量子コンピューティング

阿部 英介
理化学研究所 創発物性科学研究センター

応用物理特別講義A
2020年度春学期後半 金曜4限@~~14-202~~オンライン講義

講義内容

- **量子技術の概要**
  - 物理系の例
  - 量子コンピューティングの難しさ
  - ディビンチェンゾの要請

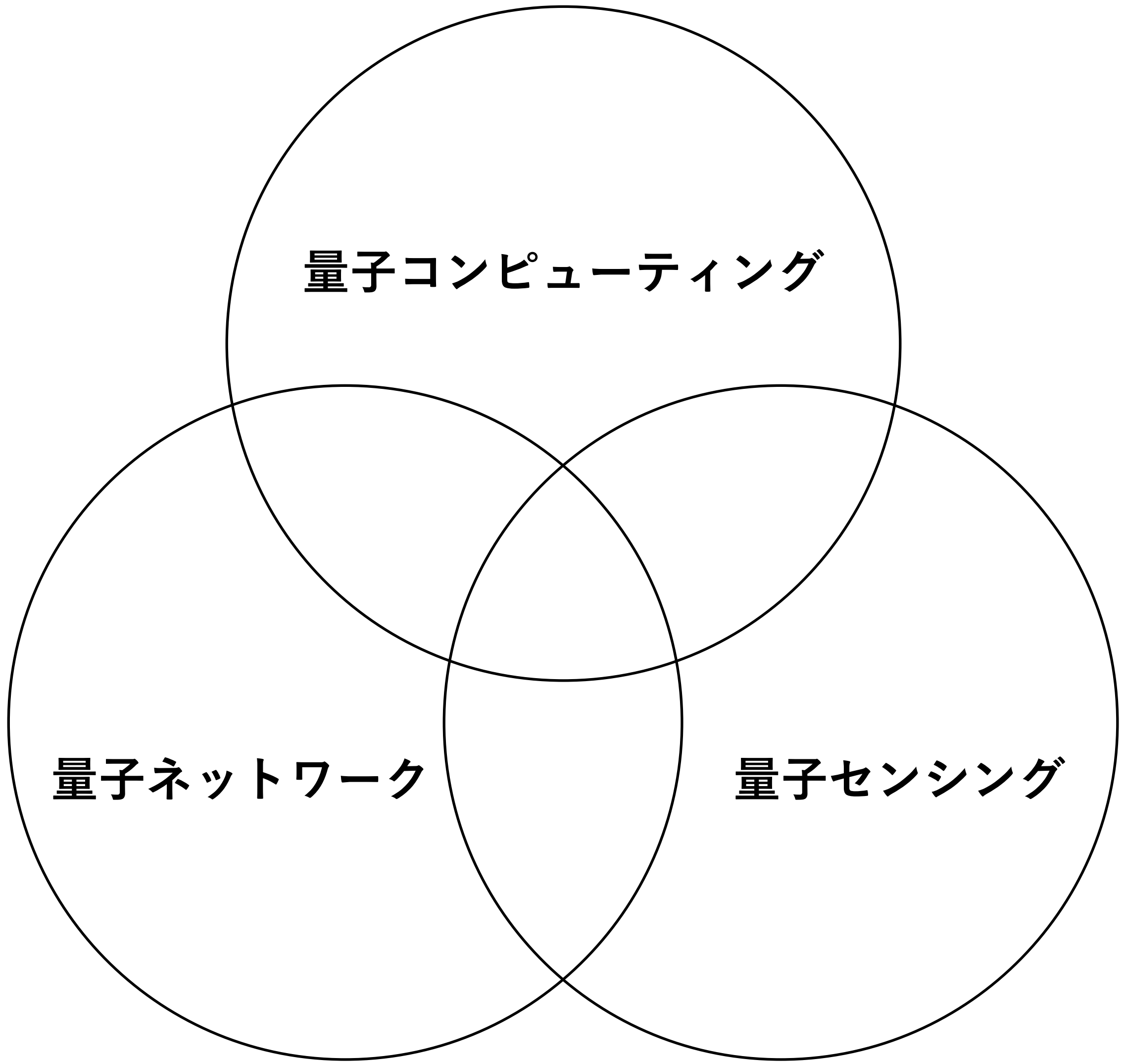
- **スピンと磁気共鳴**
  - 量子ビットとの対応
  - 磁気共鳴によるスピン操作

講義内容

- **量子技術の概要**
  - 物理系の例
  - 量子コンピューティングの難しさ
  - ディビンチェンゾの要請

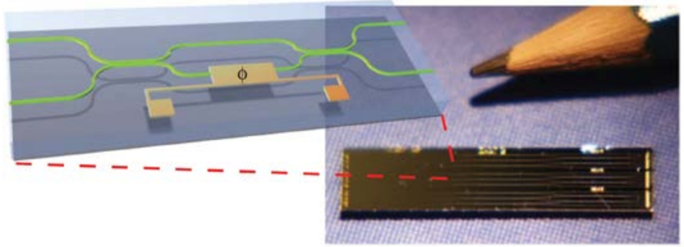
- **スピンと磁気共鳴**
  - 量子ビットとの対応
  - 磁気共鳴によるスピン操作

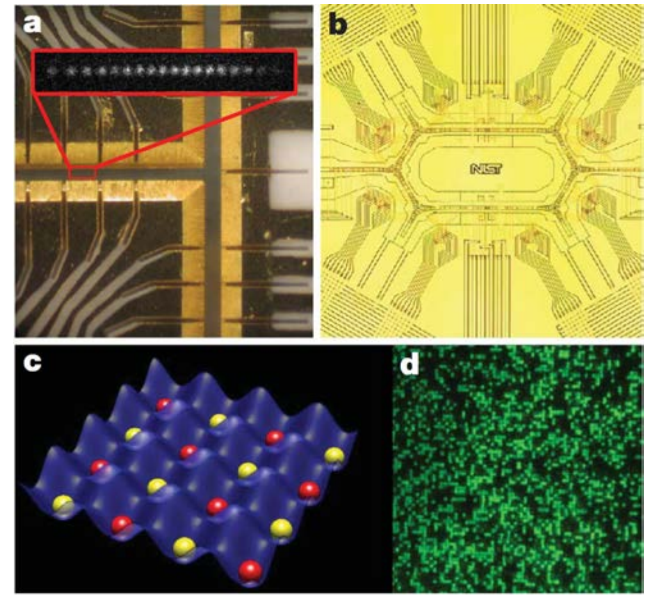
量子技術



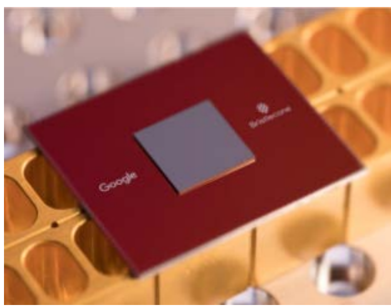
量子技術のプラットフォーム

光回路

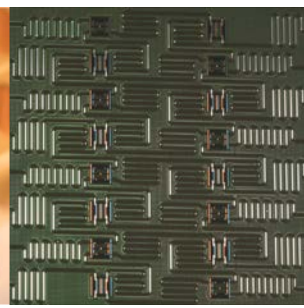




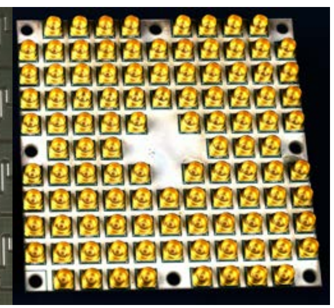
超伝導回路



©Google



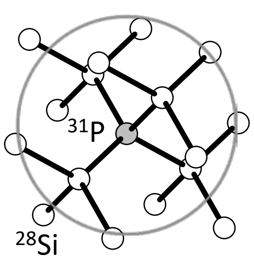
©IBM

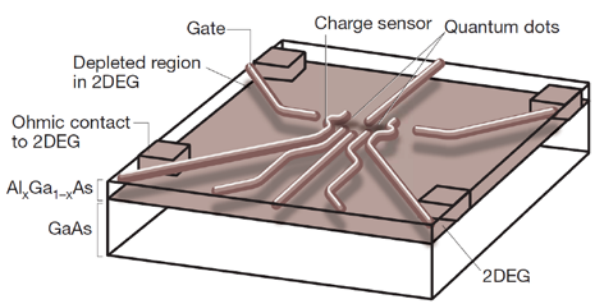


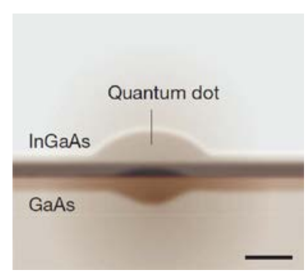
©Intel

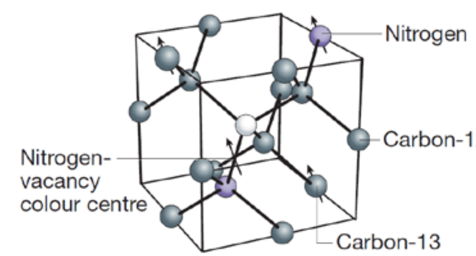
捕捉イオン/冷却原子

半導体スピン







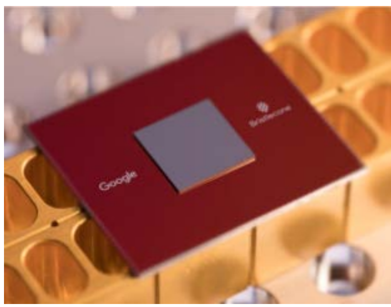


Nature **464**, 45 (2010) Ladd *et al.*

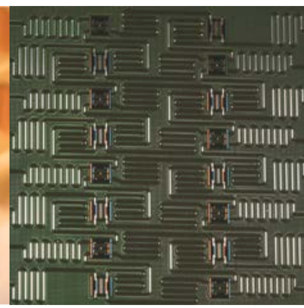
応用物理 **86 (6),** 453 (2017) 阿部 & 伊藤

“固体量子情報デバイスの現状と将来展望 —万能ディジタル量子コンピュータの実現に向けて”

