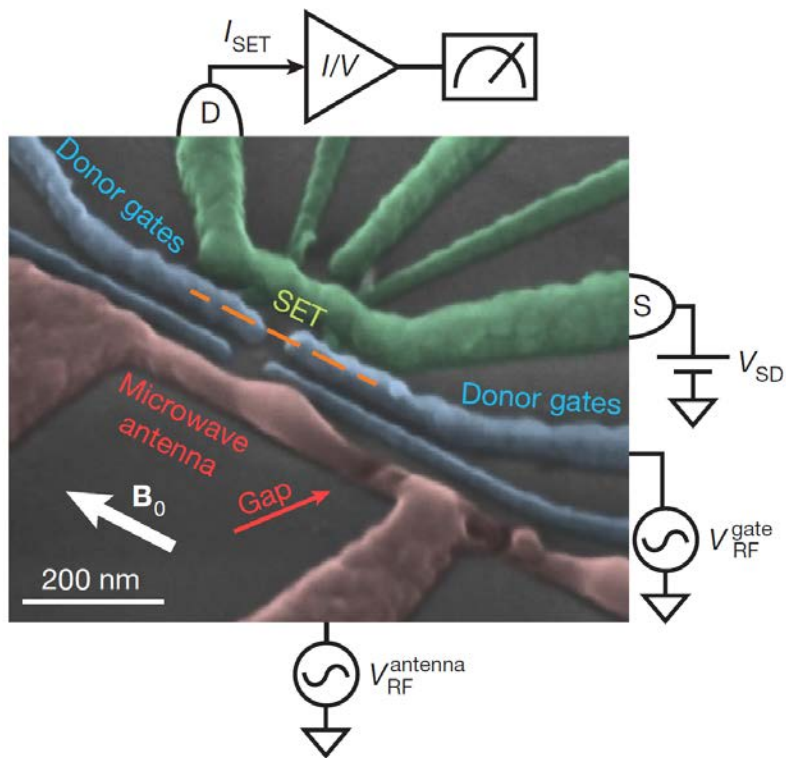
<https://doi.org/10.1038/s41586-020-2057-7>

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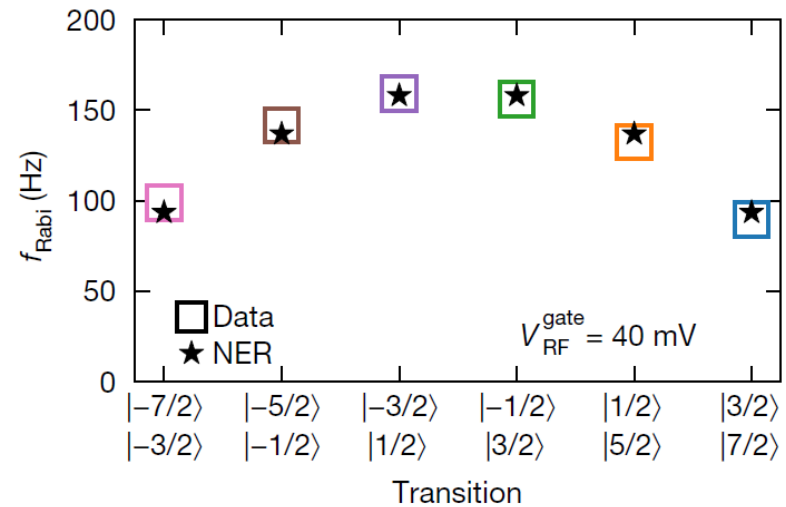
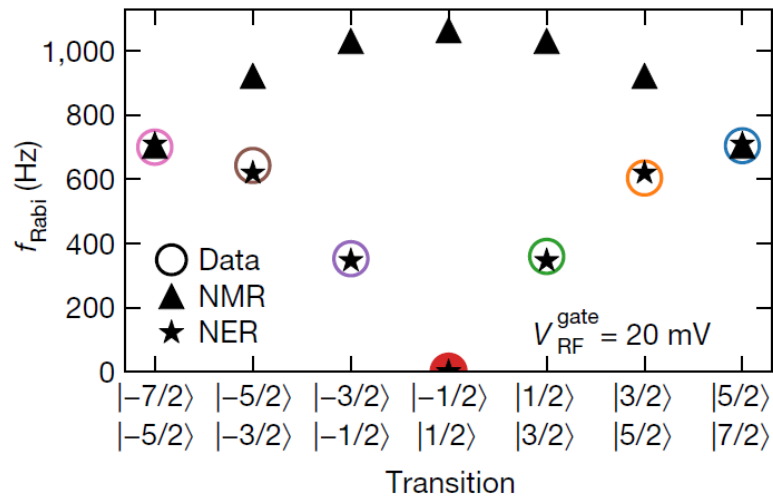
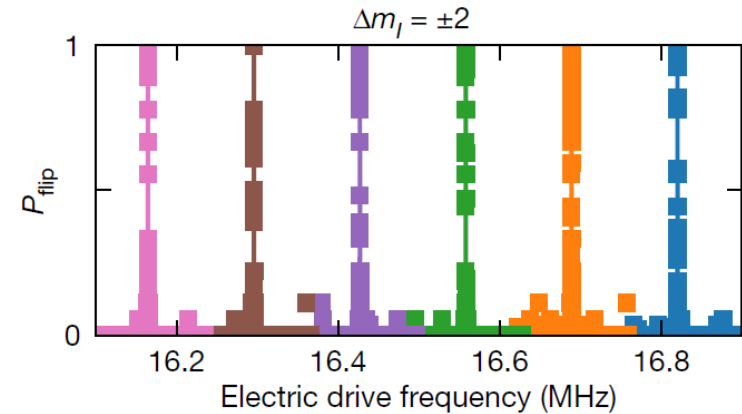
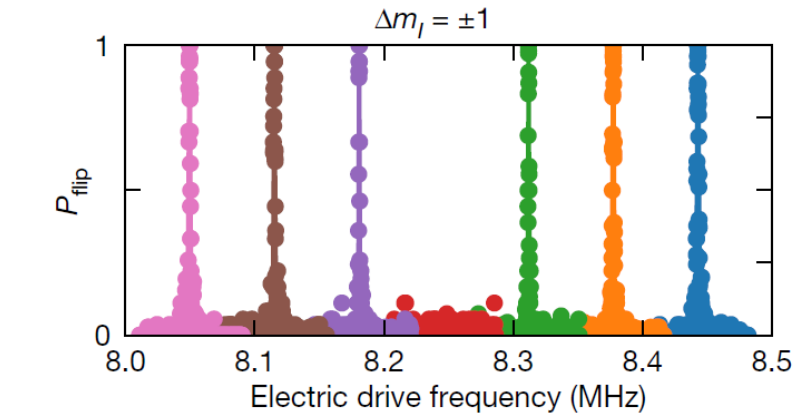
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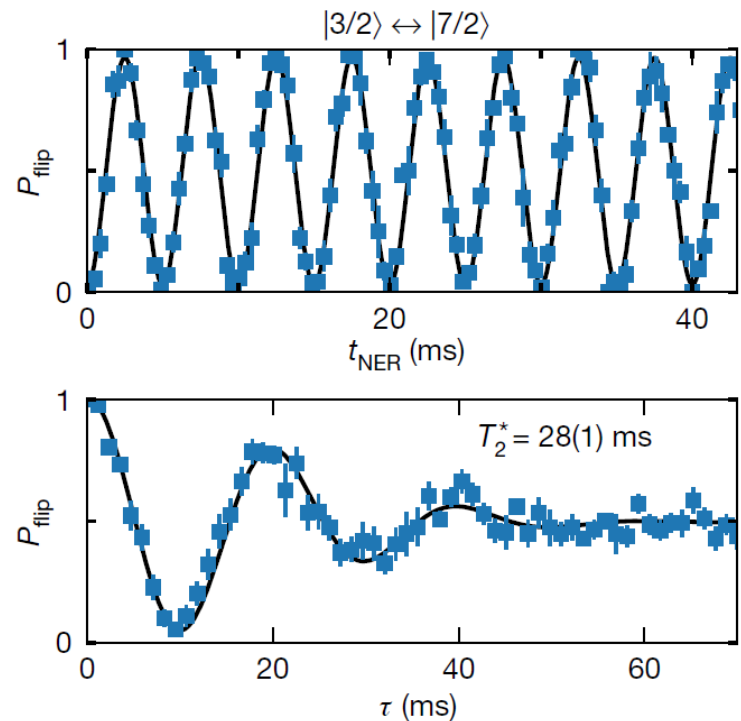
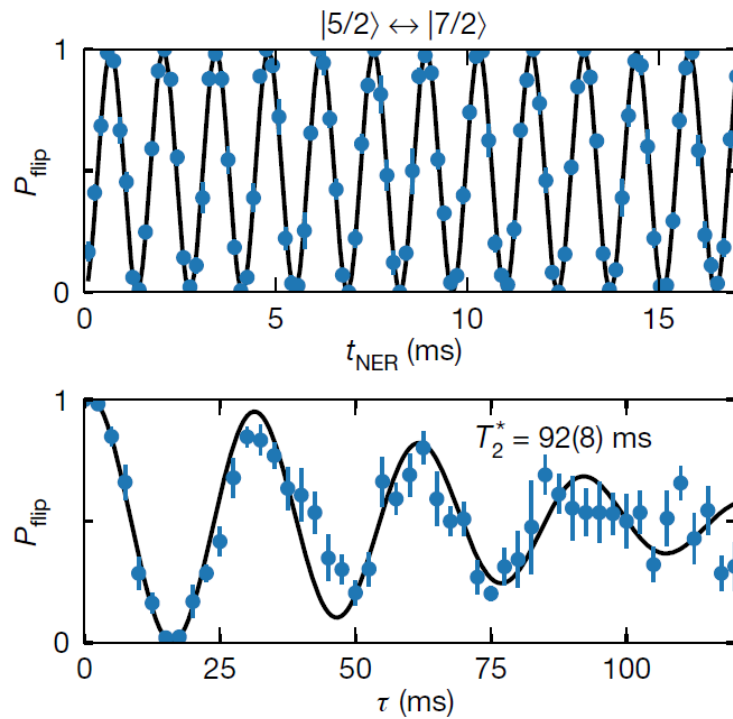
Serwan Asaad^{1,6}, Vincent Mourik^{1,6}, Benjamin Joecker¹, Mark A. I. Johnson¹, Andrew D. Baczewski², Hannes R. Firgau¹, Mateusz T. Mądzik¹, Vivien Schmitt¹, Jarryd J. Pla³, Fay E. Hudson¹, Kohei M. Itoh⁴, Jeffrey C. McCallum⁵, Andrew S. Dzurak¹, Arne Laucht¹ & Andrea Morello¹✉



Coherent electrical control of a single high-spin nucleus in silicon



Coherent electrical control of a single high-spin nucleus in silicon

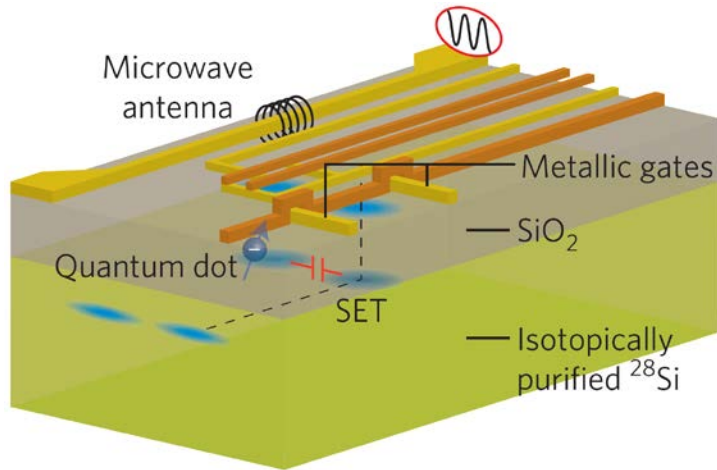


講義内容

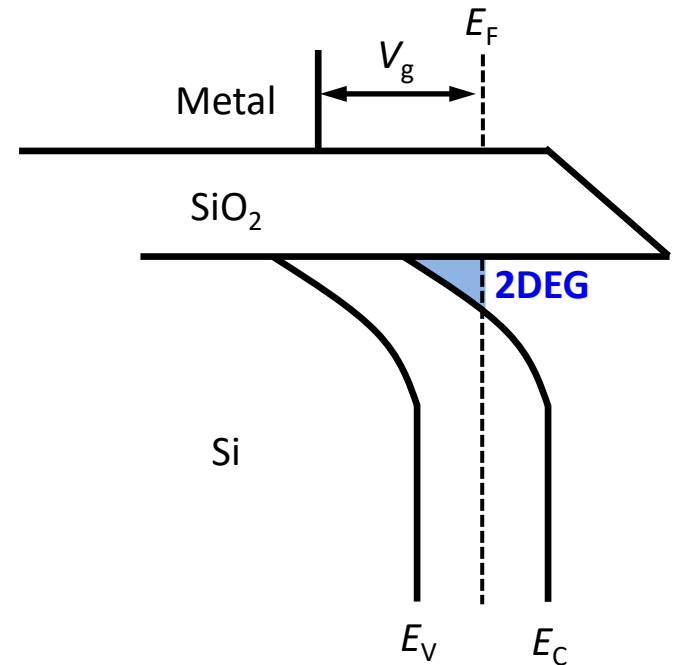
- **ドナースピン**
 - アンサンブル
 - 単一ドナー

- **量子ドットスピン**
 - MOS量子ドット
 - Si/SiGe量子ドット

界面MOS構造



反転層の形成

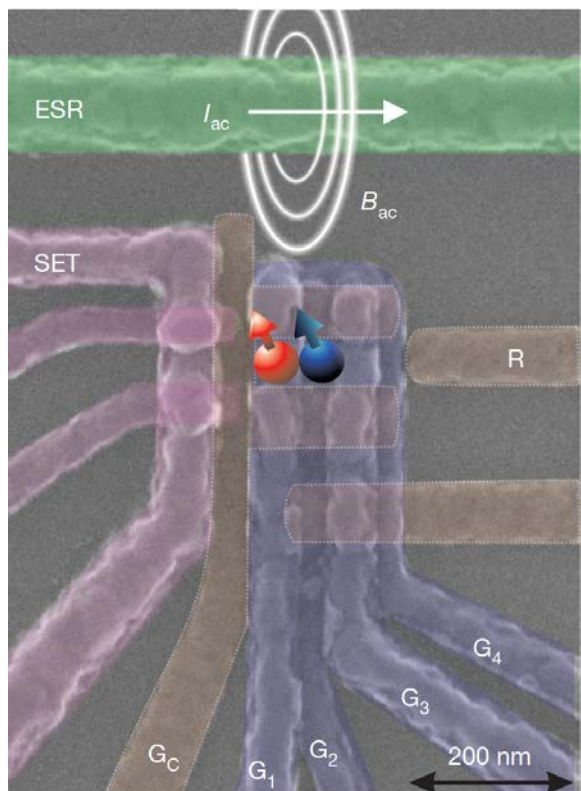
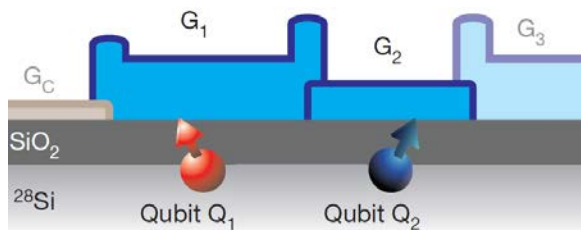


(L to R) J. Muhonen, A. Morello, M. Veldhorst, A. Dzurak

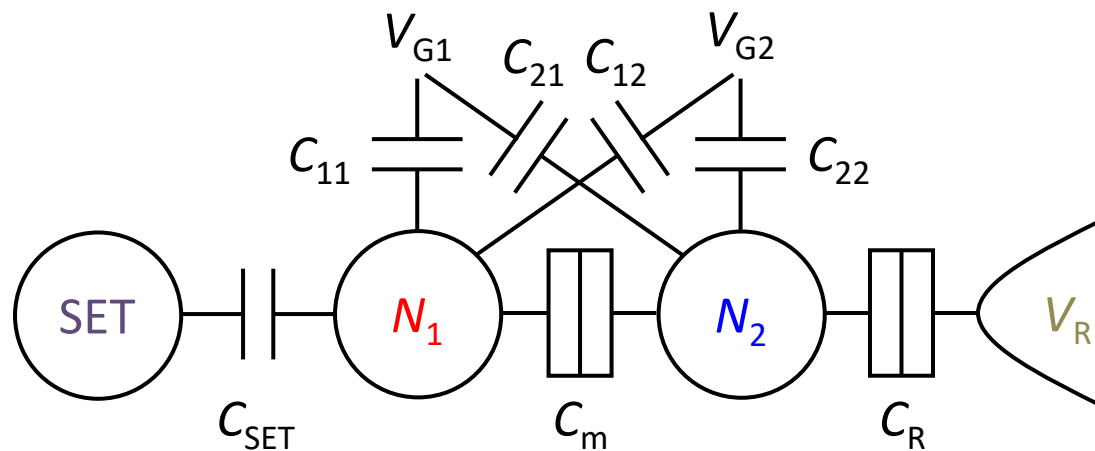
$$^{28}\text{Si} : ^{29}\text{Si} (I = \frac{1}{2}) : ^{30}\text{Si} = 92.2\% : 4.7\% : 3.1\%$$

$$\rightarrow ^{28}\text{Si} = 99.2\%$$

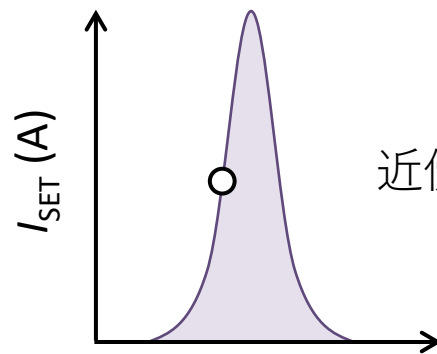
MOS型2重量子ドット



クロストーク

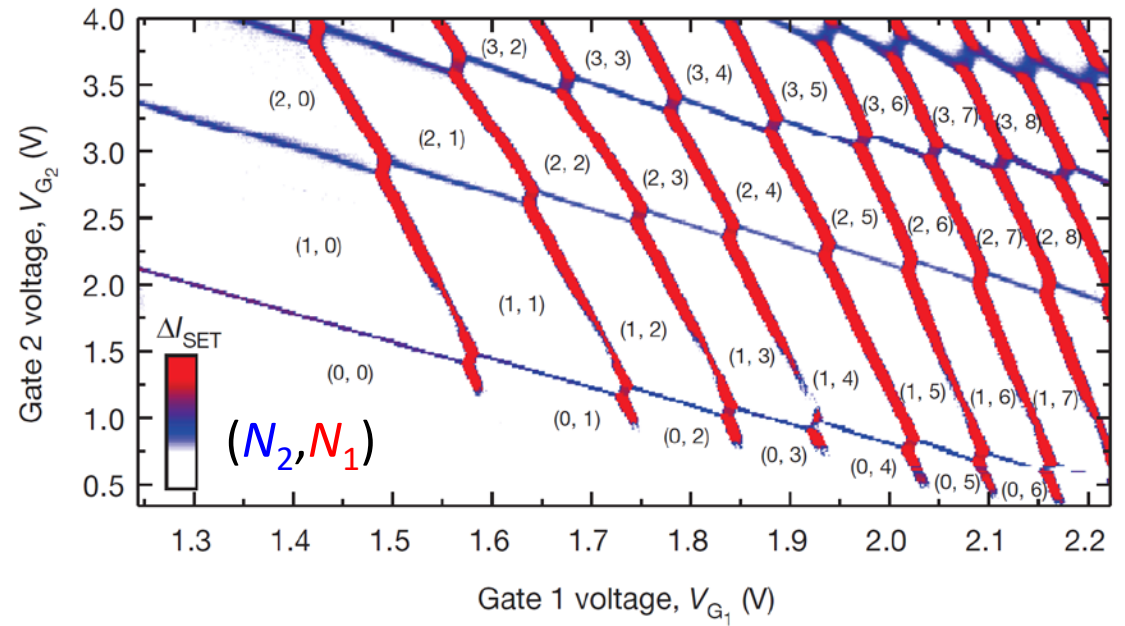
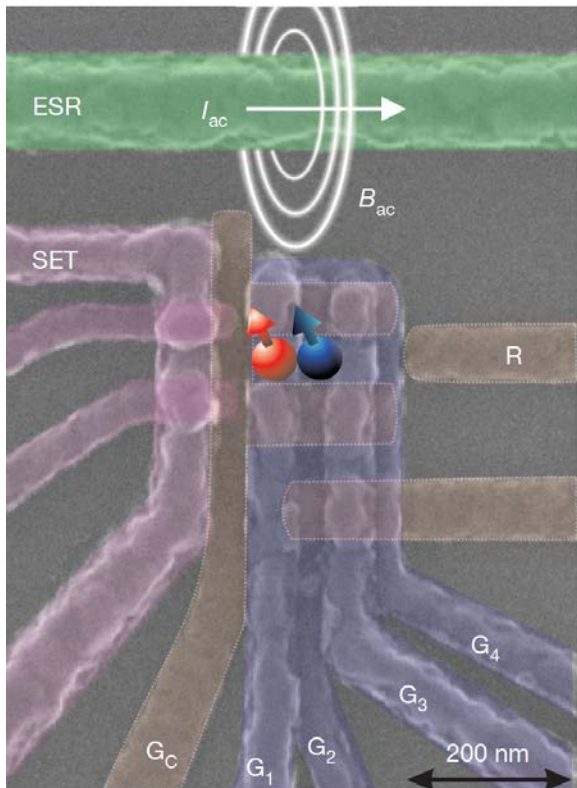
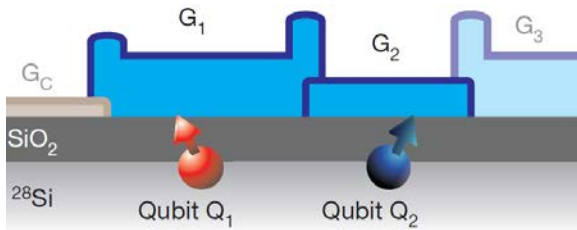


SET電荷センサ



近傍のQDsの電子数変化に
鋭敏に応答

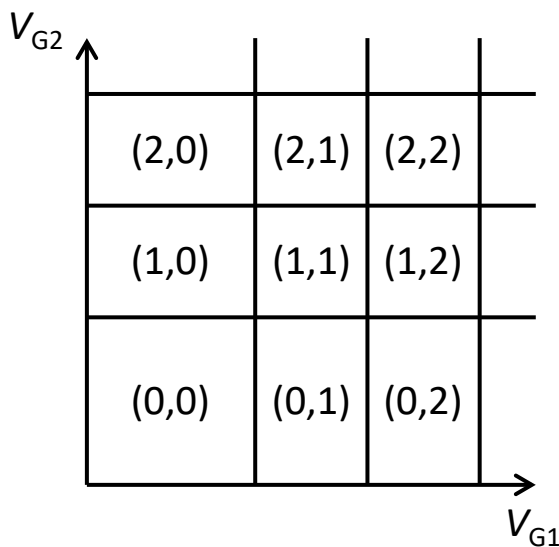
MOS型2重量子ドット



* Q_1, Q_2 の並び方と N_2, N_1 の順番が逆なので見づらい...

スタビリティダイアグラム

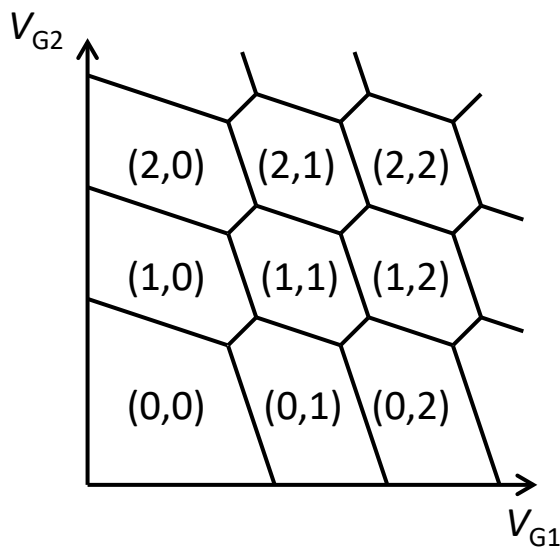
$$C_m \rightarrow 0$$



弱結合

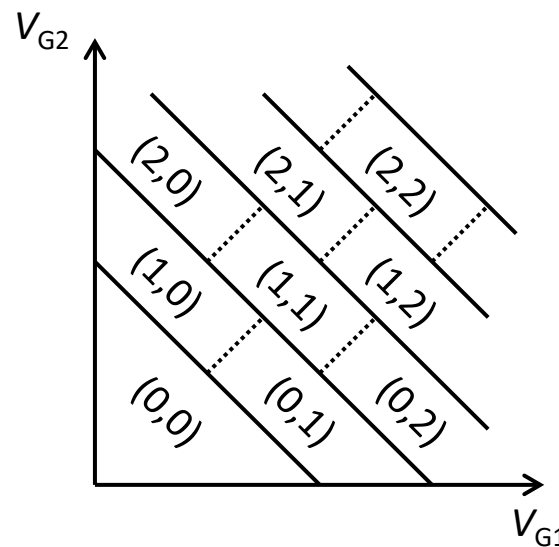
独立のドットとして
振る舞う

$$C_m \gg C_{R,11,22,12,21,SET}$$



ハニカム構造

相互に影響しあう



強結合

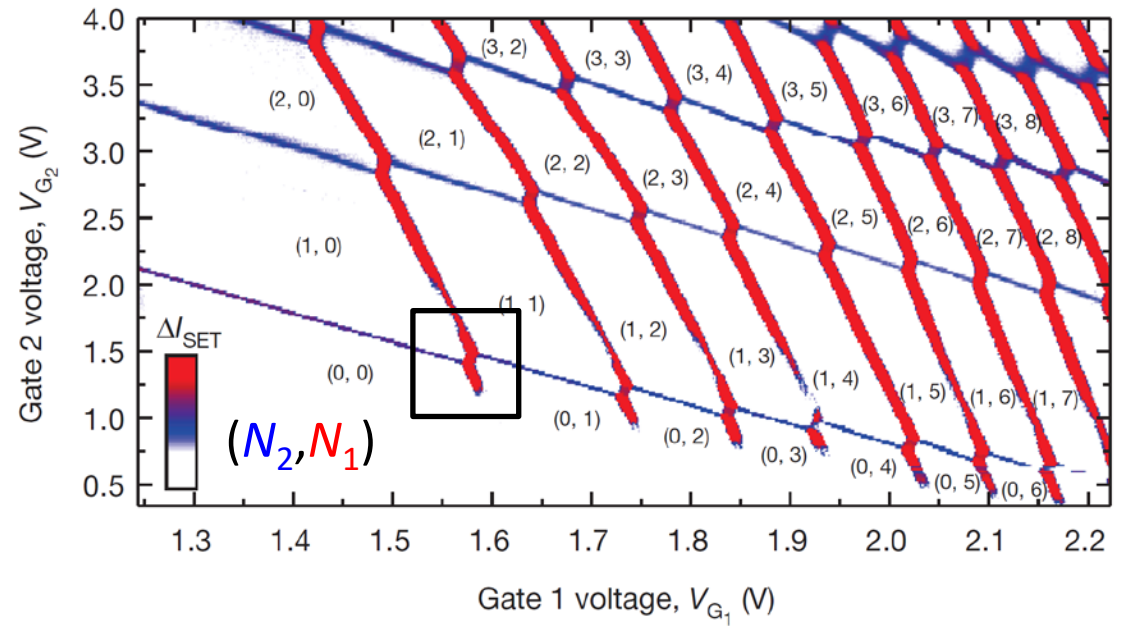
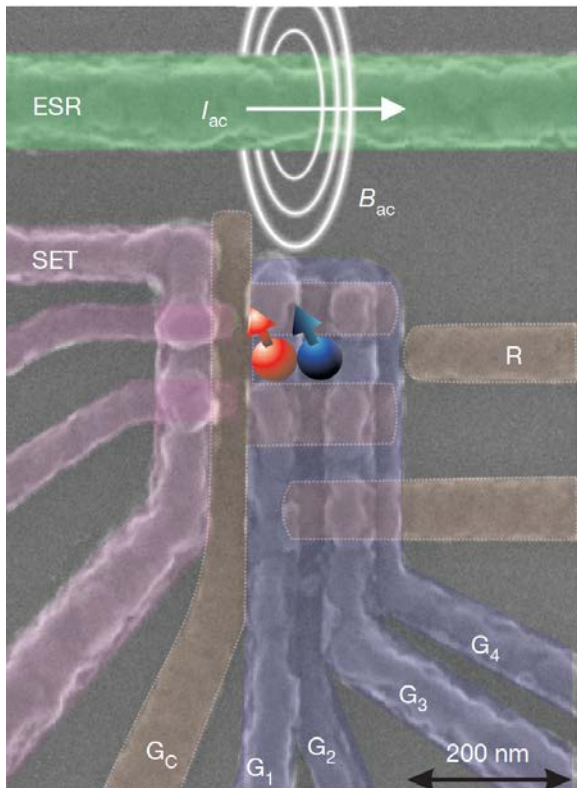
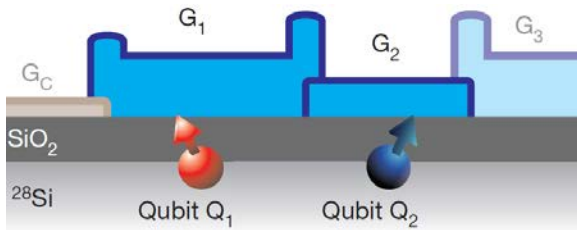
1つのドットとして
振る舞う

定相互作用モデルで理解できるがここでは省略 →

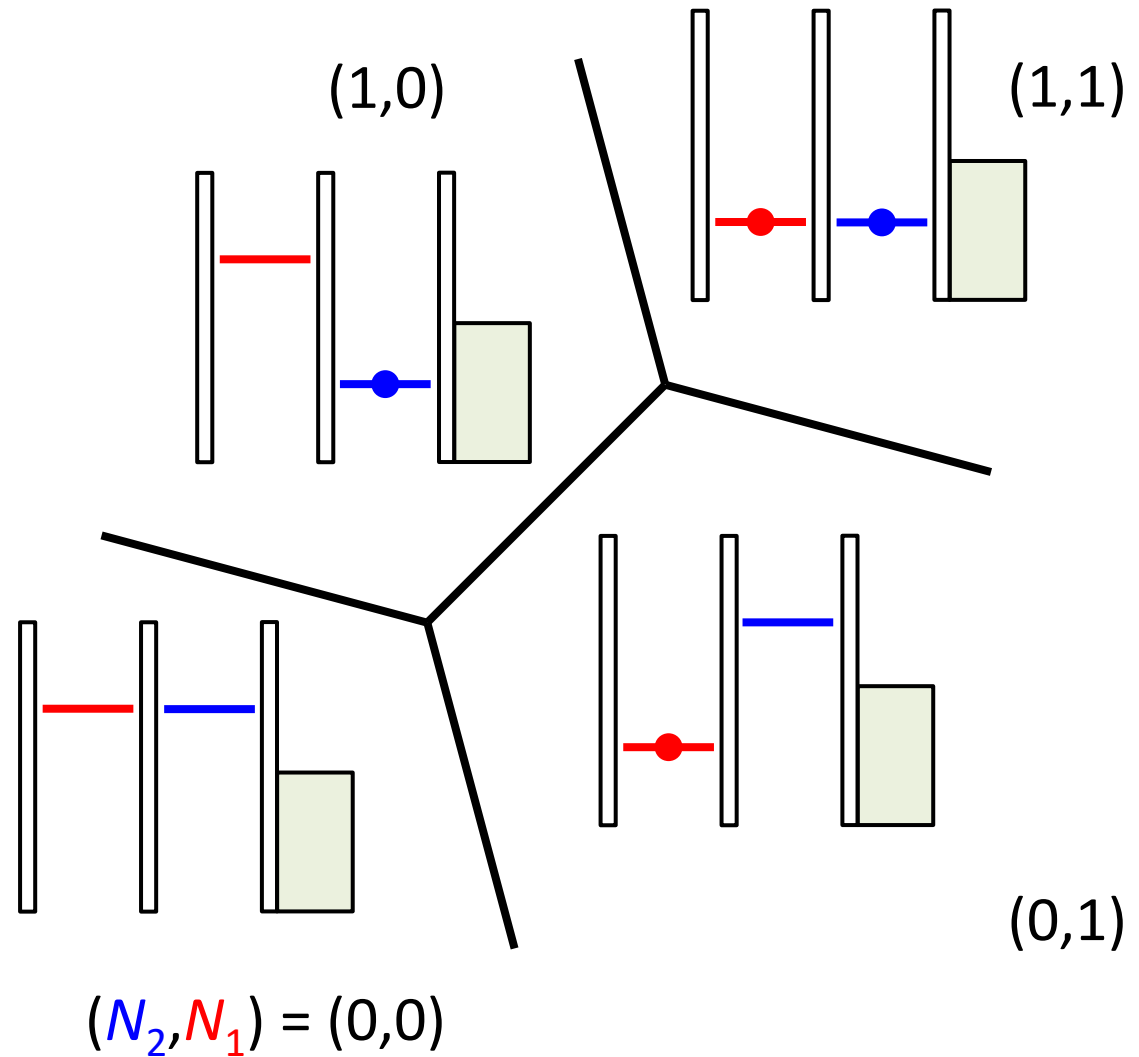
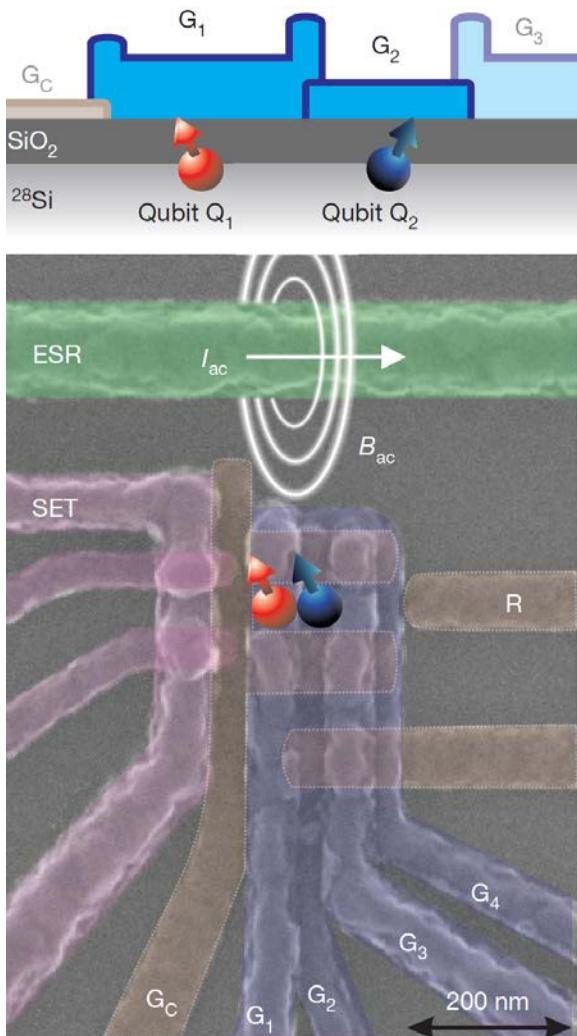
Rev. Mod. Phys. **75**, 1 (2003) van der Wiel *et al.*