超伝導回路



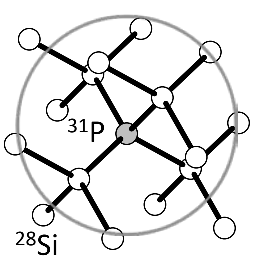
©Google

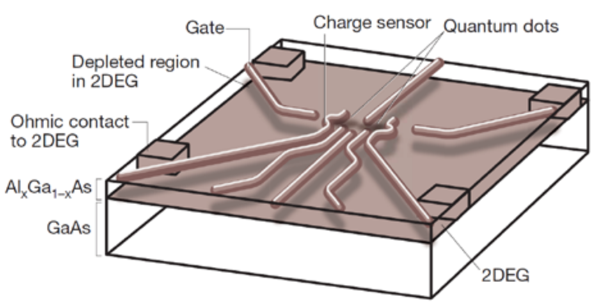


©IBM

©Intel

半導体スピン

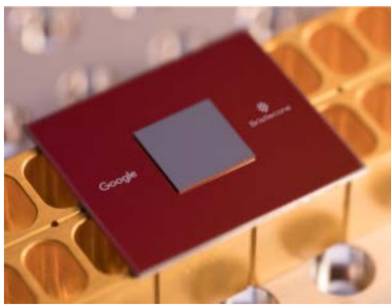




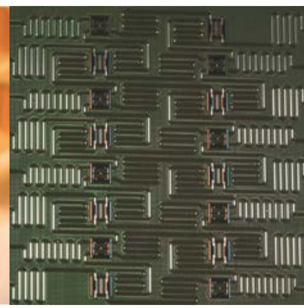
精密工学会誌 **85 (12),** 1048 (2019) 阿部 & 玉手

“超伝導量子ビット技術”

超伝導回路



©Google

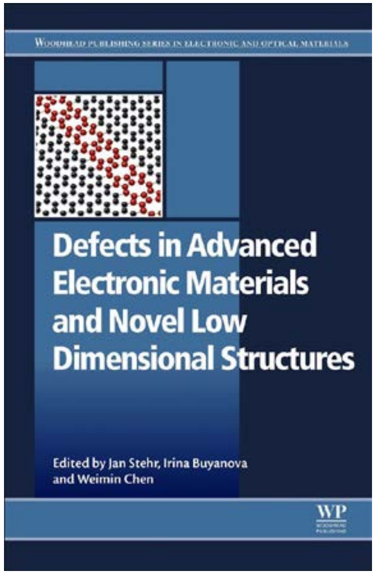


©IBM

©Intel

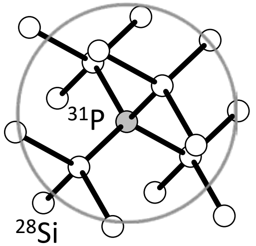
Defects in Advanced Electronic Materials and Novel Low Dimensional Structures, P.241–263, Abe & Itoh

“Defects for quantum information processing in silicon”



(June, 2018)

半導体スピン



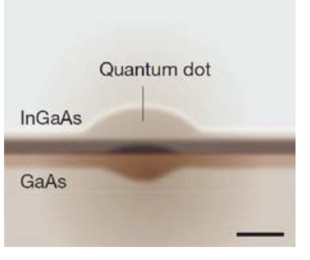
固体物理 **48 (11),** 541 (2013) 山本 & 阿部

“光制御量子ドットスピンを用いた量子情報システムの現状と将来展望”

光技術コンタクト **51 (5),** 10 (2013) 阿部

“量子中継と量子ドットスピン–光子間量子もつれ”

半導体スピン



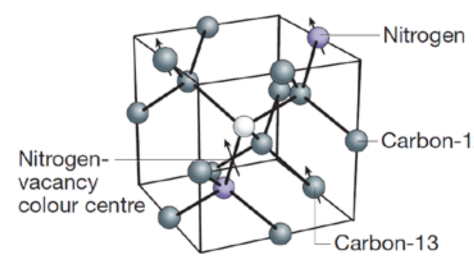
NEW DIAMOND **33 (2),** 3 (2017) 阿部 & 伊藤

“スピントロニクス研究の原点からダイヤモンドでのトレンド, 今後の展開まで”

J. Appl. Phys. **123,** 161191 (2018) Abe & Sasaki

“Tutorial: Magnetic resonance with nitrogen-vacancy centers in diamond —microwave engineering, materials science, and magnetometry”

半導体スピン



講義内容

- **量子技術の概要**
  - 物理系の例
  - 量子コンピューティングの難しさ
  - ディビンチェンゾの要請

- **スピンと磁気共鳴**
  - 量子ビットとの対応
  - 磁気共鳴によるスピン操作

量子ビット

定義: 計算基底のベクトル表示

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\langle 0|0\rangle = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$$

$$\langle 1|0\rangle = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$$

公理: 許される状態はヒルベルト空間内

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$|\alpha|^2 + |\beta|^2 = 1 \qquad \alpha, \beta \in \boldsymbol{C}$$

$$\langle \psi|\psi\rangle = \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = |\alpha|^2 + |\beta|^2 = 1$$

量子ビット

重ね合わせ状態

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$\Downarrow$$

$$|\psi\rangle = \underline{e^{i\gamma}}\left(\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle\right)$$

測定に影響しない

$$0 \leq \theta \leq \pi$$

$$0 \leq \gamma, \phi < 2\pi$$

$$\Downarrow$$

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

$$0 \leq \theta \leq \pi$$

$$0 \leq \phi < 2\pi$$

ブロッホ球

量子ビットの状態を可視化する

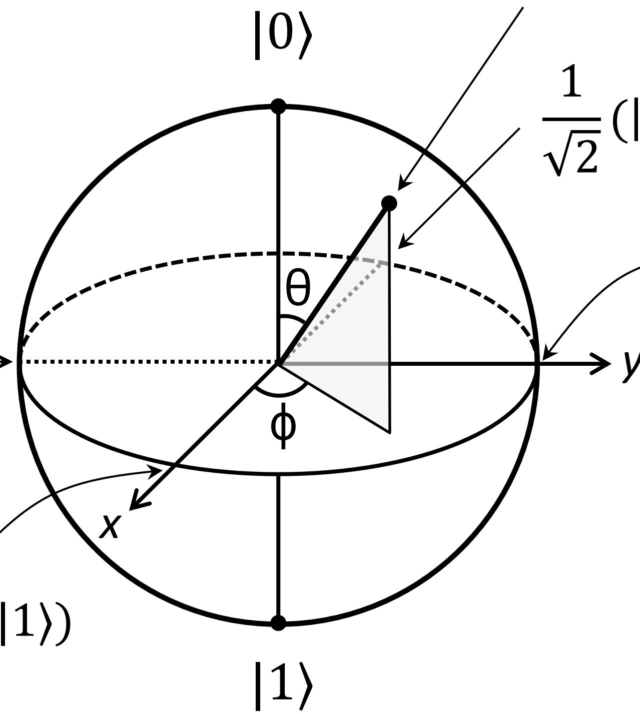
$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

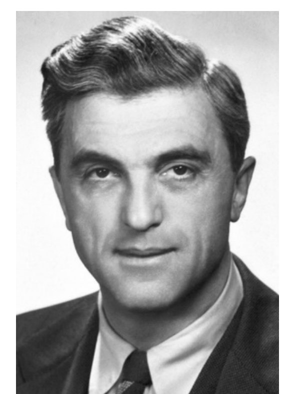
$$\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$





Felix Bloch
(1905–1983)

1量子ビットゲート

公理: 量子状態の時間発展はユニタリ

$$|\psi'\rangle = U|\psi\rangle \qquad UU^\dagger = I$$

恒等変換(何もしない)

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

アダマールゲート

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

パウリ行列

$$X = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Y = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad Z = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