Rev. Mod. Phys. **79**, 1217 (2007) Hanson *et al.*

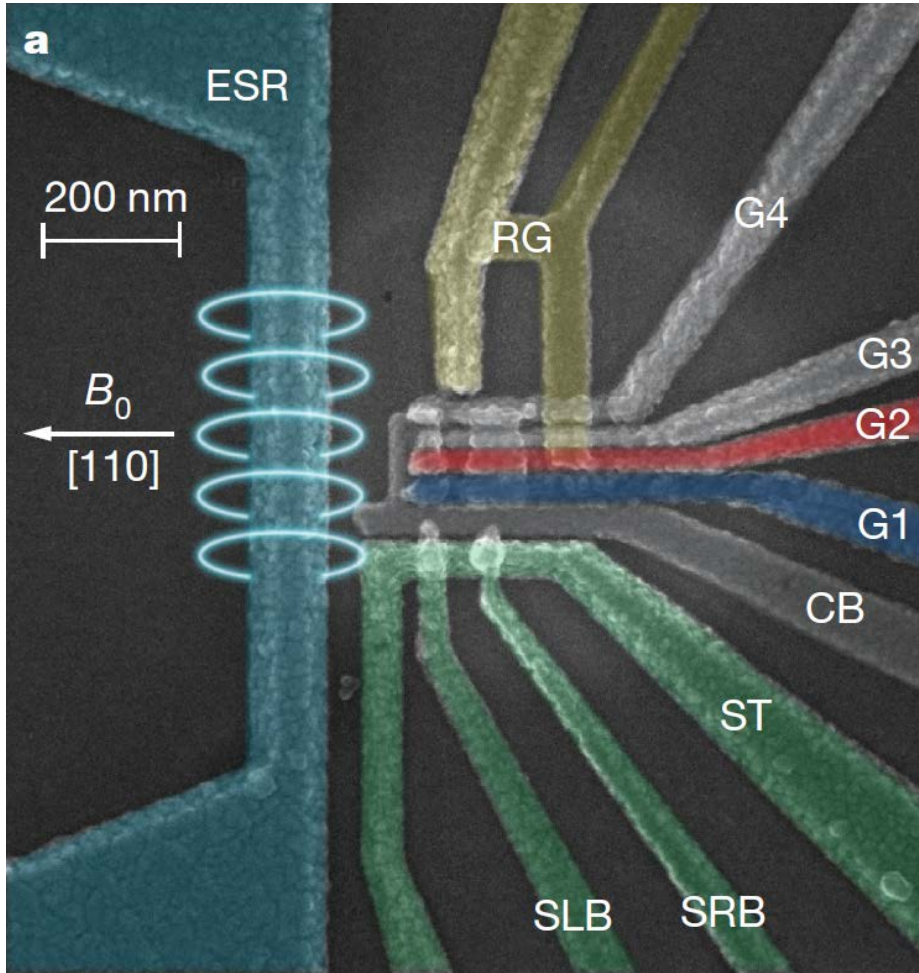
MOS型2重量子ドット



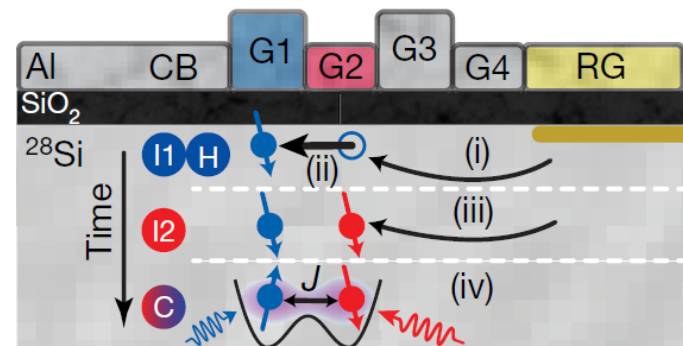
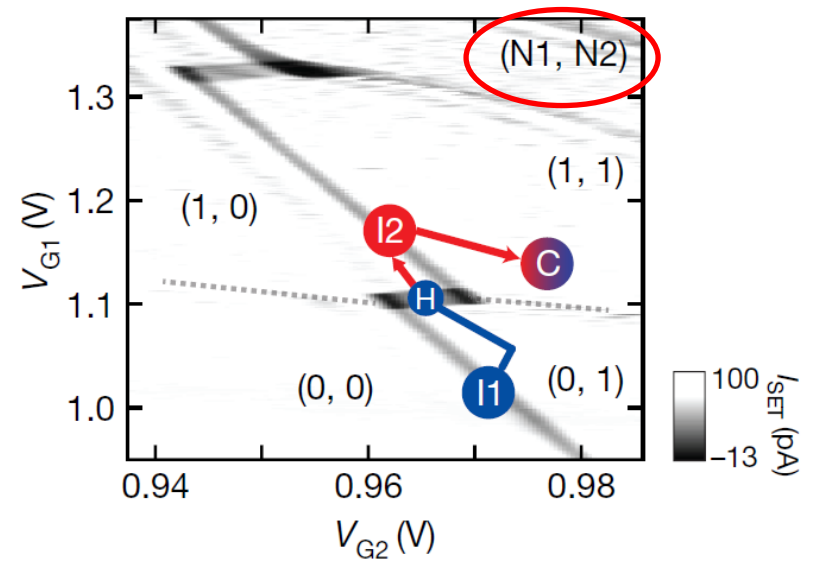
MOS型2重量子ドット



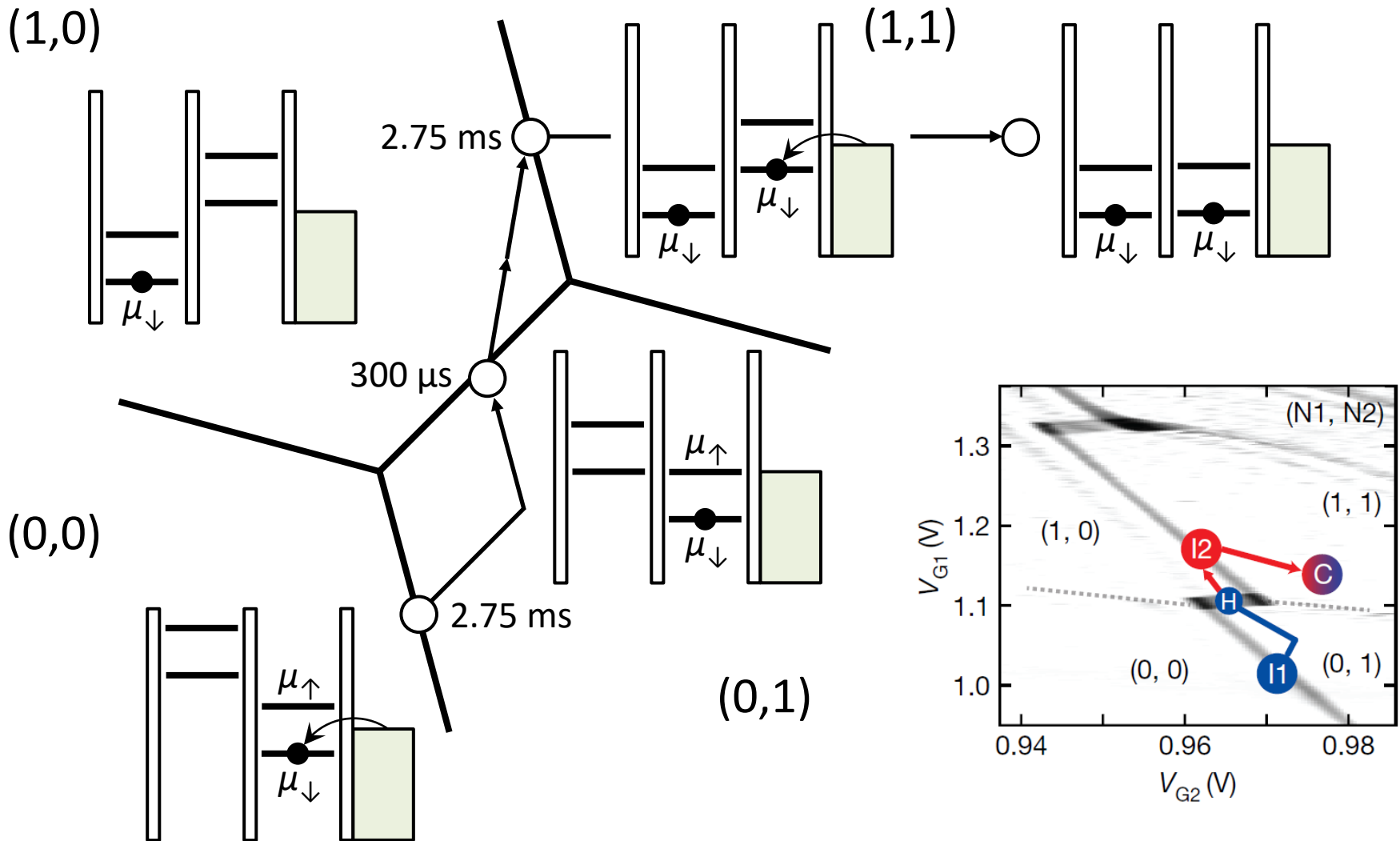
量子ビット操作



* Q_1, Q_2 の同じ並び方

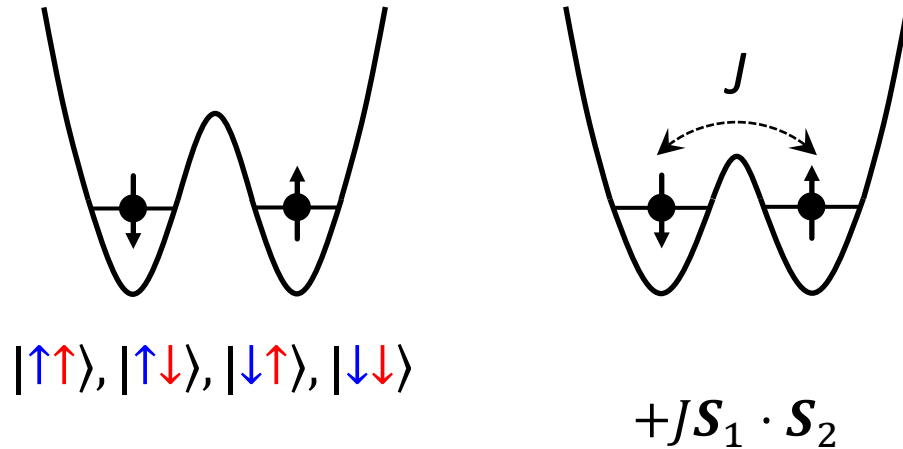


スピン初期化



2スピン状態

量子ドット閉じ込めポテンシャル

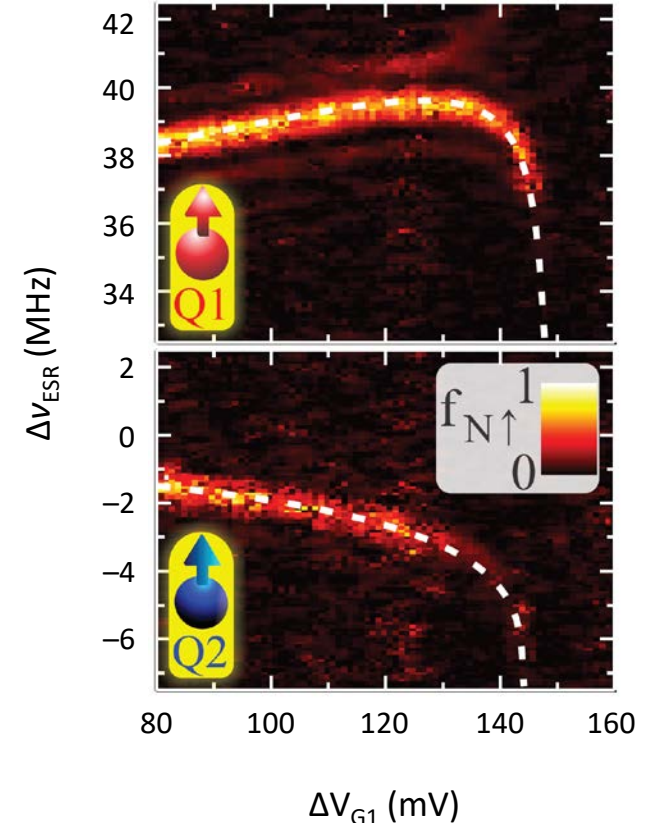


$$H = \gamma_{e,1}B_0(S_{z,1} \otimes I) + \gamma_{e,2}B_0(I \otimes S_{z,2})$$

*低磁場におけるSi:Pのスピンハミルトニアンと形式上同じ形(なので、ここで解きません)

$$H = \gamma_e B_0 S_z - \gamma_p B_0 I_z + a_0 \mathbf{S} \cdot \mathbf{I}$$

電場によるESR周波数制御



2スピン状態

$$(f_{1\uparrow}, f_{1\downarrow}, f_{2\uparrow}, f_{2\downarrow}) \rightarrow (\bar{E}_z, 0, 0, -\bar{E}_z) \rightarrow \left(\bar{E}_z, \frac{\delta E_z}{2}, -\frac{\delta E_z}{2}, -\bar{E}_z \right) \rightarrow \left(\bar{E}_z, \frac{\delta \tilde{E}_z - J}{2}, -\frac{\delta \tilde{E}_z + J}{2}, -\bar{E}_z \right)$$

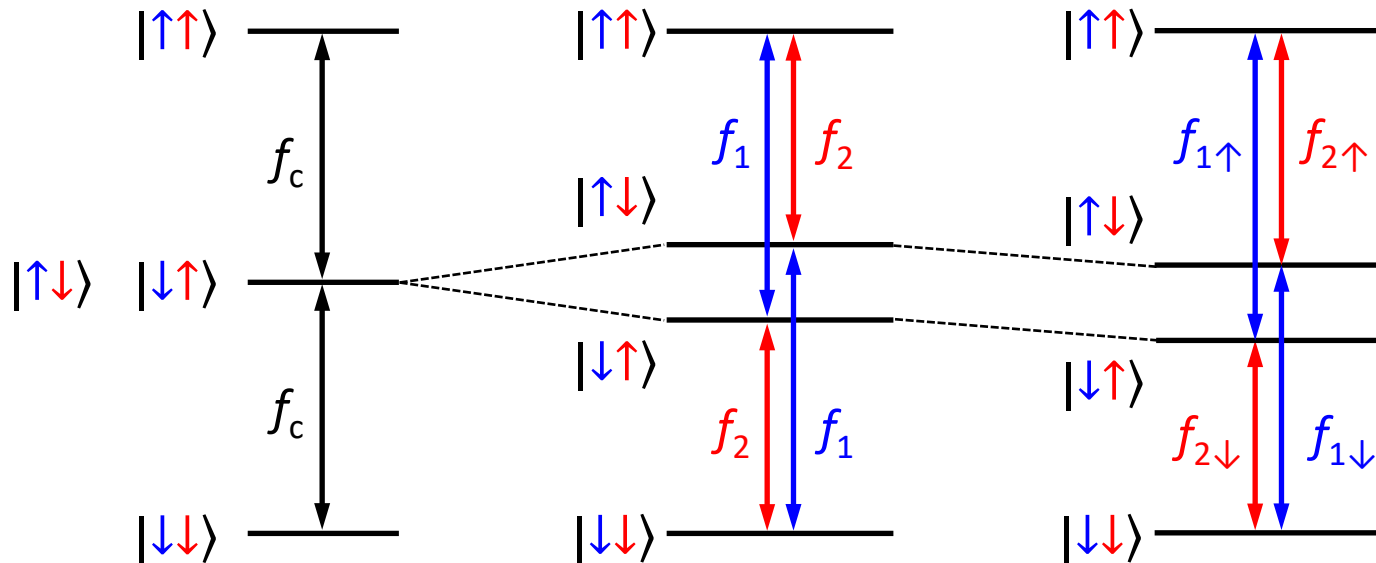
$$\bar{E}_z/h = f_c = 39.33 \text{ GHz}$$