位相(*S*)ゲート

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

***T*ゲート**

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

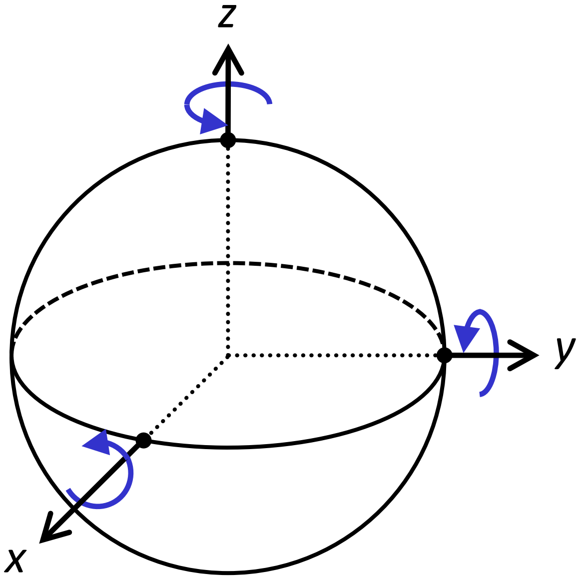
回転ゲート

***x*, *y*, *z*軸周りの回転**

$$R_x(\varphi) = e^{-i\varphi X/2} = \begin{pmatrix} \cos\frac{\varphi}{2} & -i\sin\frac{\varphi}{2} \\ -i\sin\frac{\varphi}{2} & \cos\frac{\varphi}{2} \end{pmatrix}$$

$$R_y(\varphi) = e^{-i\varphi Y/2} = \begin{pmatrix} \cos\frac{\varphi}{2} & -\sin\frac{\varphi}{2} \\ \sin\frac{\varphi}{2} & \cos\frac{\varphi}{2} \end{pmatrix}$$

$$R_z(\varphi) = e^{-i\varphi Z/2} = \begin{pmatrix} e^{-i\varphi/2} & 0 \\ 0 & e^{i\varphi/2} \end{pmatrix}$$



指数演算子

シュレディンガー方程式の解

$$|\psi(t+\Delta t)\rangle = \exp\left(-i\frac{H\Delta t}{\hbar}\right)|\psi(t)\rangle$$

指数演算子

$$e^{iAx} \equiv \sum_{n=0}^{\infty}\frac{(iAx)^n}{n!}$$

$$= \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}(Ax)^{2k} + i\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}(Ax)^{2k+1}$$

$$A^2 = I$$

$$= \cos x \cdot I + i \sin x \cdot A$$

アダマールゲート

$$|a\rangle \;—\boxed{H}—\; \frac{1}{\sqrt{2}}\sum_{b=0,1}(-1)^{a\cdot b}|b\rangle = \frac{|0\rangle + (-1)^a|1\rangle}{\sqrt{2}}$$

$$\begin{cases} H|0\rangle = \dfrac{|0\rangle + |1\rangle}{\sqrt{2}} \\[2ex] H|1\rangle = \dfrac{|0\rangle - |1\rangle}{\sqrt{2}} \end{cases} \Longleftrightarrow \begin{cases} H\begin{pmatrix}1\\0\end{pmatrix} = \dfrac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \\[2ex] H\begin{pmatrix}0\\1\end{pmatrix} = \dfrac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \end{cases} \Longleftrightarrow H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$$

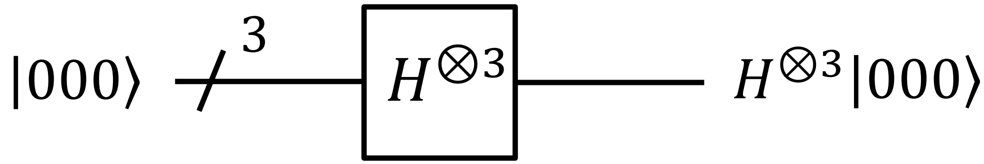
$$HH = \frac{1}{2}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$\Longleftrightarrow \; H^\dagger = H$ (自己共役)

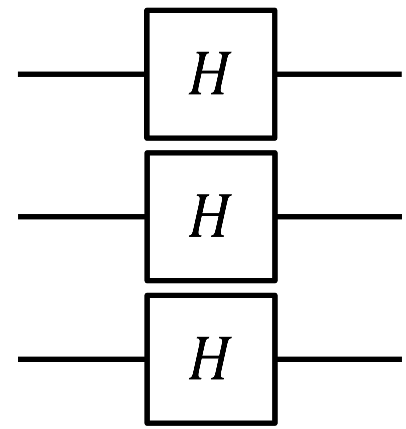
Jacques Hadamard
(1865–1965)

(from Wikipedia)

n量子ビットの重ね合わせ



公理: *n*量子ビット系 → テンソル積



$$H^{\otimes 3}|000\rangle$$

$$= \frac{1}{\sqrt{2^3}}(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

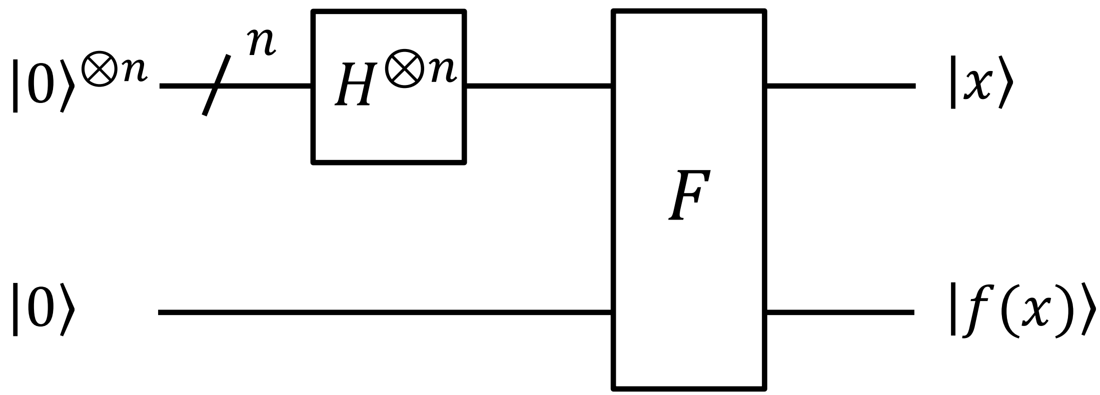
$$= \frac{1}{\sqrt{2^3}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle)$$

$$|0\rangle + |1\rangle + |2\rangle + |3\rangle + |4\rangle + |5\rangle + |6\rangle + |7\rangle$$

10進数表示

$$= \frac{1}{\sqrt{2^3}} \sum_{a,b,c=0,1} |abc\rangle = \frac{1}{\sqrt{2^3}} \sum_{x=0}^{2^3-1} |x\rangle$$

量子並列性



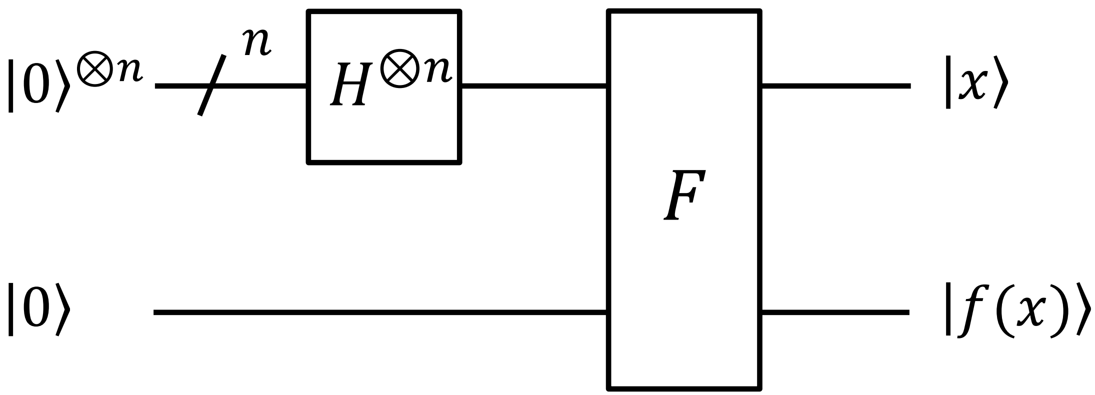
$f(x)$: 2値関数(ビットデータ列)

$$F|x\rangle|a\rangle = |x\rangle|a \oplus f(x)\rangle$$

$$FF|x\rangle|a\rangle = |x\rangle|a \oplus f(x) \oplus f(x)\rangle = |x\rangle|a\rangle$$

$$|0\rangle^{\otimes n}|0\rangle \xrightarrow{(H^{\otimes n}) \otimes I} \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle|0\rangle \xrightarrow{F} \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle|f(x)\rangle$$

量子並列性



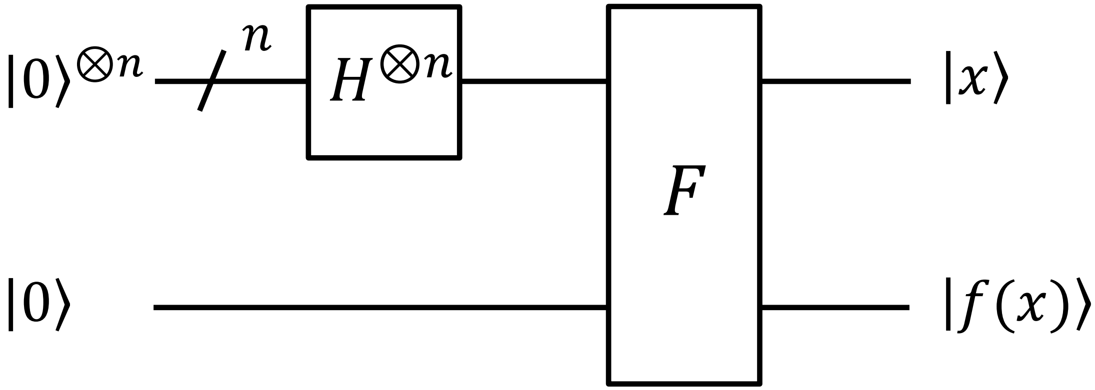
$$\frac{1}{\sqrt{2^2}}\sum_{x=0}^{2^2-1}|x\rangle|f(x)\rangle = \frac{1}{2}\left(|0\rangle|f(0)\rangle + |1\rangle|f(1)\rangle + |2\rangle|f(2)\rangle + |3\rangle|f(3)\rangle\right)$$

$f(x)$の情報を全て含んだ量子もつれ状態

⇓

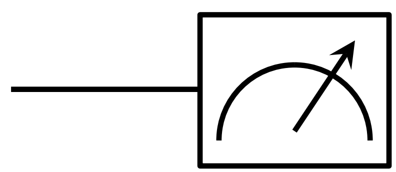
計算・情報処理の高速化に繋がる?