@ $B_0 = 1.42 \text{ T}$

$$\delta E_z/h = 13.26 \text{ MHz}$$

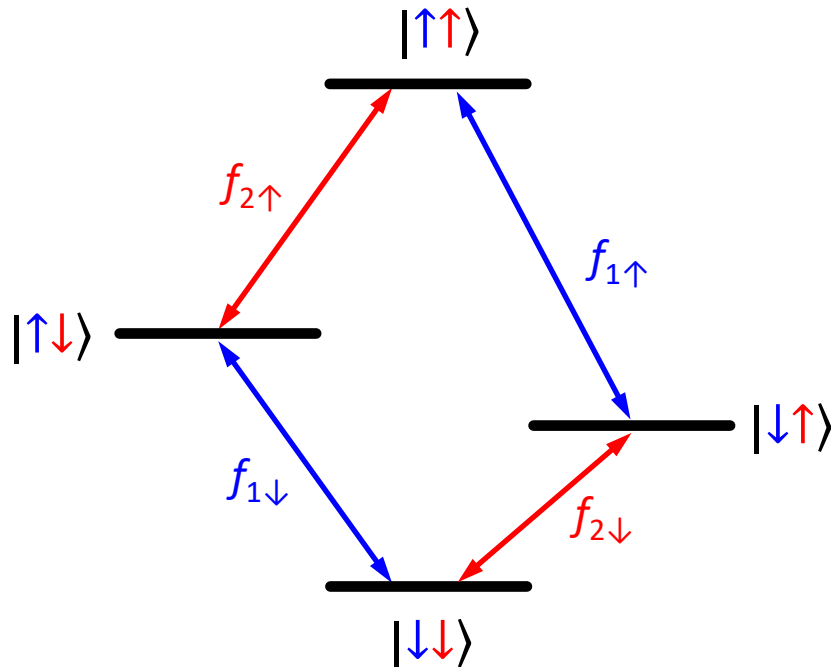
$$\delta \tilde{E}_z = (\delta E_z^2 + J^2)^{1/2}$$

$$J/h = 1.59 \text{ MHz}$$



CROTゲート

- 全ての遷移が異なる周波数を持つ



$$f_{1\uparrow} = \bar{E}_z + \frac{\delta\tilde{E}_z + J}{2}$$

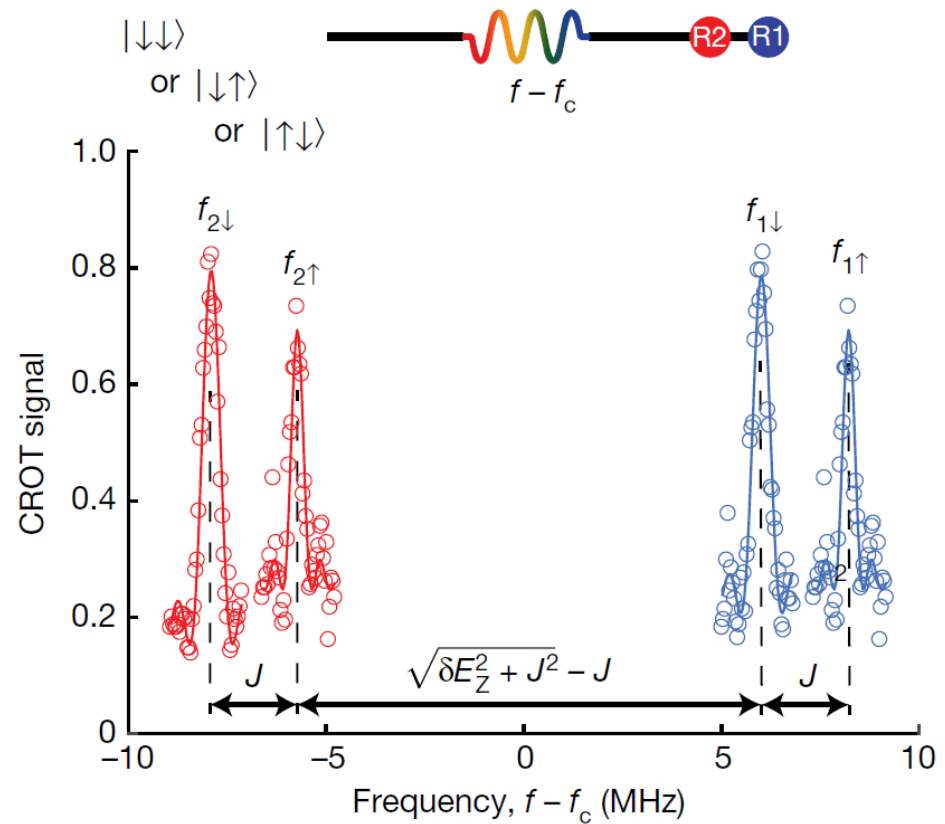
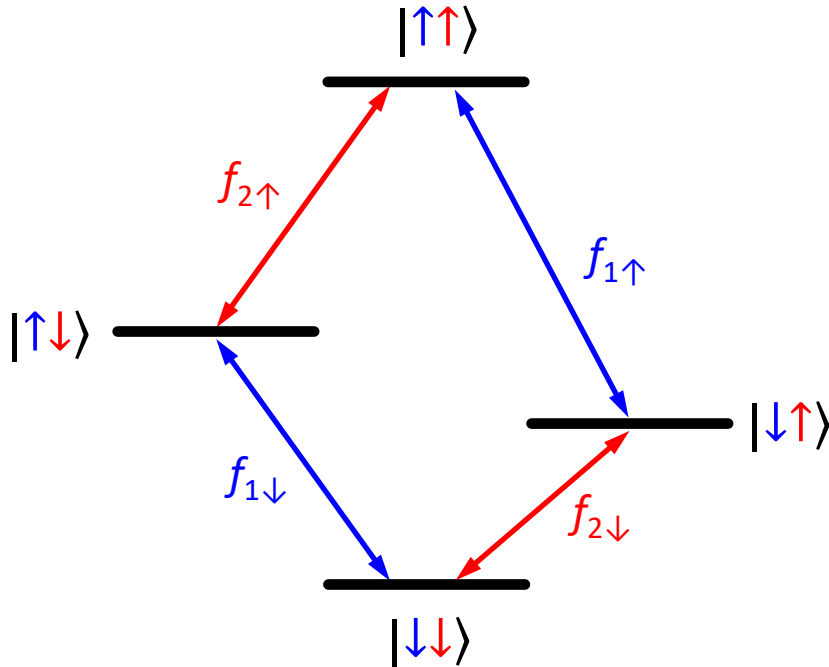
$$f_{1\downarrow} = \bar{E}_z + \frac{\delta\tilde{E}_z + J}{2}$$

$$f_{2\uparrow} = \bar{E}_z + \frac{-\delta\tilde{E}_z + J}{2}$$

$$f_{2\downarrow} = \bar{E}_z + \frac{-\delta\tilde{E}_z - J}{2}$$

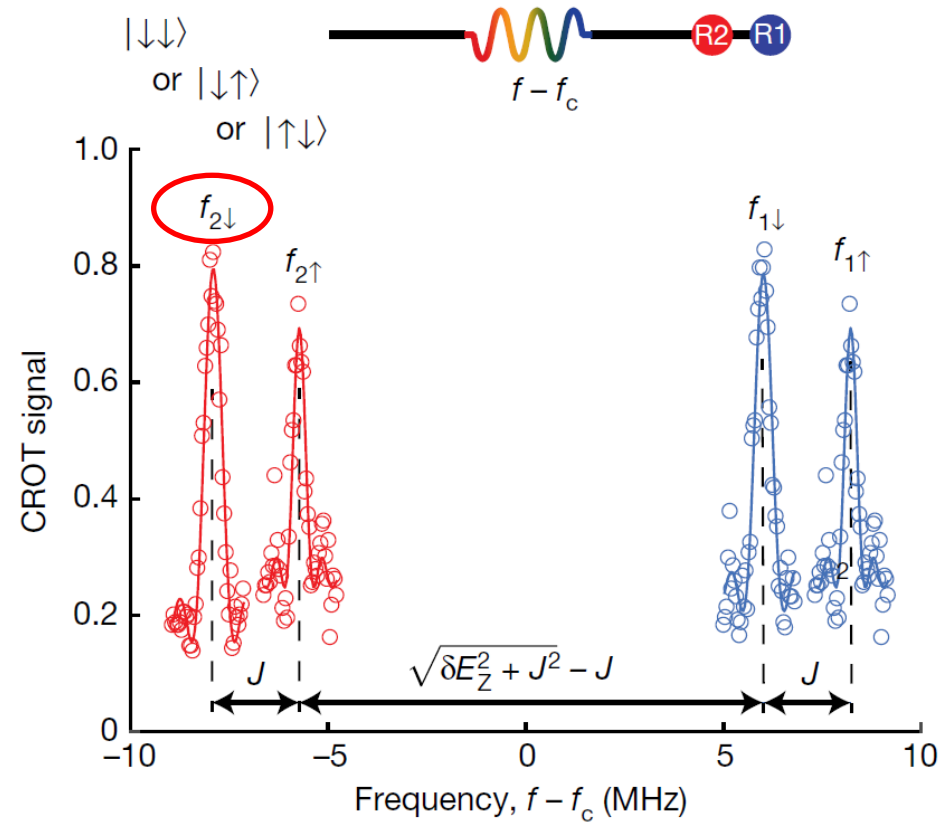
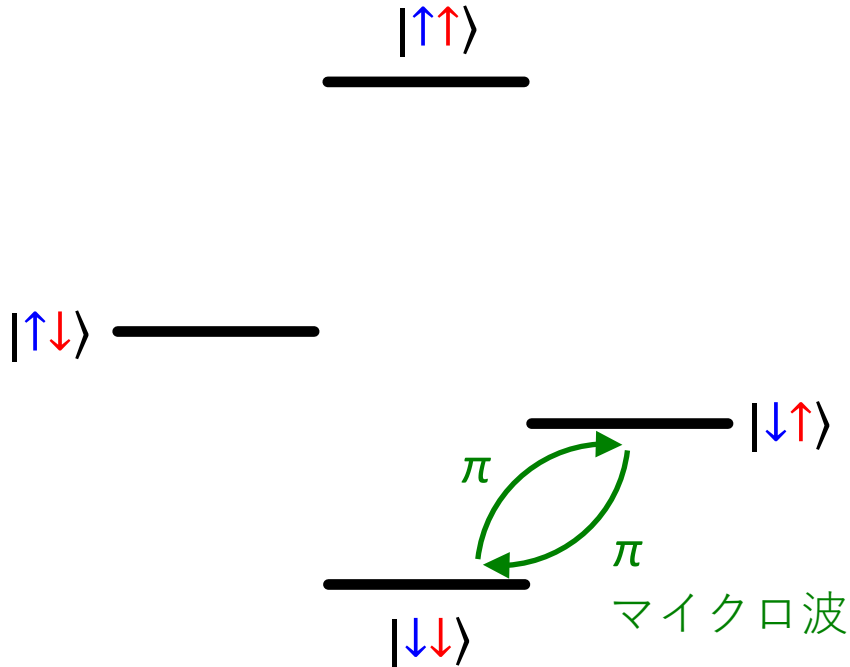
CROTゲート

- 全ての遷移が異なる周波数を持つ
- 選択励起の π パルスにより2量子ビットゲートが実現可能



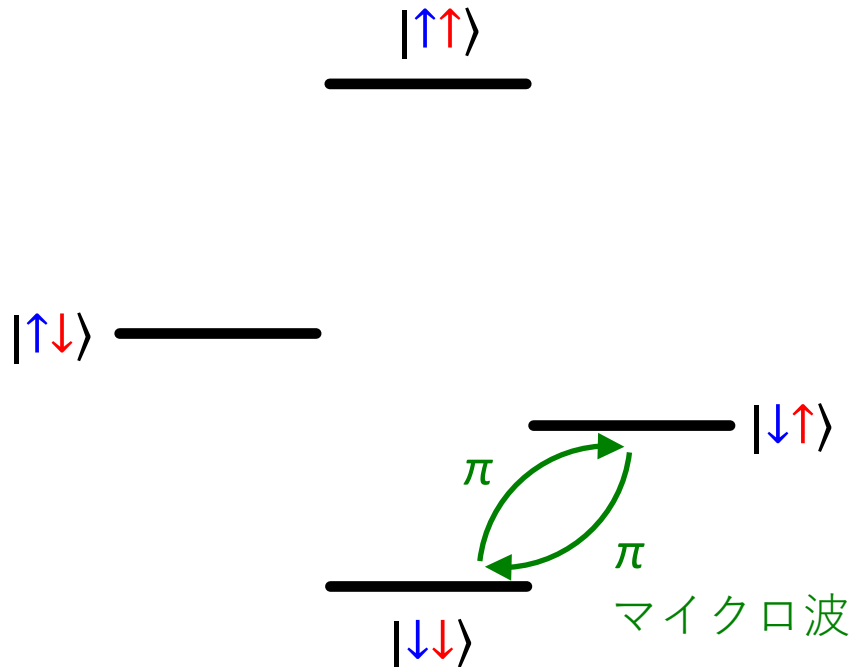
CROTゲート

- 全ての遷移が異なる周波数を持つ
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CROTゲート

- 全ての遷移が異なる周波数を持つ
- 選択励起の π パルスにより2量子ビットゲートが実現可能



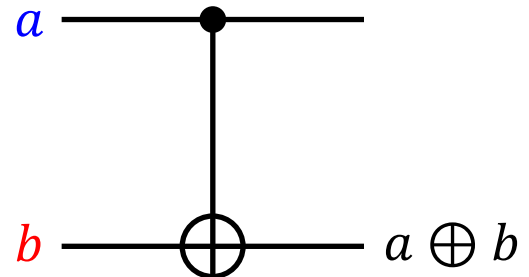
e.g. CNOTゲート

$$|\uparrow\uparrow\rangle = |00\rangle \longrightarrow |00\rangle$$

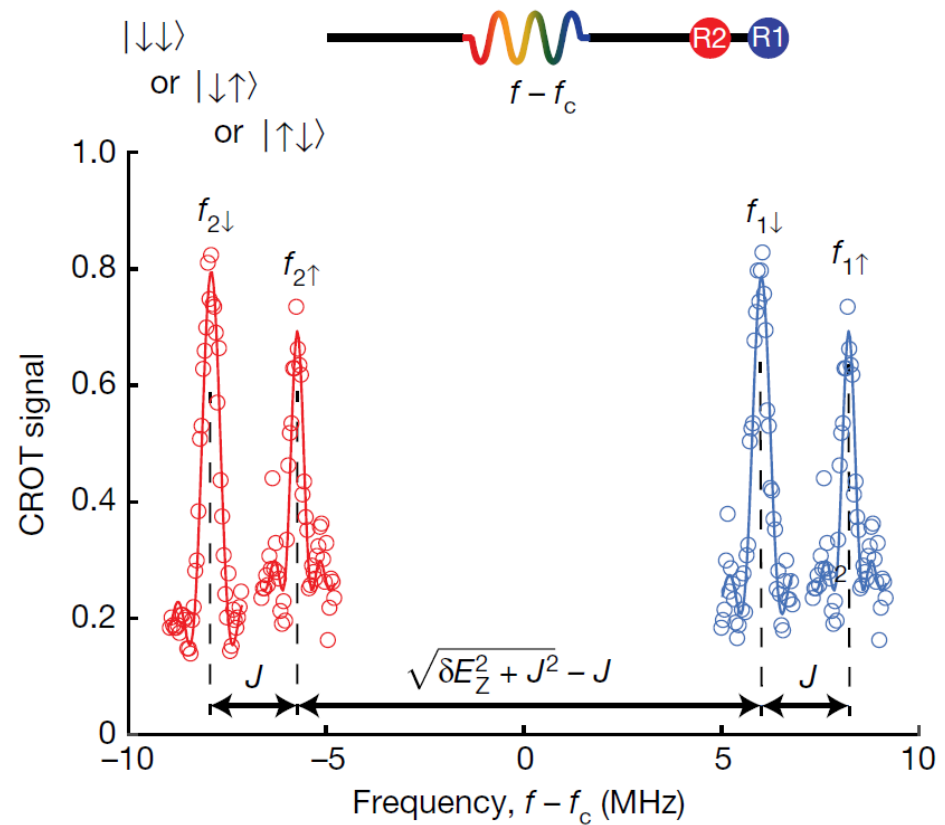
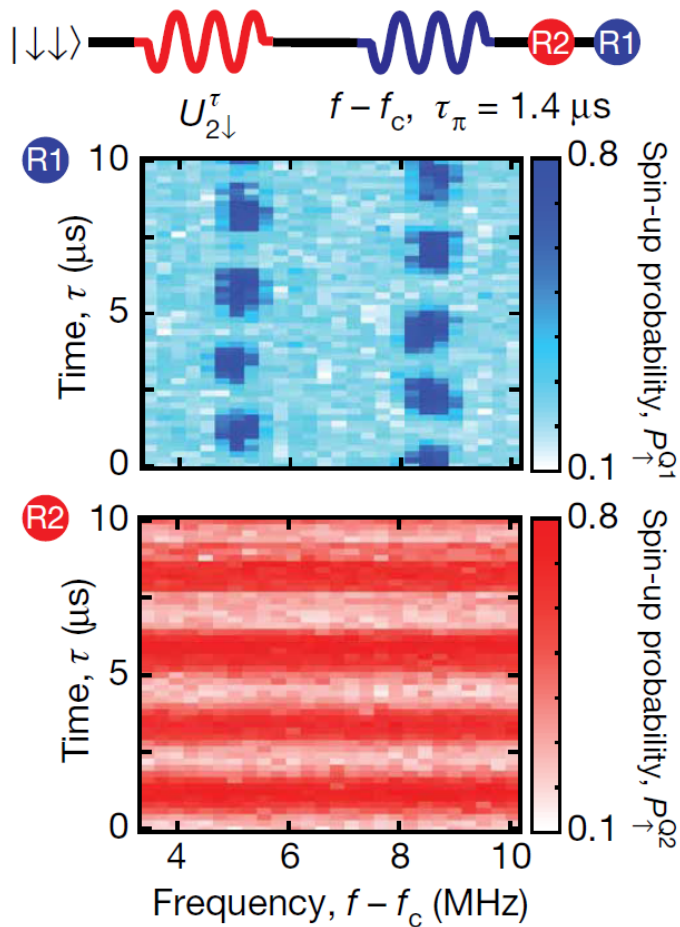
$$|\uparrow\downarrow\rangle = |01\rangle \longrightarrow |01\rangle$$

$$|\downarrow\uparrow\rangle = |10\rangle \longrightarrow |11\rangle$$

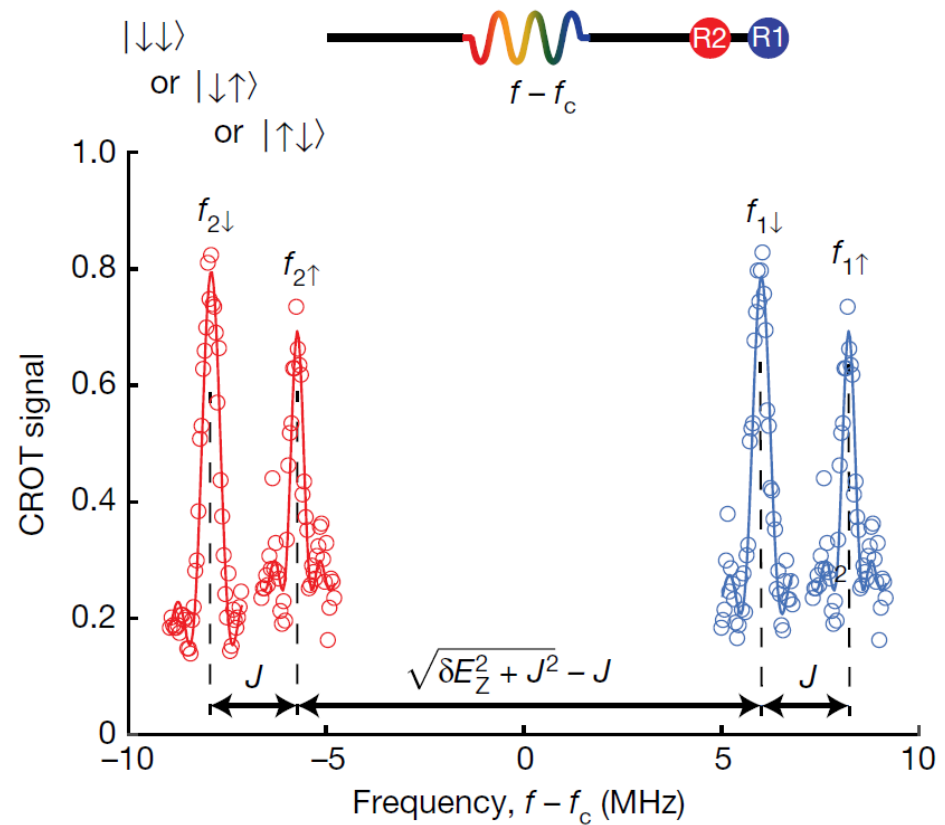
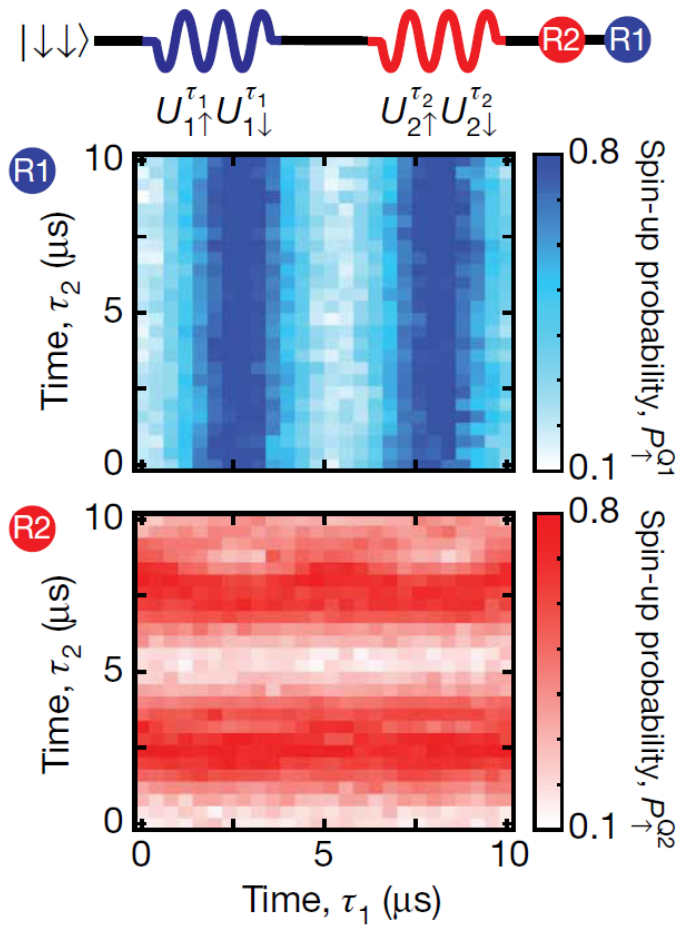
$$|\downarrow\downarrow\rangle = |11\rangle \longrightarrow |10\rangle$$



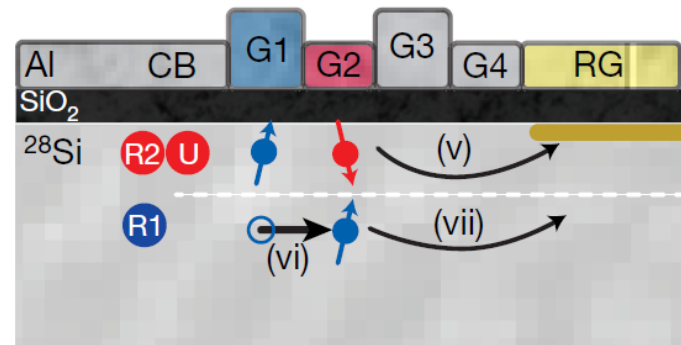
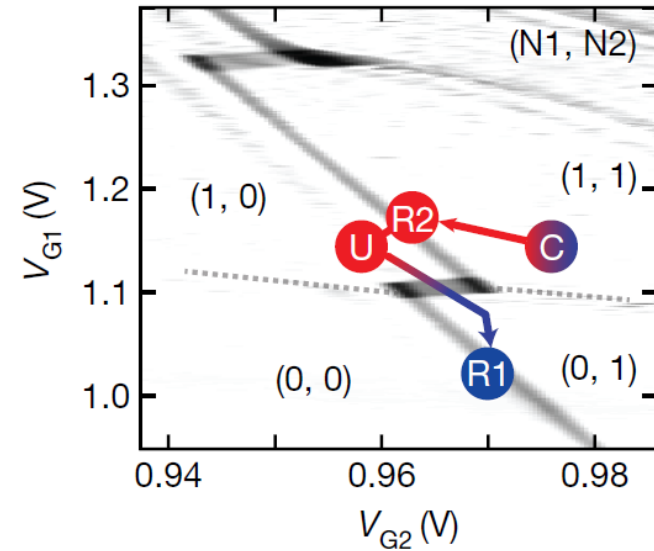
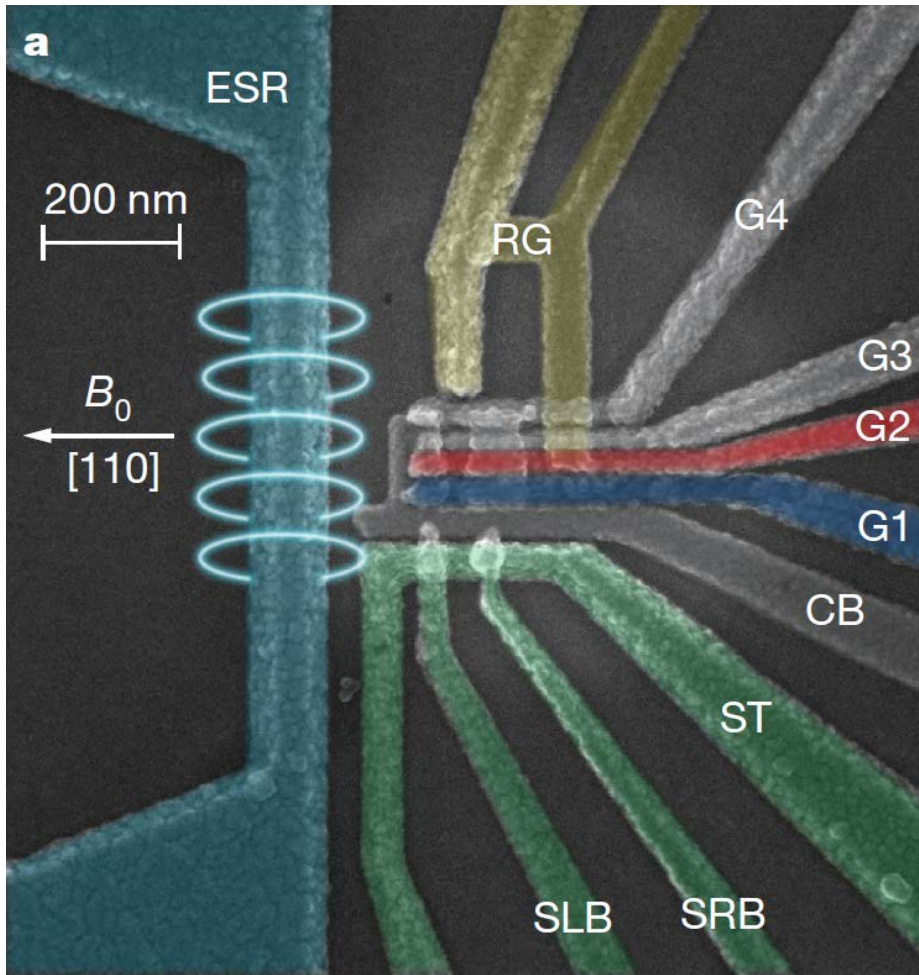
CROTゲート



1量子ビットゲート



スピン読み出し



“高温”動作

Article

Operation of a silicon quantum processor unit cell above one kelvin

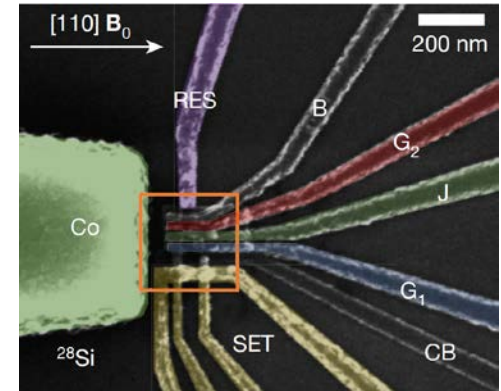
<https://doi.org/10.1038/s41586-020-2171-6>

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C. H. Yang¹✉, R. C. C. Leon¹, J. C. C. Hwang^{1,6}, A. Saraiva¹, T. Tantt¹, W. Huang¹, J. Camirand Lemyre², K. W. Chan¹, K. Y. Tan^{3,7}, F. E. Hudson¹, K. M. Itoh⁴, A. Morello¹, M. Pioro-Ladrière^{2,5}, A. Laucht¹ & A. S. Dzurak¹✉



Article

Universal quantum logic in hot silicon qubits

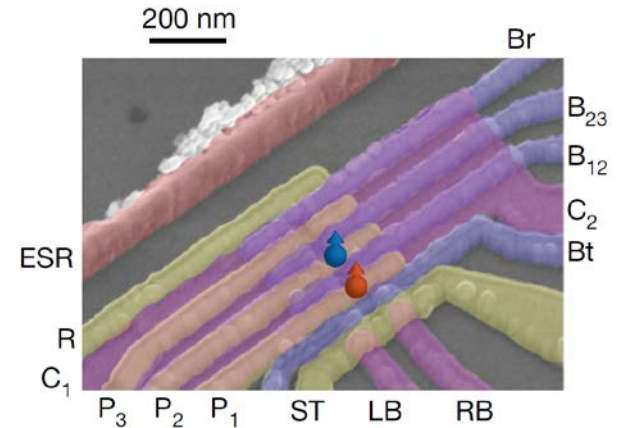
<https://doi.org/10.1038/s41586-020-2170-7>

Received: 22 October 2019

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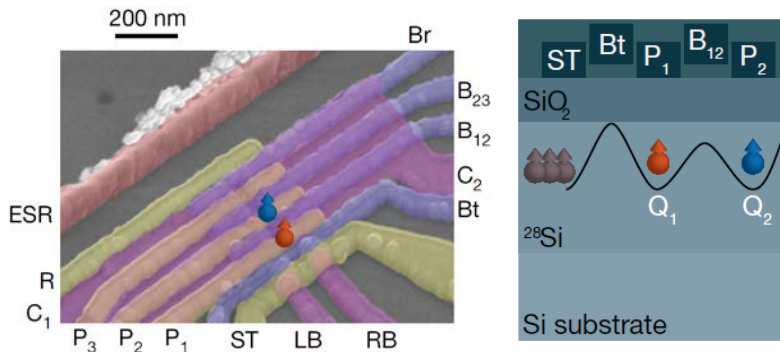
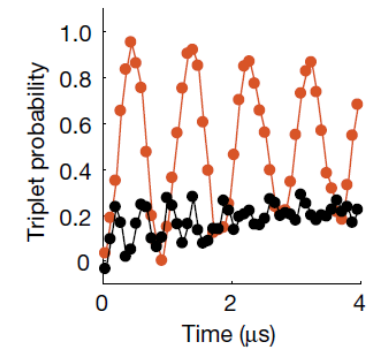
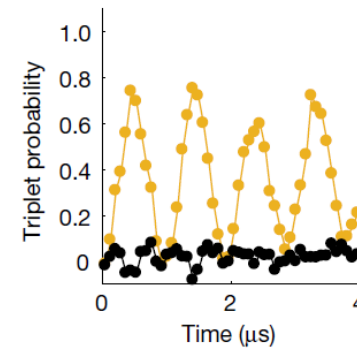
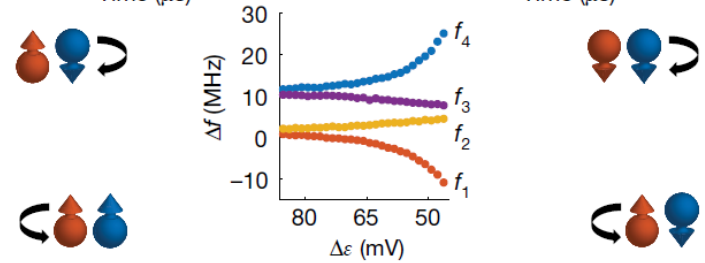
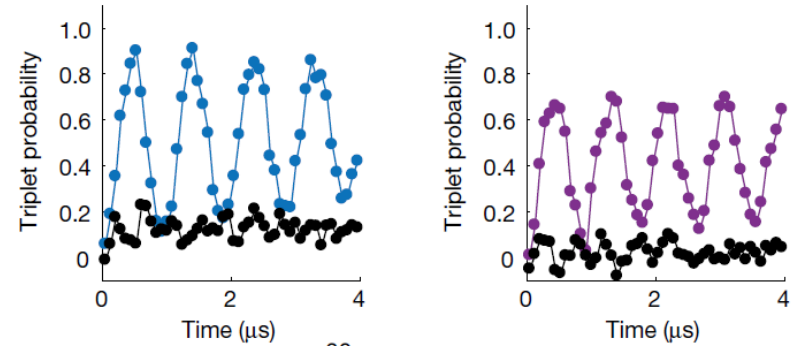
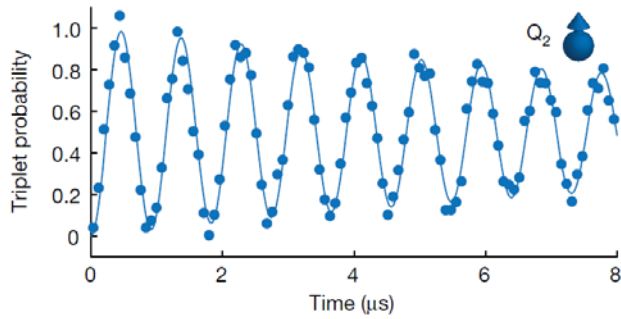
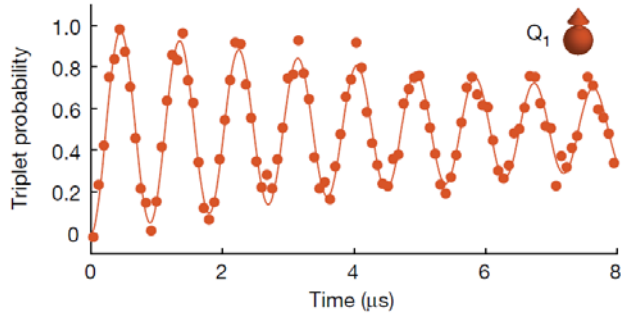
L. Petit¹, H. G. J. Eenink¹, M. Russ¹, W. I. L. Lawrie¹, N. W. Hendrickx¹, S. G. J. Philips¹, J. S. Clarke², L. M. K. Vandersypen¹ & M. Veldhorst¹✉



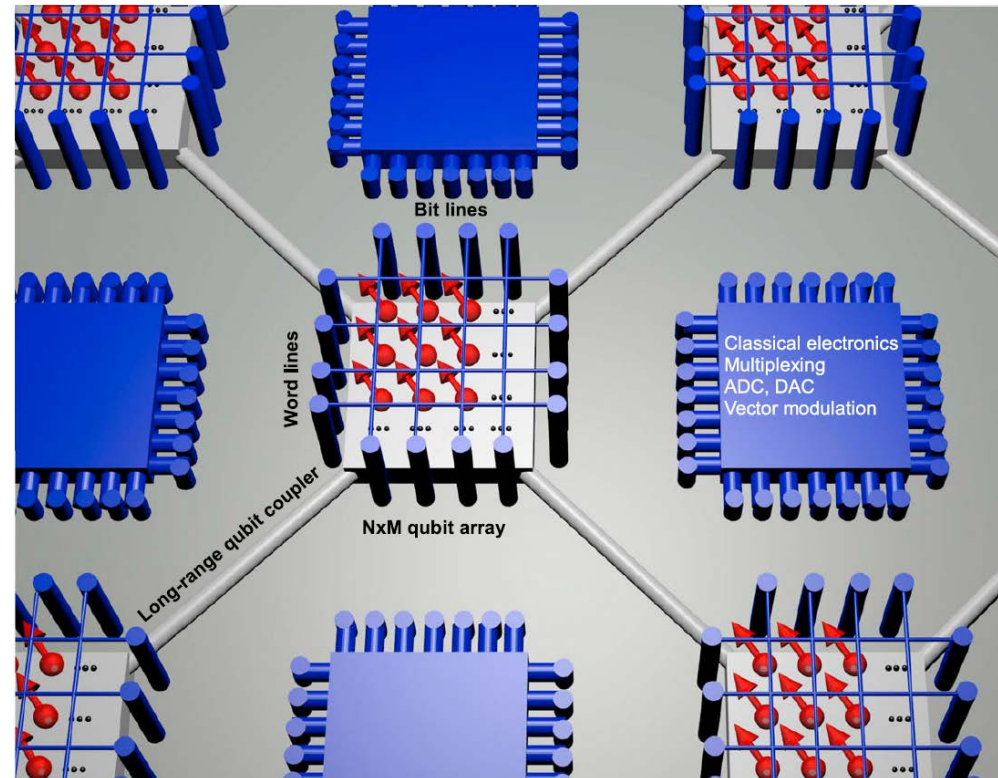
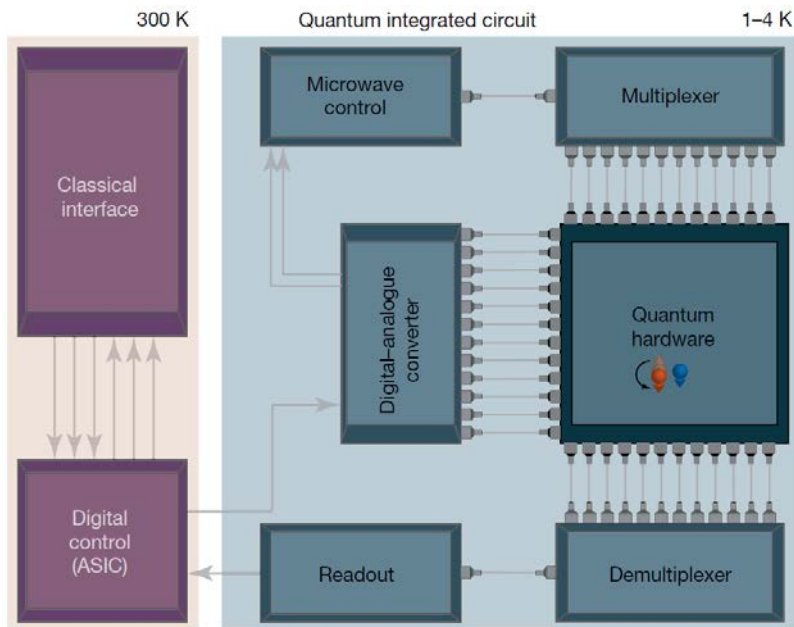
Nature **580**, 350 (2020) Yang *et al.*