量子並列性と測定



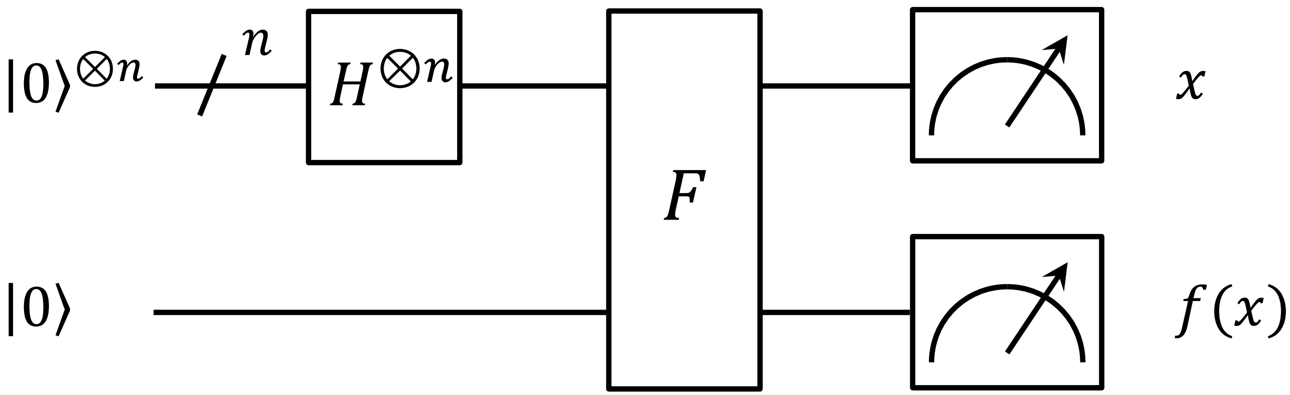
$$\frac{1}{\sqrt{2^2}}\sum_{x=0}^{2^2-1}|x\rangle|f(x)\rangle = \frac{1}{2}\left(|0\rangle|f(0)\rangle + |1\rangle|f(1)\rangle + |2\rangle|f(2)\rangle + |3\rangle|f(3)\rangle\right)$$

$\alpha|0\rangle + \beta|1\rangle$



$$\begin{cases} \text{確率}|\alpha|^2\text{で“0”を出力} \\ \text{確率}|\beta|^2\text{で“1”を出力} \end{cases}$$

量子並列性と測定



$$\frac{1}{\sqrt{2^2}}\sum_{x=0}^{2^2-1}|x\rangle|f(x)\rangle = \frac{1}{2}\left(\cancel{|0\rangle|f(0)\rangle} + |1\rangle|f(1)\rangle + \cancel{|2\rangle|f(2)\rangle} + \cancel{|3\rangle|f(3)\rangle}\right)$$

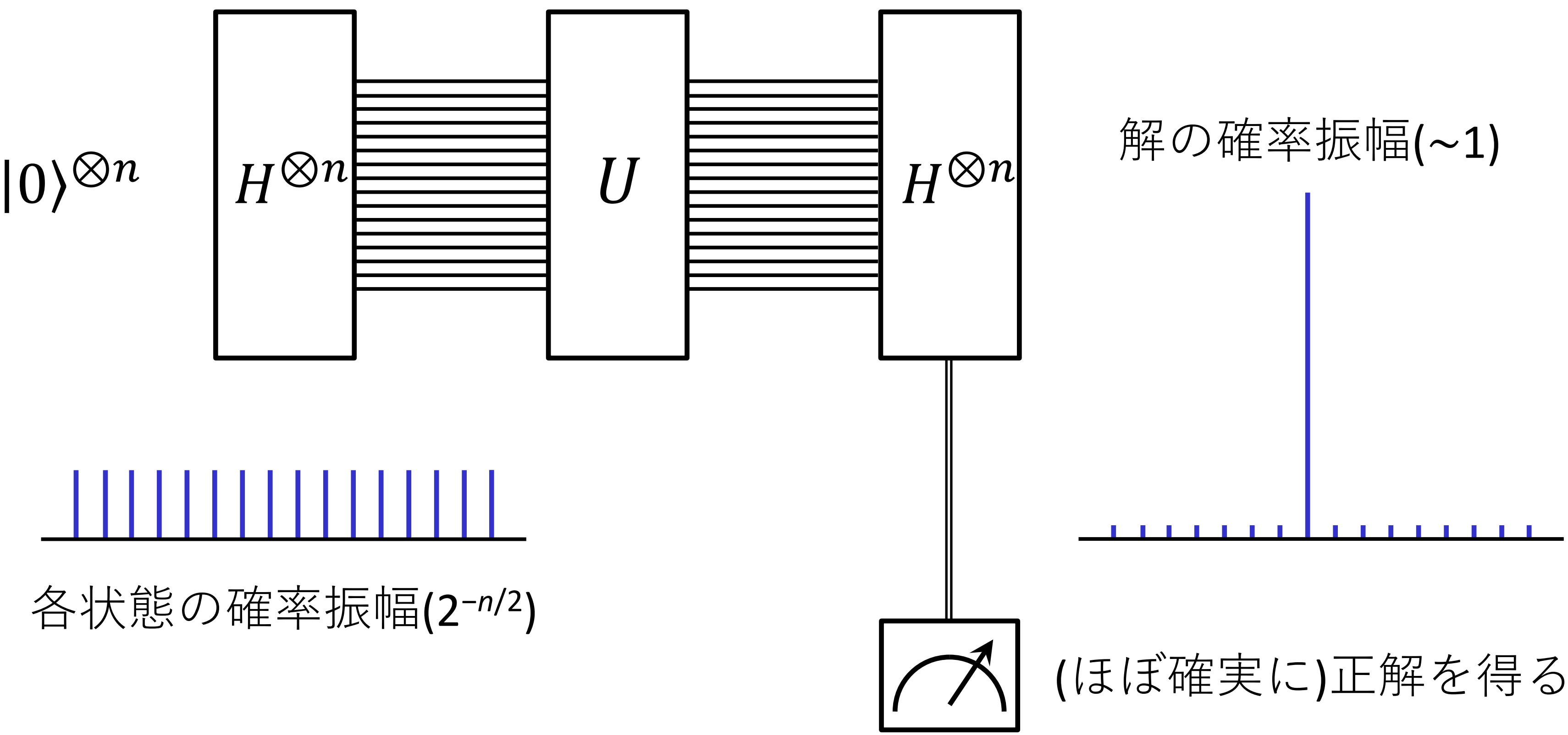
確率1/4でどれか1つの組の結果を知る

⇩

量子並列性にナイーブに期待される計算・情報処理の高速化は、
測定による状態の収縮によりキャンセルされてしまう

量子アルゴリズム

重ね合わせ状態(**量子並列性**)から始めて、解の状態の確率振幅が大きくなるよう(**量子干渉**)にユニタリ変換し、最後に**測定**



量子コンピューティングの難しさ

実験

- 量子情報を**位相**に書き込み、**量子干渉**により解の状態を抜き出す
  → 計算中に**量子コヒーレンス**を保つことが必要

- 量子状態は複製できない(**複製禁止定理**)
  → **量子誤り訂正符号 & 誤り耐性量子計算**

(フォールトトレラント, fault tolerant)

複製禁止定理

任意の状態$|\psi\rangle$に対して$U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$となる
ユニタリ演算子Uは存在しない

A single quantum cannot be cloned

W. K. Wootters*

Center for Theoretical Physics, The University of Texas at Austin, Austin, Texas 78712, USA

W. H. Zurek

Theoretical Astrophysics 130-33, California Institute of Technology, Pasadena, California 91125, USA

If a photon of definite polarization encounters an excited atom, there is typically some nonvanishing probability that the atom will emit a second photon by stimulated emission. Such a photon is guaranteed to have the same polarization as the original photon. But is it possible by this or any other process to amplify a quantum state, that is, to produce several copies of a quantum system (the polarized photon in the present case) each having the same state as the original? If it were, the amplifying process could be used to ascertain the exact state of a quantum system: in the case of a photon, one could determine its polarization by first producing a beam of identically polarized copies and then measuring the Stokes parameters[1]. We show here that the linearity of quantum mechanics forbids such replication and that this conclusion holds for all quantum systems.