Nature **580**, 355 (2020) Petit *et al.*

“高温”動作



“高温”動作



npj Quant. Info. **3**, 34 (2017) Vandersypen *et al.*

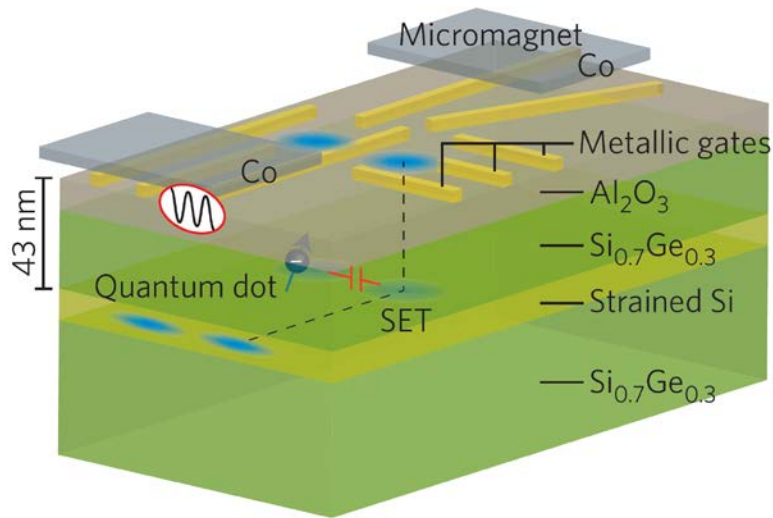
Nature **580**, 355 (2020) Petit *et al.*

講義内容

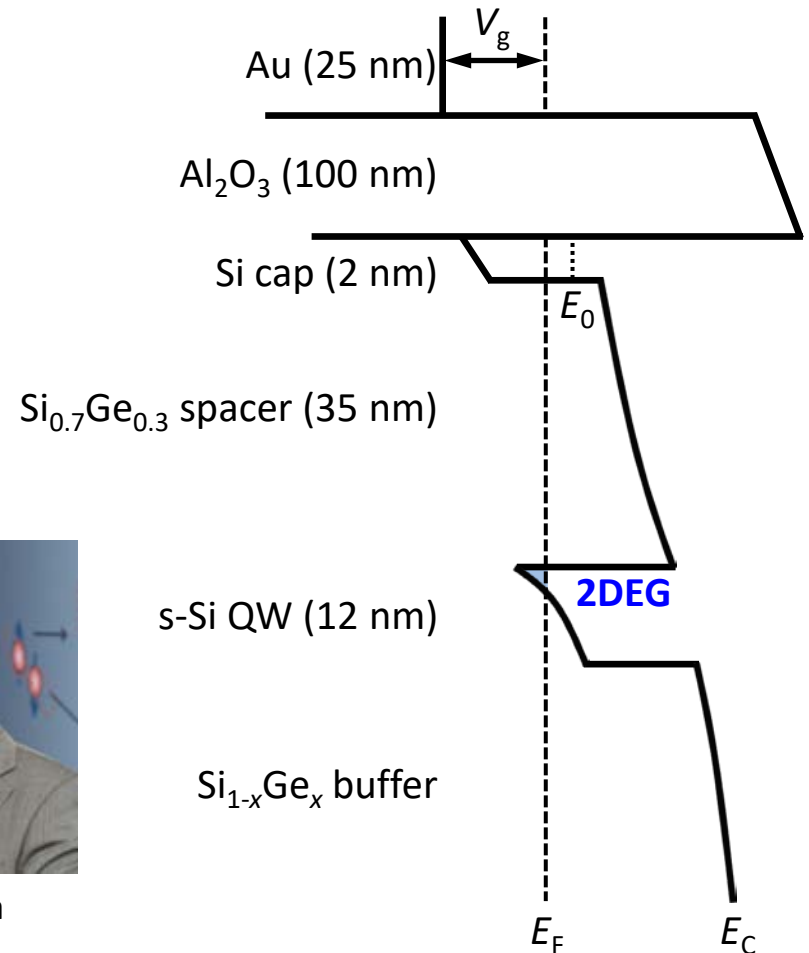
- **ドナースピン**
 - アンサンブル
 - 単一ドナー

- **量子ドットスピン**
 - MOS量子ドット
 - Si/SiGe量子ドット

Si/SiGeへテロ構造



ノンドープ構造による蓄積型QD



L. Vandersypen

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J. Petta

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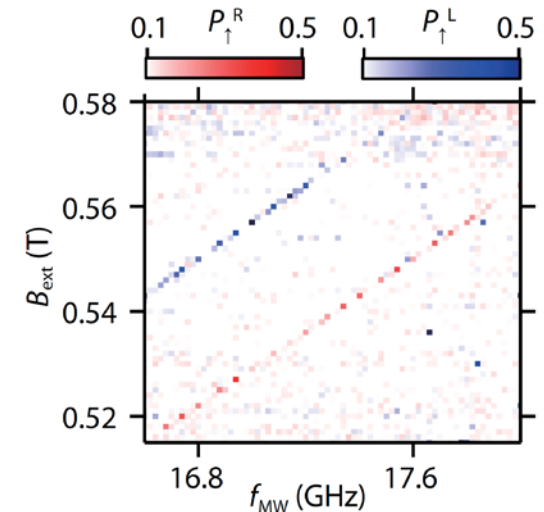
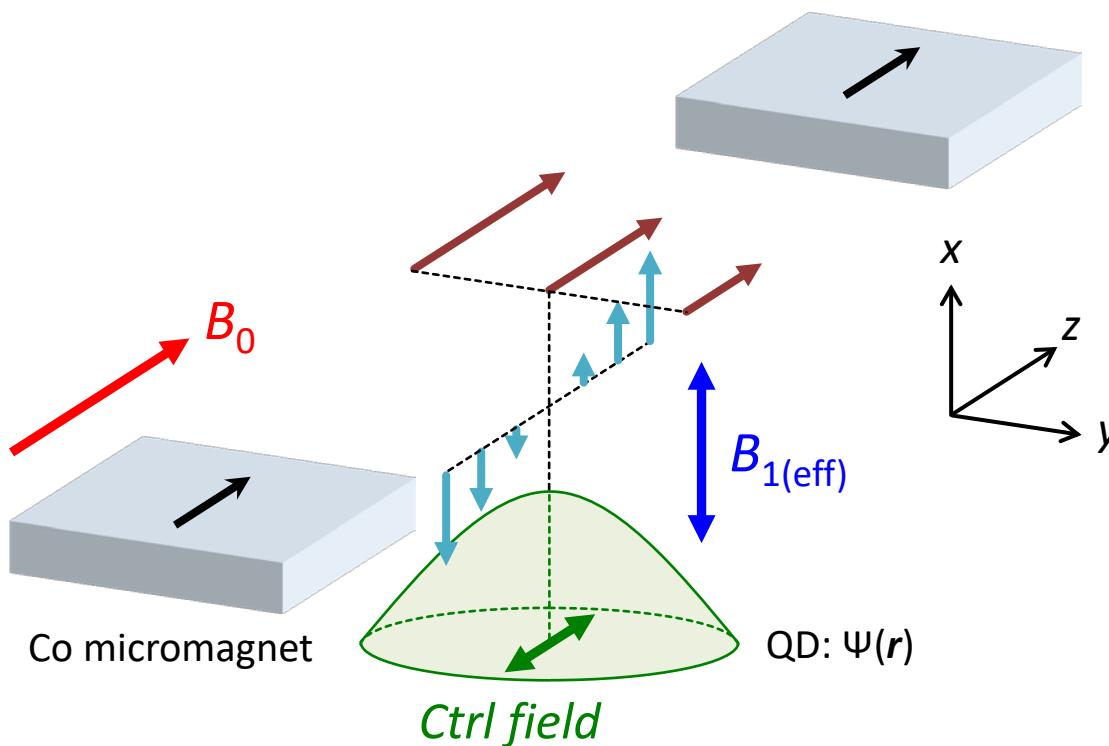


S. Tarucha

©RIKEN

電気双極子スピン共鳴

- **y方向の磁場勾配**によって共鳴周波数を制御
- **±z方向に電子波動関数を“揺する”**ことで**x方向に実効的な交流磁場**を生成



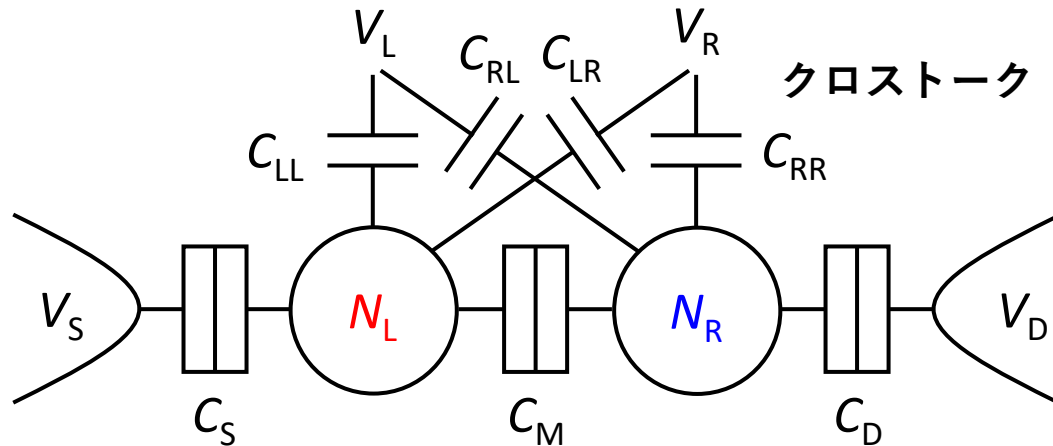
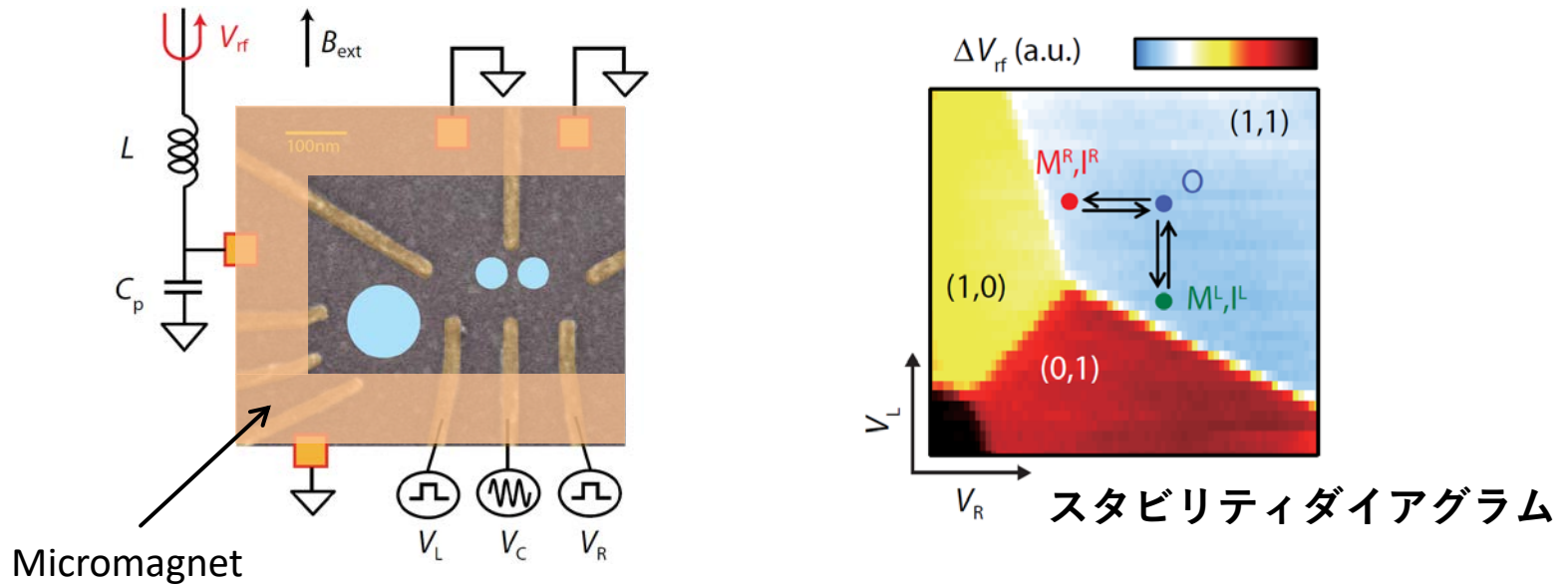
Sci. Adv. **2**, e1600694 (2016) Takeda *et al.*

(Theory) Phys. Rev. Lett. **96**, 047202 (2006) Tokura *et al.*

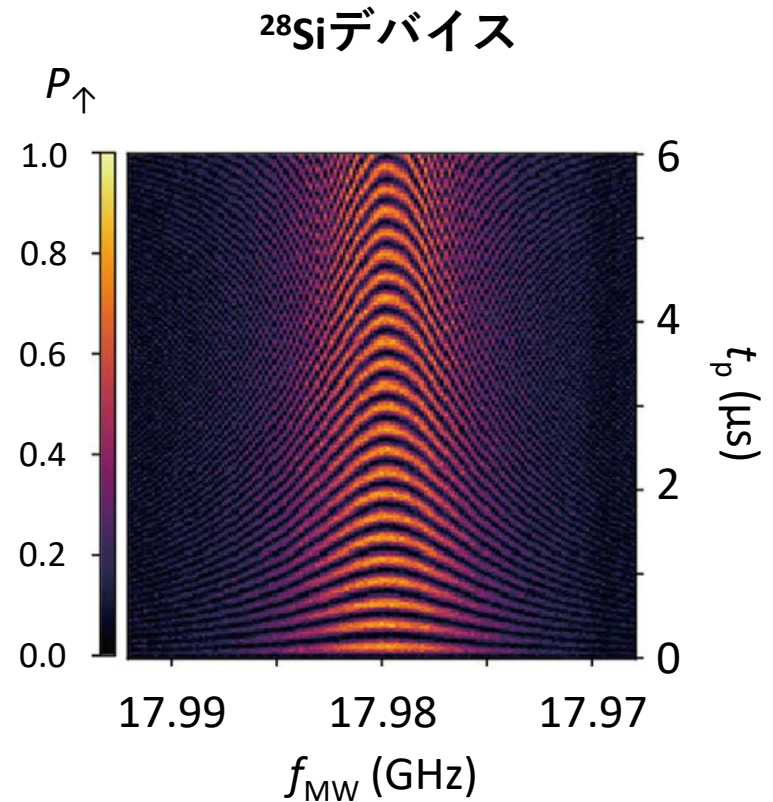
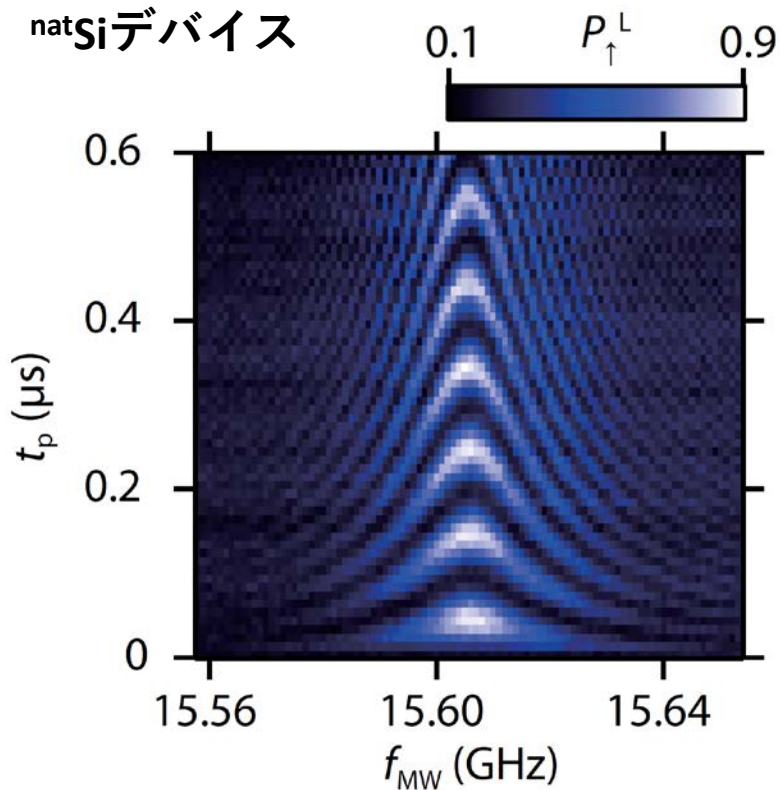
(GaAs QD) Nature Phys. **4**, 776 (2008) Pioro-Ladrière *et al.*