Note that if photons could be cloned, a plausible argument could be made for the possibility of faster-than-light communication[2]. It is well known that for certain non-separably correlated Einstein–Podolsky–Rosen pairs of photons, once an observer has made a polarization measurement (say, vertical versus horizontal) on one member of the pair, the other one, which may be far away, can be for all purposes of prediction regarded as having the same polarization[3]. If this second photon could be replicated and its precise polarization measured as above, it would be possible to ascertain whether, for example, the first photon had been subjected to a measurement of linear or circular polarization. In this way the first observer would be able to transmit information faster than light by encoding his message into his choice of measurement. The actual impossibility of cloning photons, shown below, thus prohibits superluminal communication by this scheme. That such a scheme must fail for some reason despite the well-established existence of long-range quantum correlations[6–8], is a general consequence of quantum mechanics[9].

A perfect amplifying device would have the following effect on an incoming photon with polarization state $|s\rangle$:

$$|A_0\rangle|s\rangle \rightarrow |A_s\rangle|ss\rangle \tag{1}$$

Here $|A_0\rangle$ is the 'ready' state of the apparatus, and $|A_s\rangle$ is its final state, which may or may not depend on the polarization of the original photon. The symbol $|ss\rangle$ refers to the state of the radiation field in which there are two photons each having the polarization $|s\rangle$. Let us suppose that such an amplification can in fact be accomplished for the vertical polarization $|\updownarrow\rangle$ and for the horizontal polarization $|\leftrightarrow\rangle$. That is,

$$|A_0\rangle|\updownarrow\rangle \rightarrow |A_{\mathrm{vert}}\rangle|\updownarrow\updownarrow\rangle \tag{2}$$

and

$$|A_0\rangle|\leftrightarrow\rangle \rightarrow |A_{\mathrm{hor}}\rangle|\leftrightarrow\leftrightarrow\rangle \tag{3}$$

According to quantum mechanics this transformation should be representable by a linear (in fact unitary) operator. It therefore follows that if the incoming photon has the polarization given by the linear combination $\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle$—for example, it could be linearly polarized in a direction 45° from the vertical, so that $\alpha=\beta=2^{-1/2}$—the result of its interaction with the apparatus will be the superposition of equations (2) and (3):

$$|A_0\rangle(\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle) \rightarrow \alpha|A_{\mathrm{vert}}\rangle|\updownarrow\updownarrow\rangle+\beta|A_{\mathrm{hor}}\rangle|\leftrightarrow\leftrightarrow\rangle \tag{4}$$

If the apparatus states $|A_{\mathrm{vert}}\rangle$ and $|A_{\mathrm{hor}}\rangle$ are not identical, then the two photons emerging from the apparatus are in a mixed state of polarization. If these apparatus states are identical, then the two photons are in the pure state

$$\alpha|\updownarrow\updownarrow\rangle+\beta|\leftrightarrow\leftrightarrow\rangle \tag{5}$$

In neither of these cases is the final state the same as the state with two photons both having the polarization $\alpha|\updownarrow\rangle+\beta|\leftrightarrow\rangle$. That state, the one which would be required if the apparatus were to be a perfect amplifier, can be written as

$$2^{-1/2}(\alpha a^{+}_{\mathrm{vert}}+\beta a^{+}_{\mathrm{hor}})^2|0\rangle = \alpha^2|\updownarrow\updownarrow\rangle+2^{1/2}\alpha\beta|\updownarrow\leftrightarrow\rangle+\beta^2|\leftrightarrow\leftrightarrow\rangle$$

which is a pure state different from the one obtained above by superposition [equation (5)].

Thus no apparatus exists which will amplify an arbitrary polarization. The above argument does not rule out the possibility of a device which can amplify two special polarizations, such as vertical and horizontal. Indeed, any measuring device which distinguishes between these two polarizations, a Nicol prism for example, could be used to trigger such an amplification.

The same argument can be applied to any other kind of quantum system. As in the case of photons, linearity does not forbid the amplification of any given state by a device designed especially for that state, but it does rule out the existence of a device capable of amplifying an arbitrary state.

* Present address: Department of Physics and Astronomy, Williams College, Williamstown, Massachusetts 01267, USA.

Milonni (unpublished work) has shown that the process of stimulated emission does not lead to quantum amplification, because if there is stimulated emission there must also be—with equal probability in the case of one incoming photon—spontaneous emission, and the polarization of a spontaneously emitted photon is entirely independent of the polarization of the original.

It is conceivable that a more sophisticated amplifying apparatus could get around Milonni's argument. We have therefore presented the above simple argument, based on the linearity of quantum mechanics, to show that no apparatus, however complicated, can amplify an arbitrary polarization.

We stress that the question of replicating individual photons is of practical interest. It is obviously closely related to the quantum limits on the noise in amplifiers[10,11]. Moreover, an experiment devised to establish the extent to which polarization of single photons can be replicated through the process of stimulated emission is under way (A. Gozzini, personal communication; and see ref. 12). The quantum mechanical prediction is quite definite; for each perfect clone there is also one randomly polarized, spontaneously emitted, photon.

We thank Alain Aspect, Carl Caves, Ron Dickman, Ted Jacobson, Peter Milonni, Marlan Scully, Pierre Meystre, Don Page and John Archibald Wheeler for enjoyable and stimulating discussions.

This work was supported in part by the NSF (PHY 78-26592 and AST 79-22012-A1). W.H.Z. acknowledges a Richard Chace Tolman Fellowship.

Received 11 August; accepted 7 September 1982.

1. Born, M. & Wolf, E. *Principles of Optics* 4th edn (Pergamon, New York, 1970).
2. Herbert, N. *Found. Phys.* (in the press).
3. Einstein, A., Podolsky, B. & Rosen, N. *Phys. Rev.* **47,** 777–780 (1935).
4. Bohm, D. *Quantum Theory*, 611–623 (Prentice-Hall, Englewood Cliffs, 1951).
5. Kocher, C. A. & Commins, E. D. *Phys. Rev. Lett.* **18,** 575–578 (1967).
6. Freedman, S. J. & Clauser, J. R. *Phys. Rev. Lett.* **28,** 938–941 (1972).
7. Fry, E. S. & Thompson, R. C. *Phys. Rev. Lett.* **37,** 465–468 (1976).
8. Aspect, A., Grangier, P. & Roger, G. *Phys. Rev. Lett.* **47,** 460–463 (1981).
9. Bussey, P. J. *Phys. Lett.* **90A,** 9–12 (1982).
10. Haus, H. A. & Mullen, J. A. *Phys. Rev.* **128,** 2407–2410 (1962).
11. Caves, C. M. *Phys. Rev.* **D15,** (in the press).
12. Gozzini, A. *Proc. Symp. on Wave-Particle Dualism* (eds Diner, S., Fargue, D., Lochak, G. & Selleri, F) (Reidel, Dordrecht, in the press).

Nature **299**, 802 (1982) Wootters & Zurek

複製禁止定理

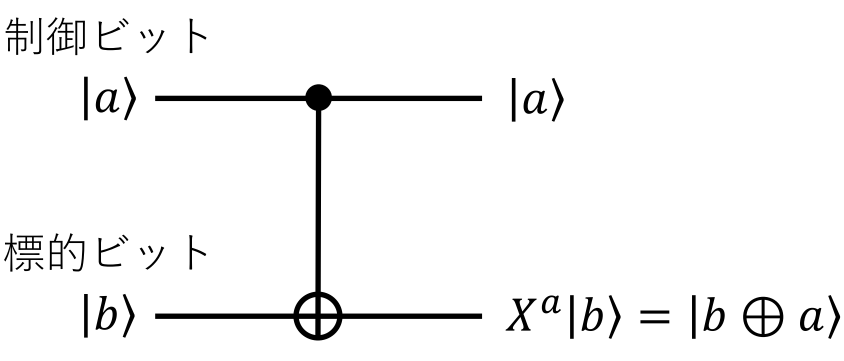
任意の状態$|\psi\rangle$に対して$U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$となる
ユニタリ演算子Uは存在しない

証明1: 存在するならば…

$$U|0\rangle|0\rangle = |0\rangle|0\rangle$$

$$U|1\rangle|0\rangle = |1\rangle|1\rangle$$

← 例えばCNOT



$$\mathrm{CNOT}|00\rangle = |00\rangle$$
$$\mathrm{CNOT}|01\rangle = |01\rangle$$
$$\mathrm{CNOT}|10\rangle = |11\rangle$$
$$\mathrm{CNOT}|11\rangle = |10\rangle$$

$$\Leftrightarrow \quad \mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

複製禁止定理

任意の状態$|\psi\rangle$に対して$U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$となる
ユニタリ演算子Uは存在しない

証明1: 存在するならば…

$$U|0\rangle|0\rangle = |0\rangle|0\rangle$$

$$U|1\rangle|0\rangle = |1\rangle|1\rangle$$

$$\Downarrow$$

$$\begin{aligned}
U(\alpha|0\rangle + \beta|1\rangle)|0\rangle &= \alpha U|0\rangle|0\rangle + \beta U|1\rangle|0\rangle \\
&= \alpha|0\rangle|0\rangle + \beta|1\rangle|1\rangle \\
&\neq (\alpha|0\rangle + \beta|1\rangle)(\alpha|0\rangle + \beta|1\rangle)
\end{aligned}$$