(Magnet design) Appl. Phys. Express **8**, 084401 (2015) Yoneda *et al.*

2重量子ドット



電気双極子スピン共鳴

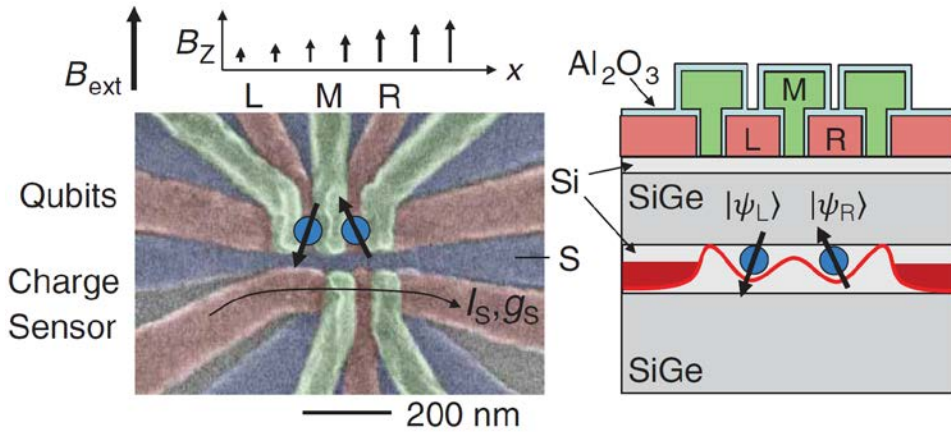


- ラビ周波数 $f_R \approx 30$ MHz
- フィデリティ $F^{RB} = 99.6\% \rightarrow > 99.9\%$
- $T_2^* = 2 \mu\text{s} \rightarrow 20 \mu\text{s}$, $T_2^{\text{CPMG}} = 3.1$ ms

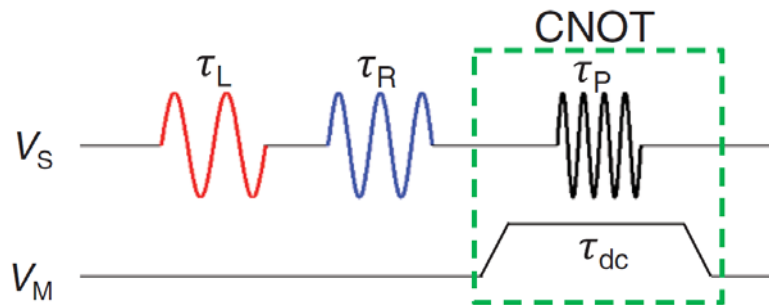
Sci. Adv. **2**, e1600694 (2016) Takeda *et al.*

Nature Nano. **13**, 102 (2018) Yoneda *et al.*

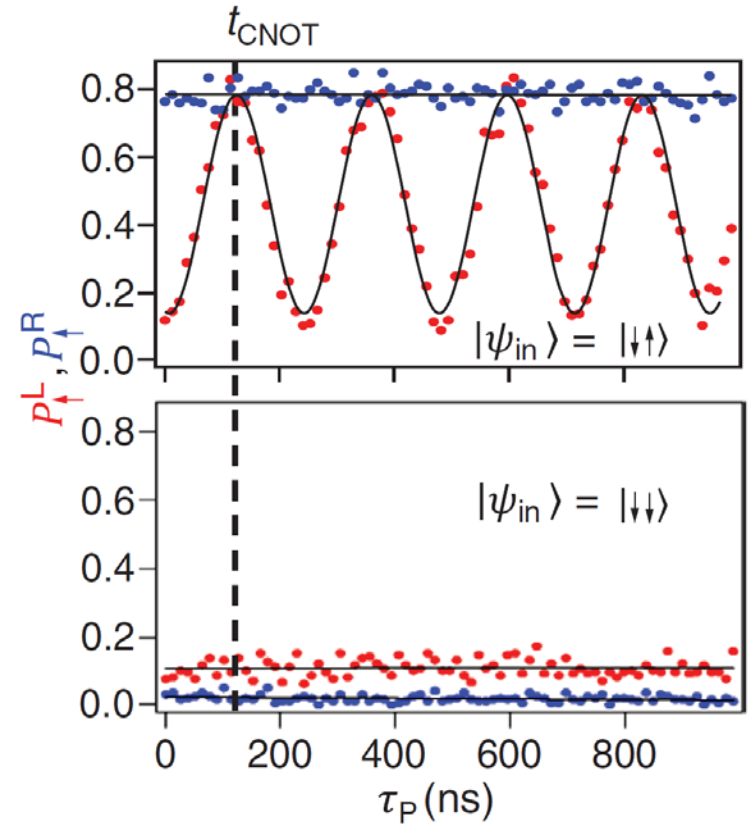
CNOTゲート



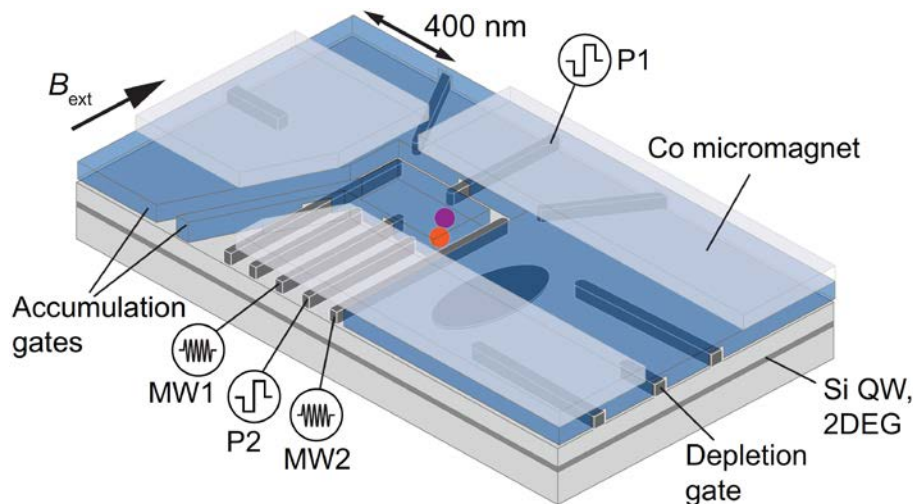
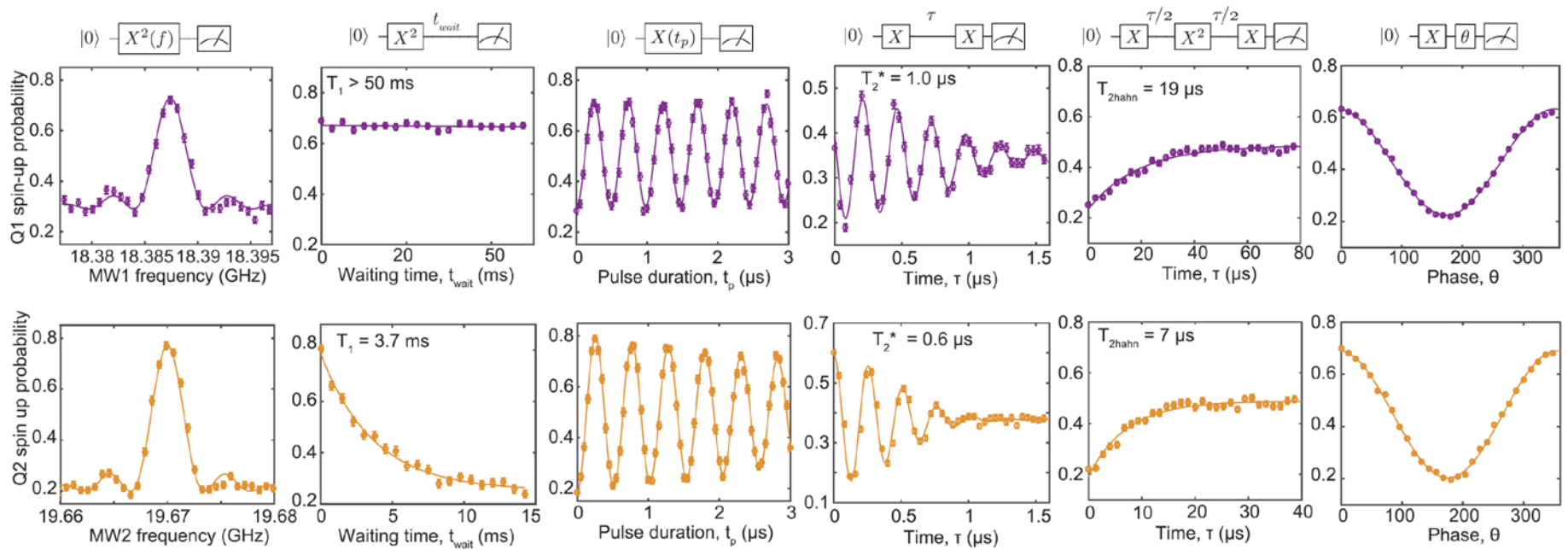
J : 可変