複製禁止定理

任意の状態$|\psi\rangle$に対して$U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$となる
ユニタリ演算子Uは存在しない

証明2: 存在するならば…

$$U|\varphi\rangle|0\rangle = |\varphi\rangle|\varphi\rangle$$

$$U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$$

$$\Downarrow$$

$$\langle\varphi|\langle 0|U^\dagger U|\psi\rangle|0\rangle = \langle\varphi|\psi\rangle\langle\varphi|\psi\rangle$$

$$\Downarrow$$

$$\langle\varphi|\psi\rangle = (\langle\varphi|\psi\rangle)^2$$

自分自身か直交状態

$$\therefore \langle\varphi|\psi\rangle = 0{,}1$$

講義内容

- **量子技術の概要**
  - 物理系の例
  - 量子コンピューティングの難しさ
  - ディビンチェンゾの要請
- **スピンと磁気共鳴**
  - 量子ビットとの対応
  - 磁気共鳴によるスピン操作

ディビンチェンゾの要請

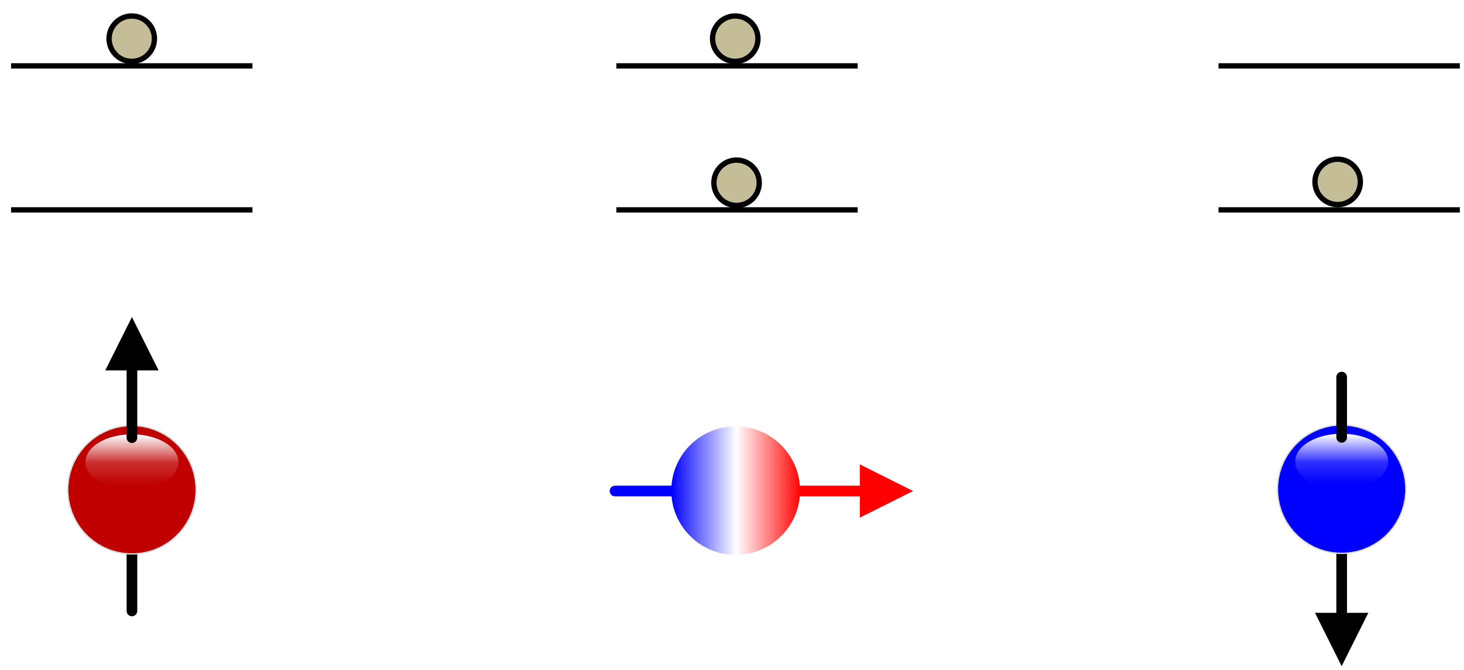
量子コンピューティングを実現する物理系に求められる条件

1. **スケーラブルな量子ビット列**
2. **初期化**
3. **長いコヒーレンス時間(T_2)**
4. **ユニバーサル量子ゲート**
5. **射影測定**

D. DiVincenzo

Fortschr. Phys. **48**, 771 (2000) DiVincenzo

スピン1/2 = 量子ビット

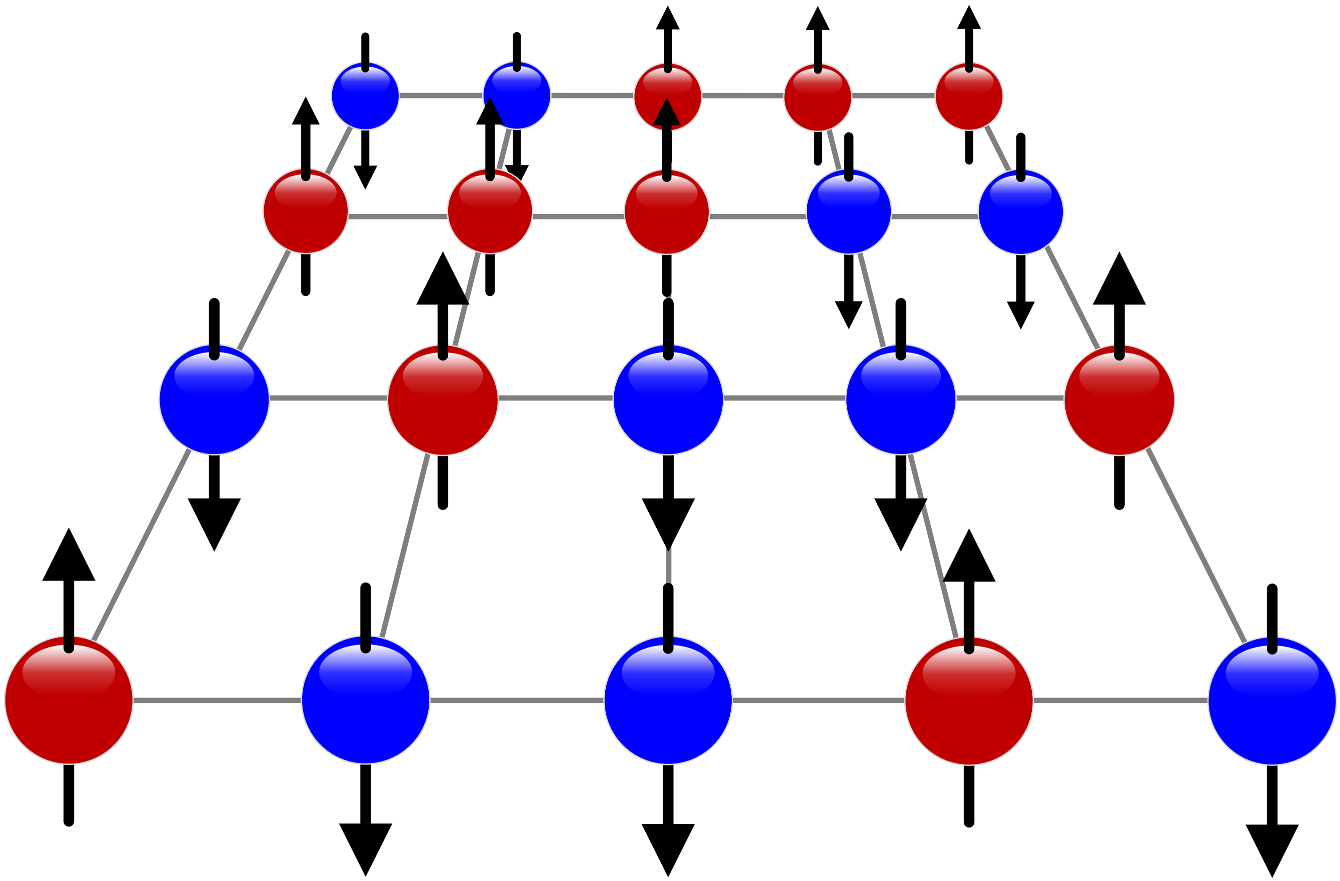


$|0\rangle \equiv |\uparrow\rangle$ $\quad |\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$ $\quad |1\rangle \equiv |\downarrow\rangle$

重ね合わせ状態

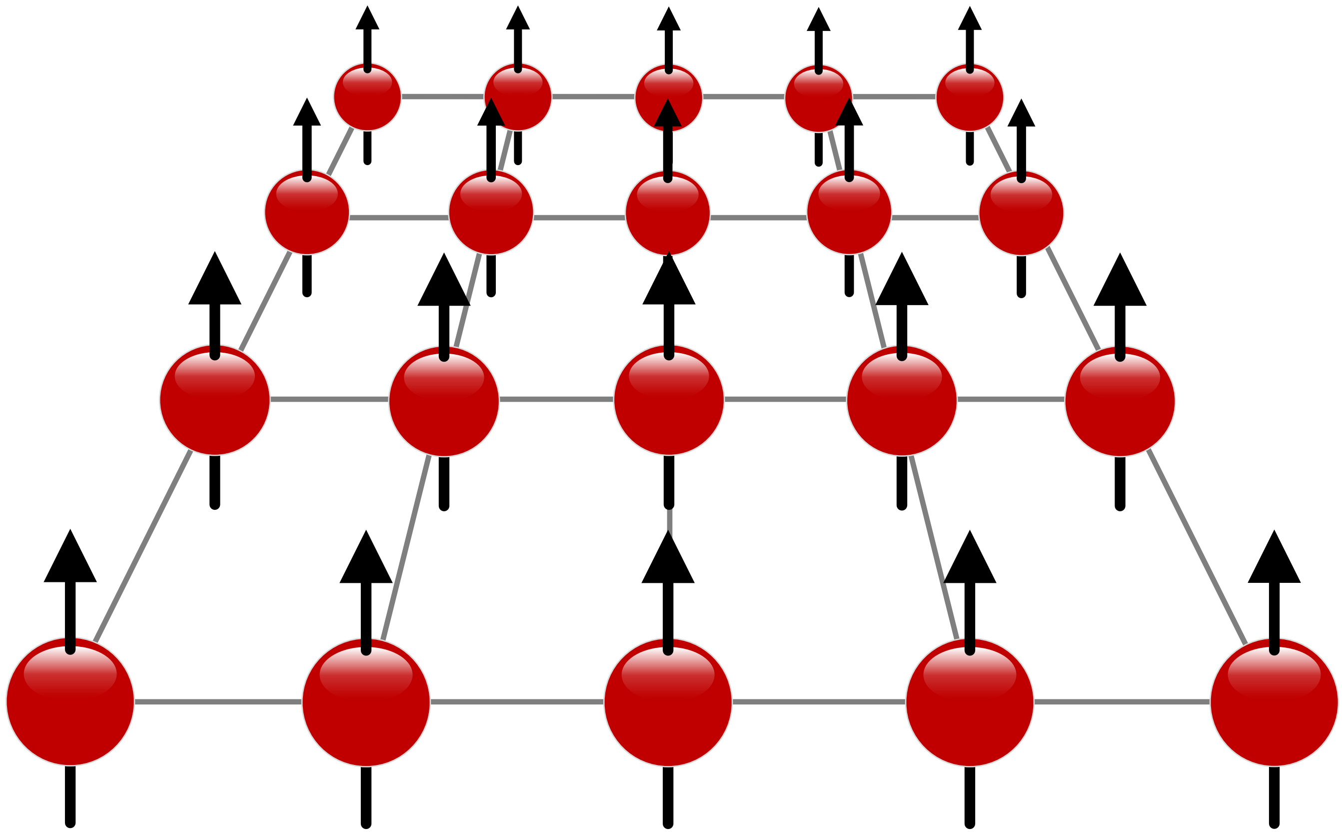
T_2: 重ね合わせ状態の持続時間 → 有限!!

量子計算の流れ(イメージ)



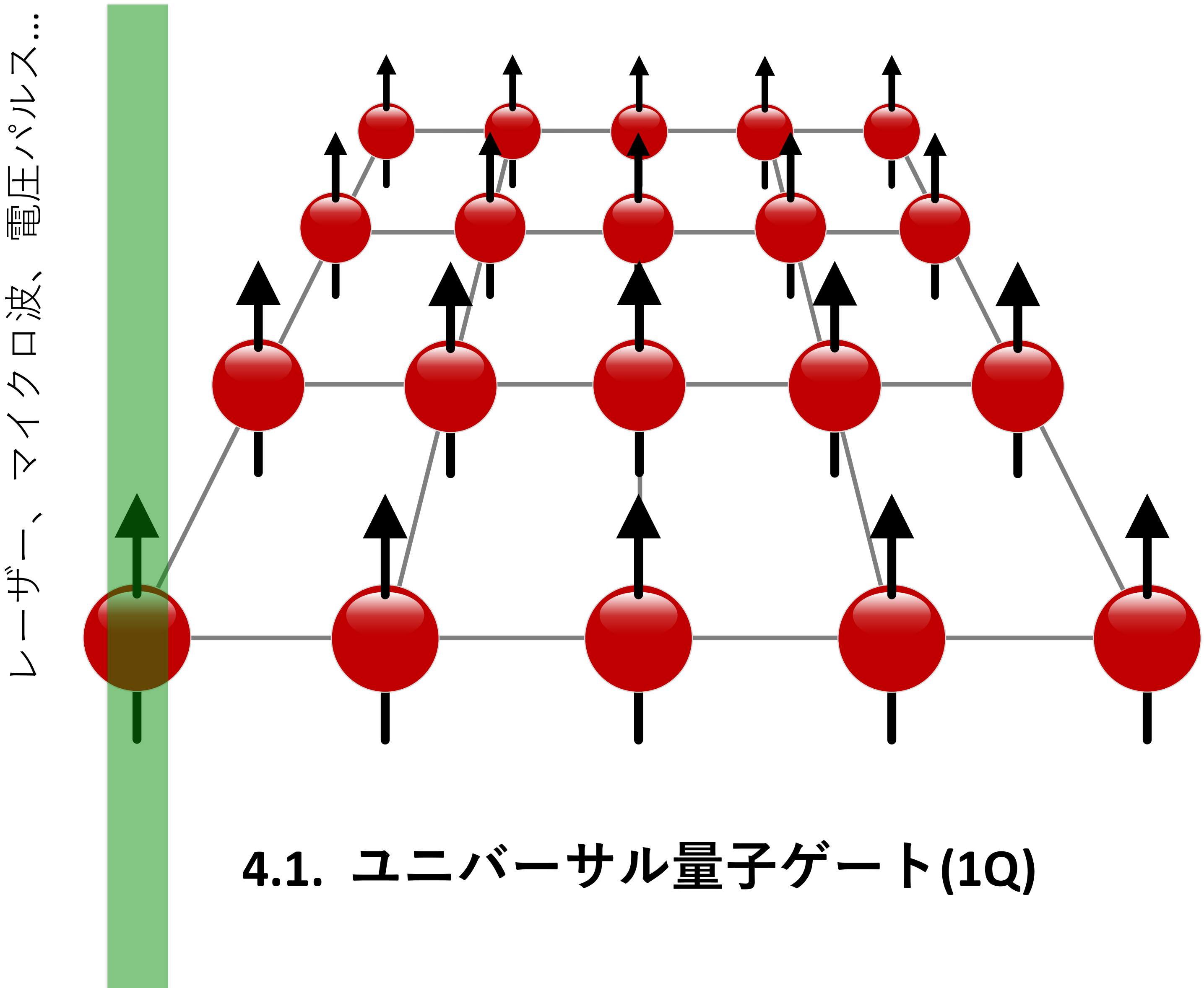
1. スケーラブルな量子ビット列

量子計算の流れ(イメージ)