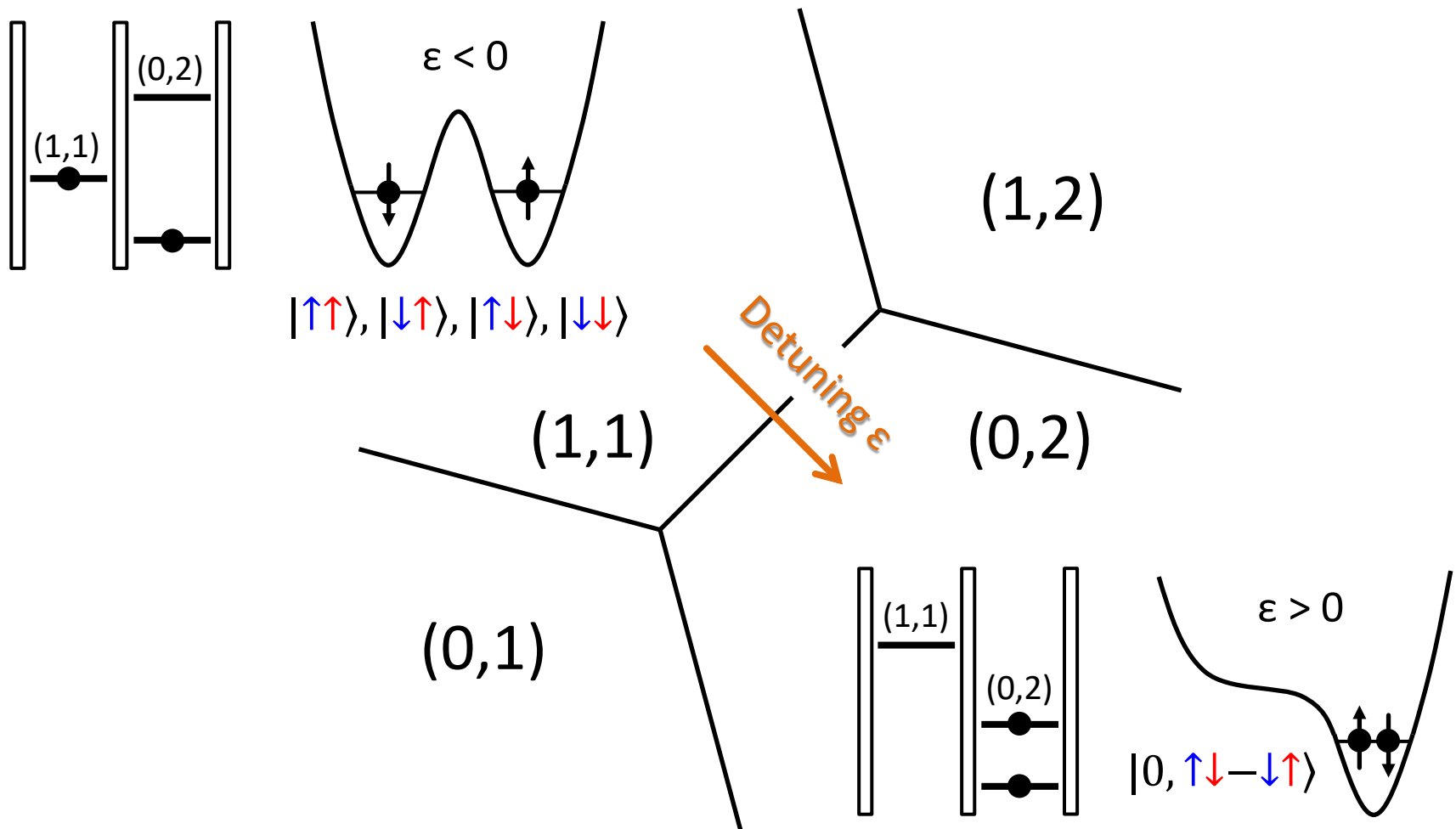
$F^{Bell} = 78\%$



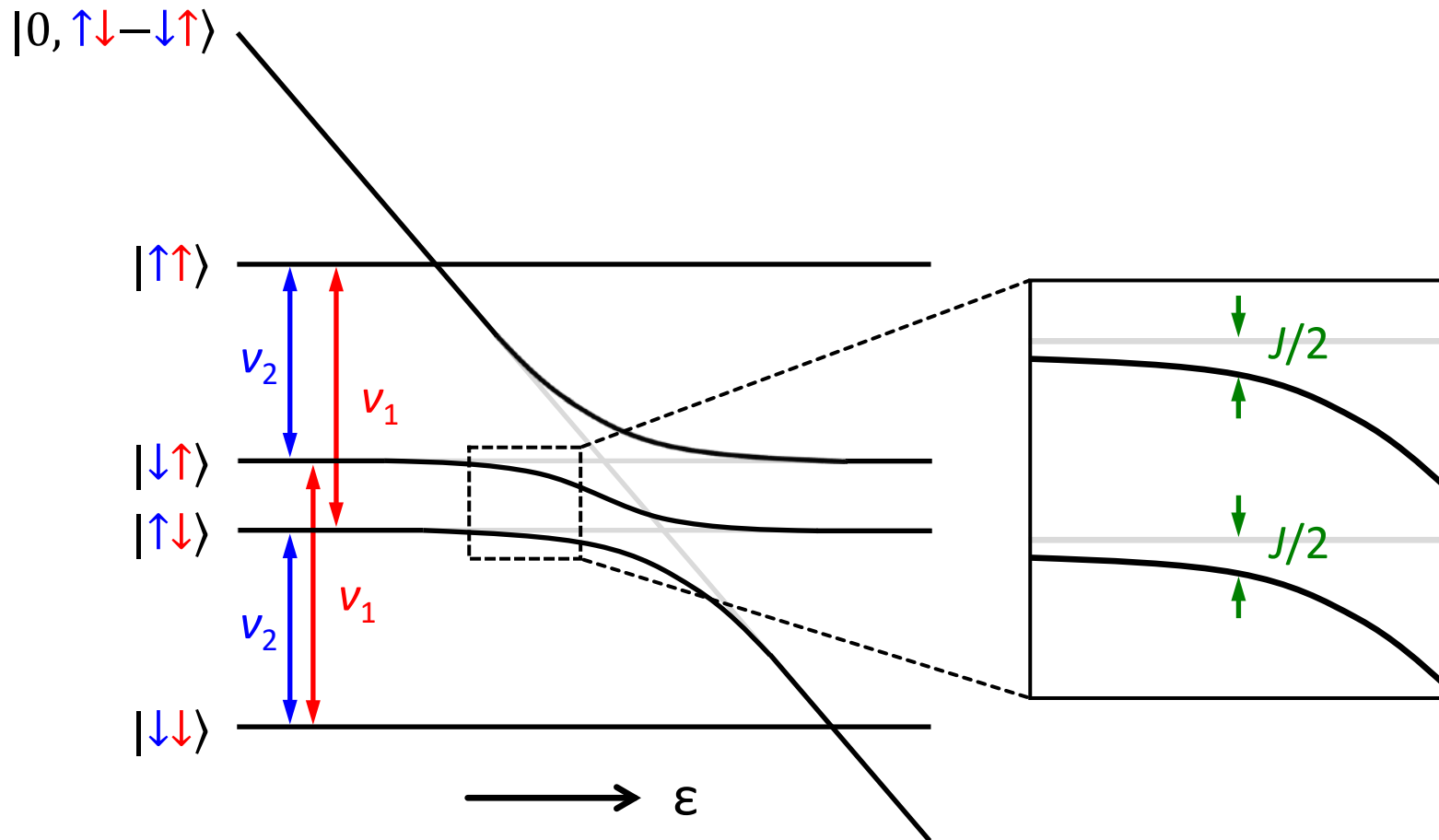
2量子ビットプロセッサ



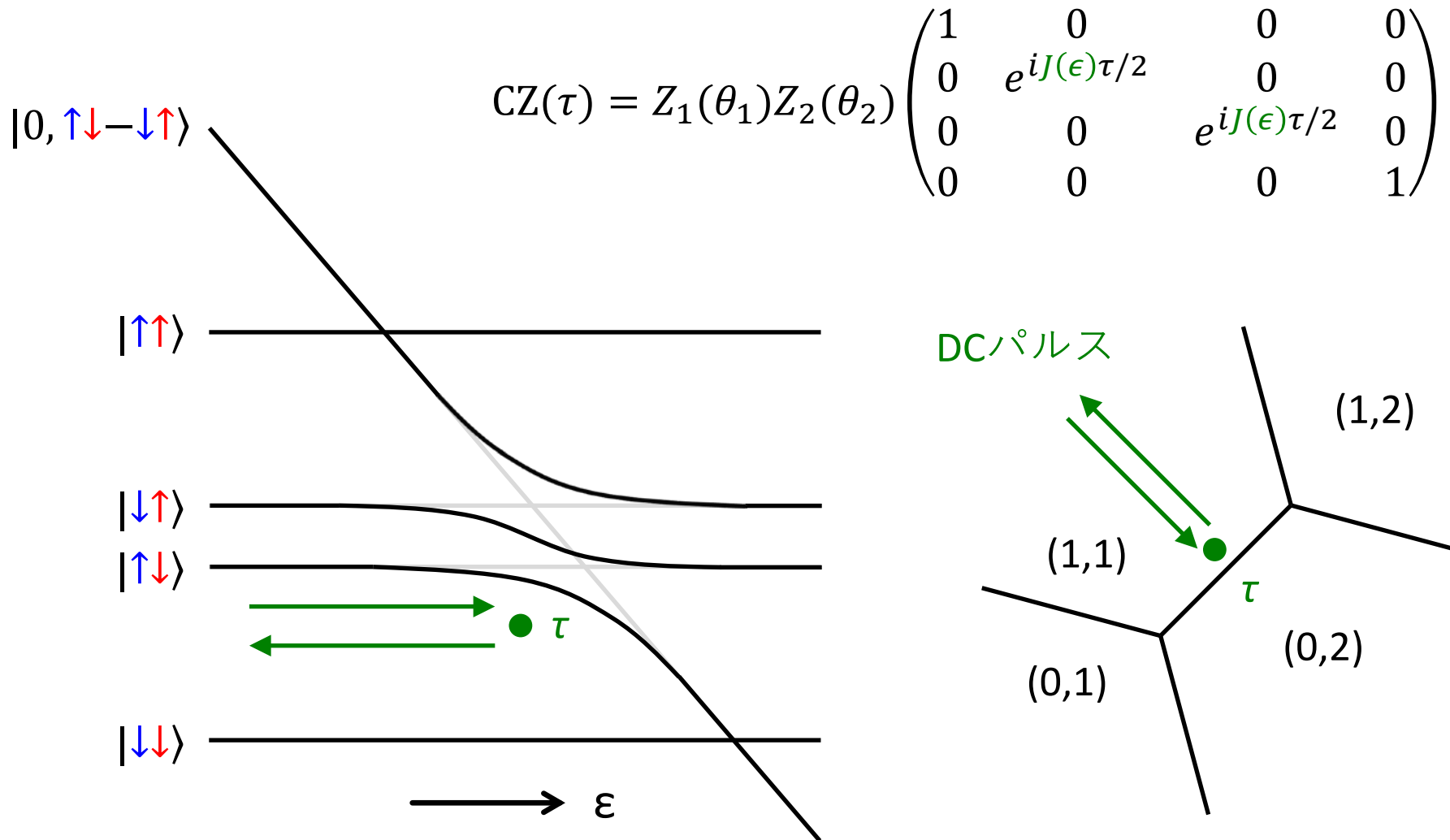
2電子状態



2電子状態



CZゲート



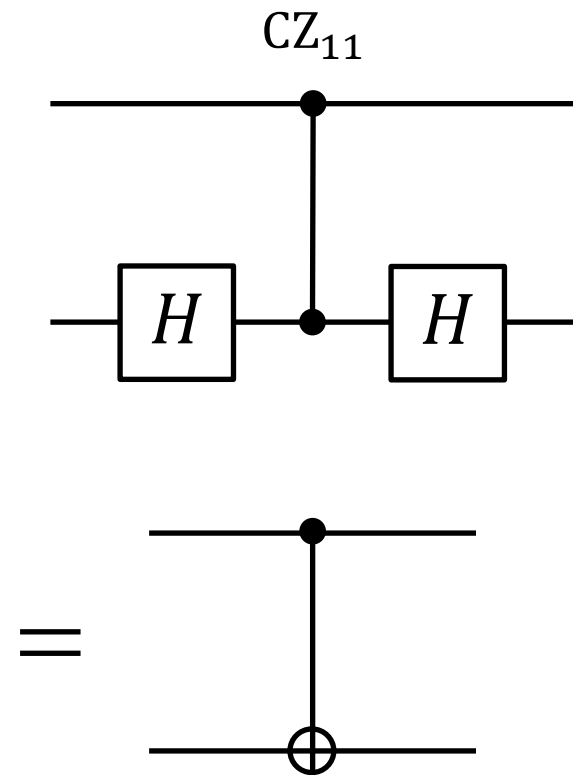
CZゲート

$$CZ_{00} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$CZ_{01} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$CZ_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$CZ_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



$$CNOT = H_2 CZ_{11} H_2$$

CZゲート

$$\text{CZ}_{00} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \approx Z_1\left(\frac{\pi}{2}\right)Z_2\left(\frac{\pi}{2}\right)\text{CZ}_J$$

$$\text{CZ}(\tau = 1/J) = Z_1(\theta_1)Z_2(\theta_2) \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{CZ}_J} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$Z(\theta) = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix} \longrightarrow Z\left(\frac{\pi}{2}\right) = \begin{pmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$Z_1\left(\frac{\pi}{2}\right) \otimes Z_2\left(\frac{\pi}{2}\right) = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

CZゲート

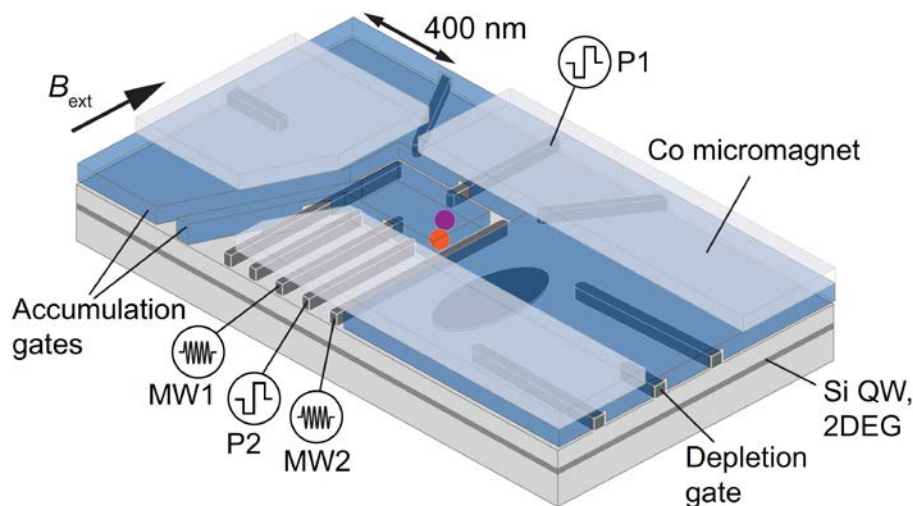
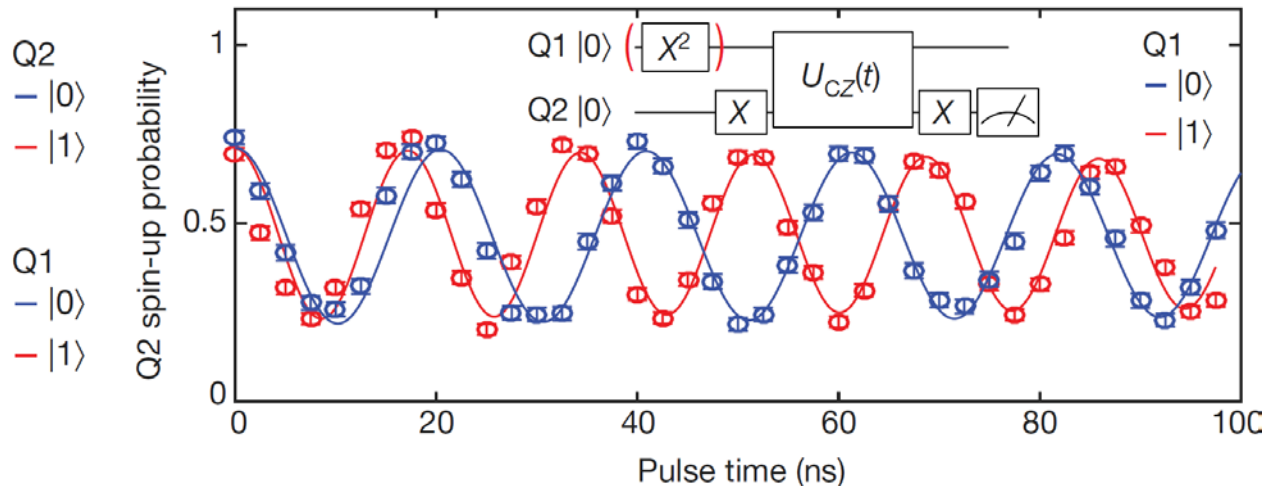
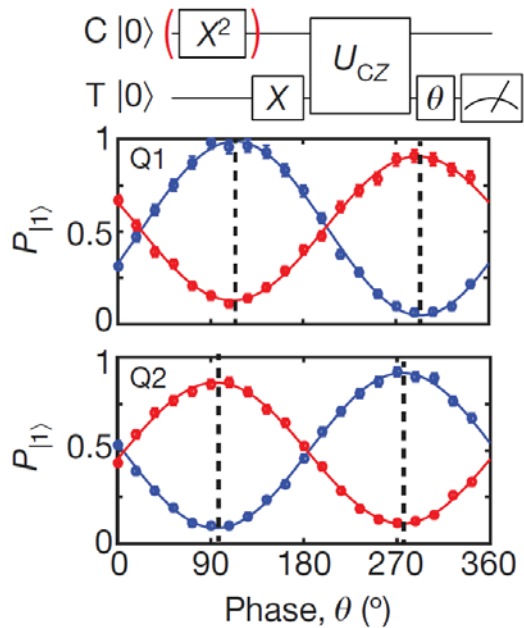
$$\text{CZ}_{00} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \approx Z_1 \left(\frac{\pi}{2} \right) Z_2 \left(\frac{\pi}{2} \right) \text{CZ}_J \quad Z \left(-\frac{\pi}{2} \right) \approx \begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}$$

$$\text{CZ}_{01} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \approx Z_1 \left(-\frac{\pi}{2} \right) Z_2 \left(\frac{\pi}{2} \right) \text{CZ}_J$$

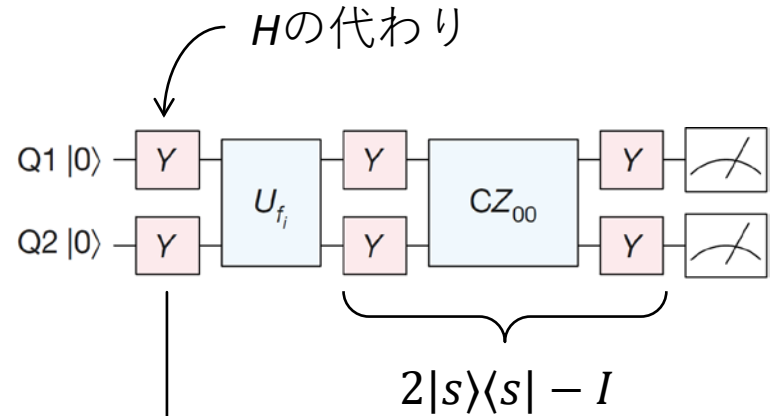
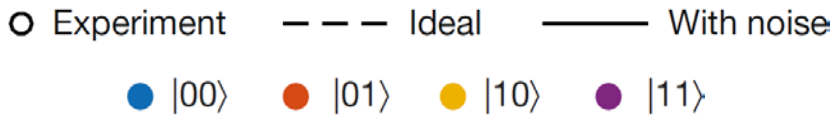
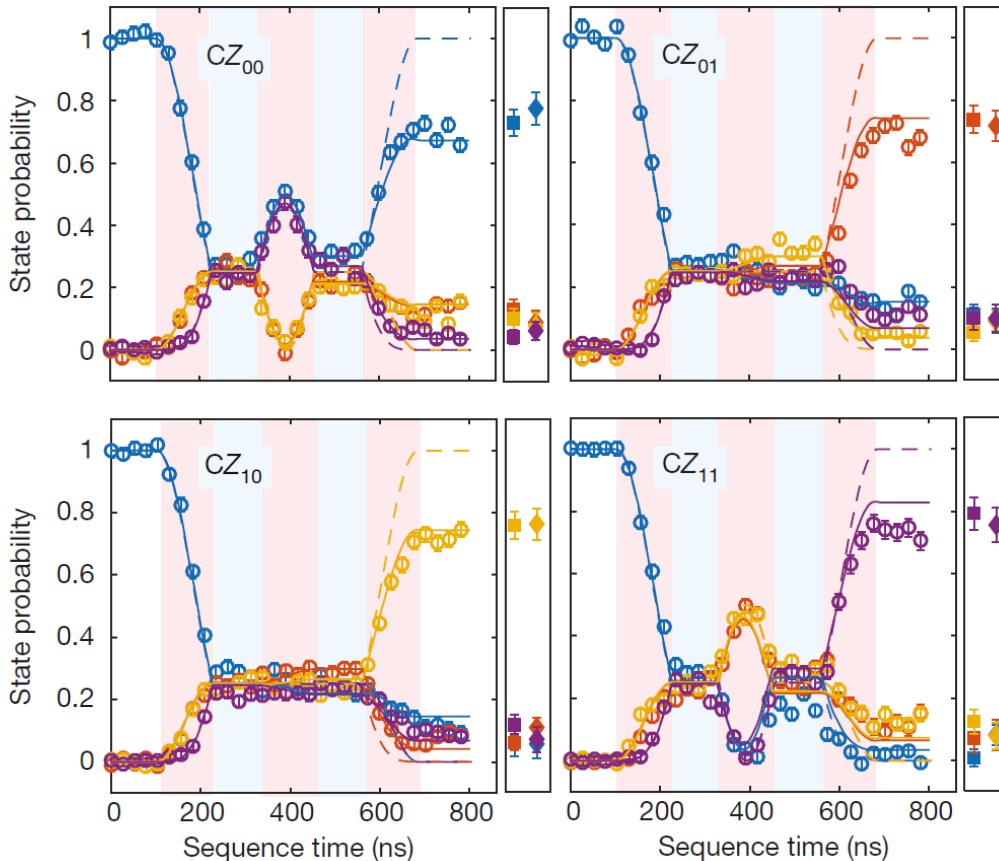
$$\text{CZ}_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \approx Z_1 \left(\frac{\pi}{2} \right) Z_2 \left(-\frac{\pi}{2} \right) \text{CZ}_J$$

$$\text{CZ}_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \approx Z_1 \left(-\frac{\pi}{2} \right) Z_2 \left(-\frac{\pi}{2} \right) \text{CZ}_J$$

2量子ビットプロセッサ



グローバラーのアルゴリズムの実行



$$|s\rangle = \frac{1}{2} \sum_{i,j=0,1} |ij\rangle$$

シリコンスピンの実験の現状

方式	1量子ビット	2量子ビット	多量子ビット化
単一リンドナー	$T_{2e}^{\text{CPMG}} = 559 \text{ ms}$ $T_{2n+}^{\text{CPMG}} = 35.6 \text{ s}$ $F_{2n+} = 99.99\% (*1)$	cf. $F_{ZZ, \sqrt{S}} = 90\% (*4)$ cf. $F^{\text{Bell}(e-n)} = 97\% (*5)$	“フリップフロップ” 量子ビット(*8)
MOS量子ドット	$T_2^{\text{CPMG}} = 28 \text{ ms}$ $F^{\text{RB}} = 99.6\% (*2)$	$F^{\text{Bell}} = 89\%$ $F^{\text{RB}} = 98\% (*6)$	CMOS/DRAM技術 との融合, 高温動作 (*9,10)
Si/SiGe量子ドット	$T_2^{\text{CPMG}} = 3.1 \text{ ms}$ $F^{\text{RB}} > 99.9\% (*3)$	$F^{\text{Bell}} = 89\% (*7)$	スピン-MW光子結合 による回路QED(*11,12)

- *1: Nature Nano. **9**, 986 (2014) Muhonen *et al.*
- *2: Nature Nano. **9**, 981 (2014) Veldhorst *et al.*
- *3: Nature Nano. **13**, 102 (2018) Yoneda *et al.*
- *4: Nature **571**, 371 (2019) He *et al.* (donor QDs)
- *5: Nature Nano. **11**, 242 (2016) Dehollain *et al.*
- *6: Nature **569**, 532 (2019) Huang *et al.*

- *7: Nature **555**, 633 (2018) Watson *et al.*
- *8: Nature Commun. **8**, 450 (2017) Tosi *et al.*
- *9: Nature Commun. **8**, 1766 (2017) Veldhorst *et al.*
- *10: Sci. Adv. **4**, eaar3960 (2018) Li *et al.*
- *11: Nature **555**, 599 (2018) Mi *et al.*
- *12: Science **359**, 1123 (2018) Samkharadze *et al.*

レポート課題 3 (10点)

CZ_{00} の例にならって、以下を示せ。

$$CZ_{01} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \approx Z_1 \left(-\frac{\pi}{2} \right) Z_2 \left(\frac{\pi}{2} \right) CZ_J$$

$$CZ_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \approx Z_1 \left(\frac{\pi}{2} \right) Z_2 \left(-\frac{\pi}{2} \right) CZ_J$$

$$CZ_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \approx Z_1 \left(-\frac{\pi}{2} \right) Z_2 \left(-\frac{\pi}{2} \right) CZ_J$$