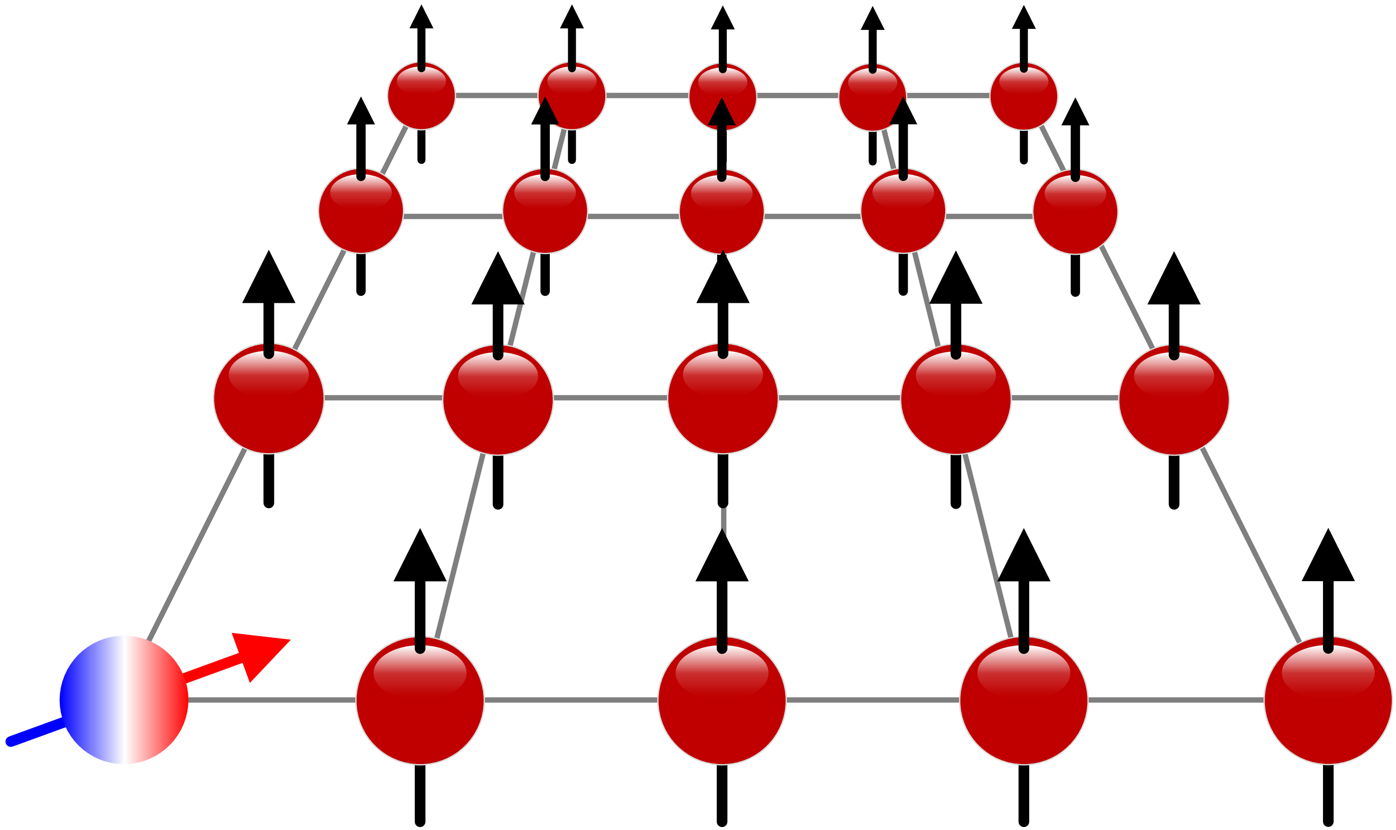
2. 初期化 $|0\rangle^{\otimes n}$

量子計算の流れ(イメージ)



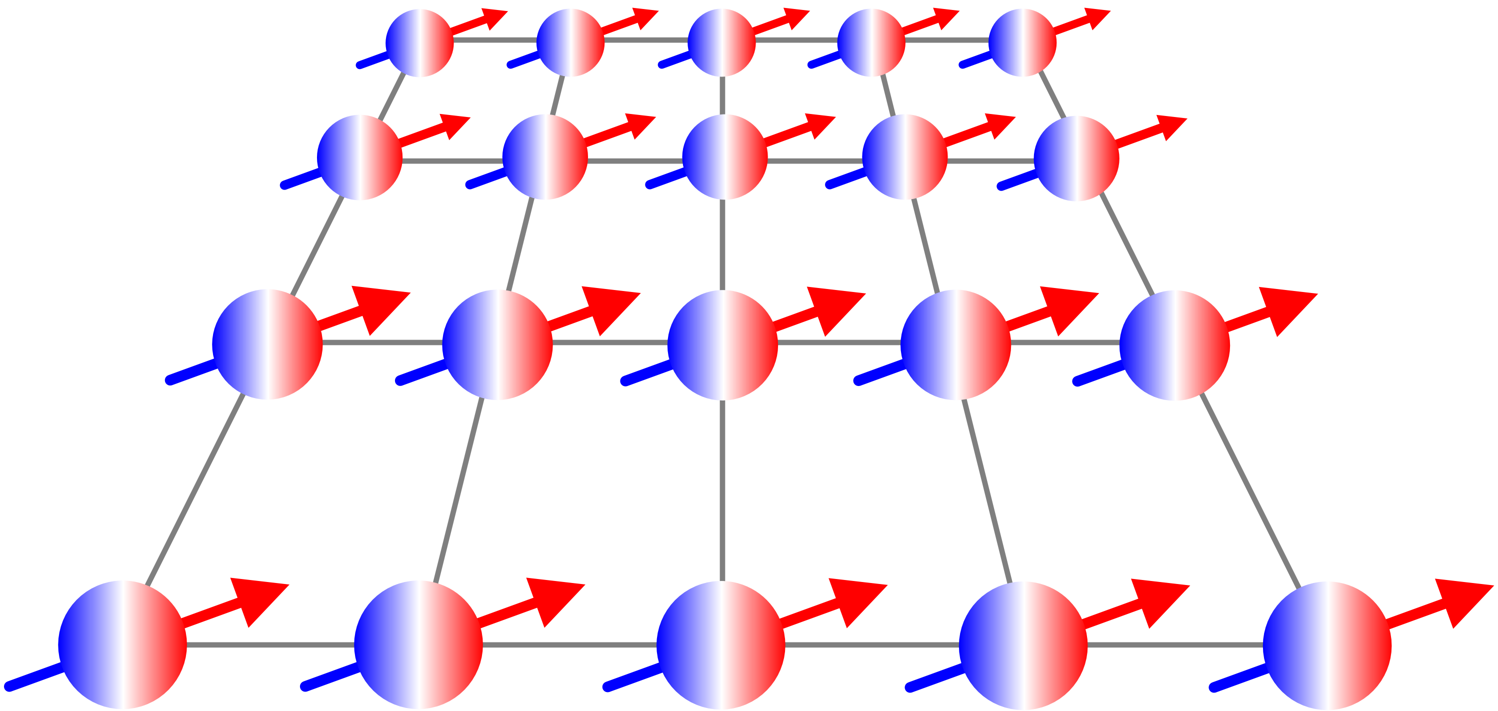
4.1. ユニバーサル量子ゲート(1Q)

量子計算の流れ(イメージ)



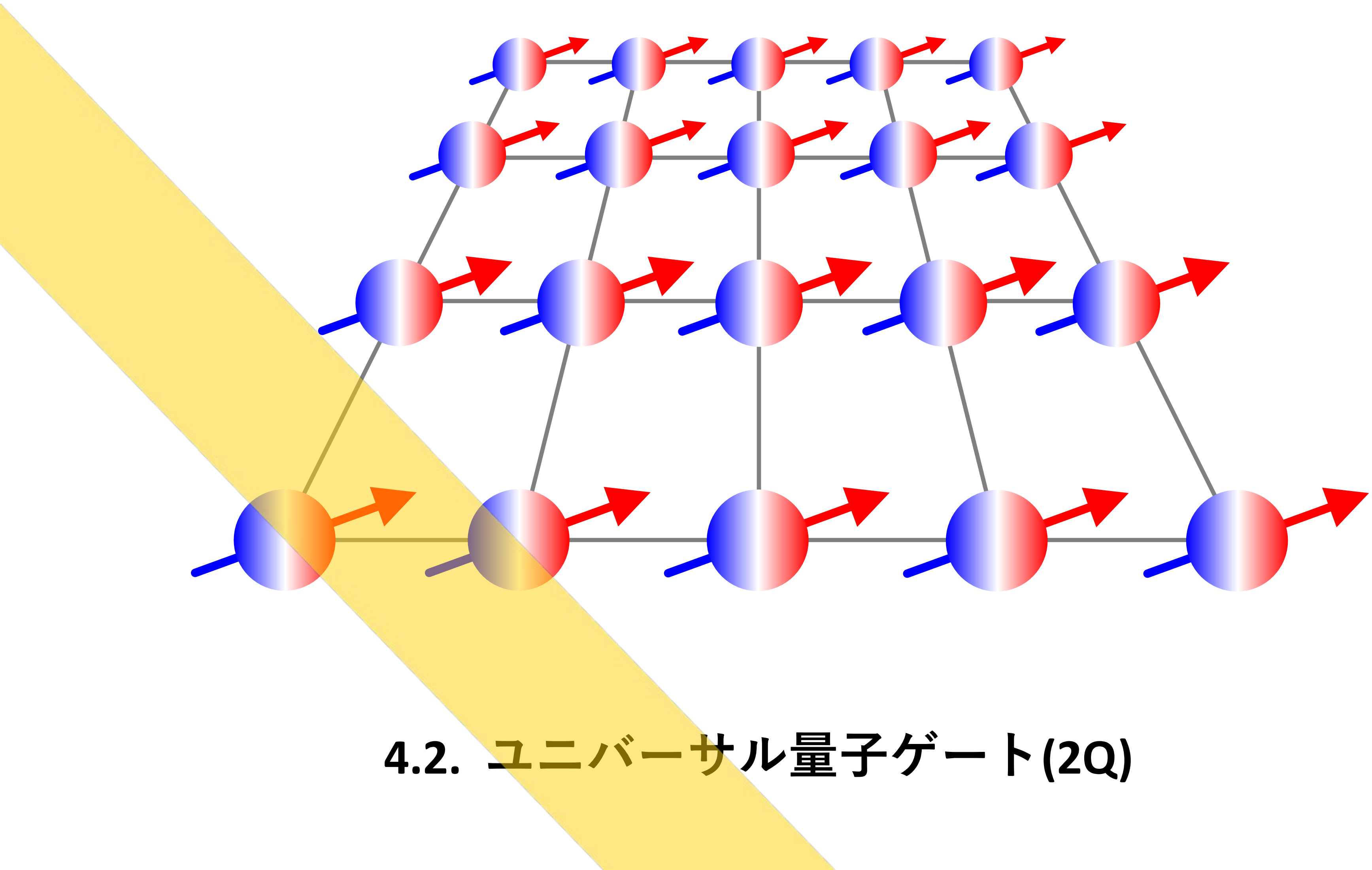
重ね合わせ状態生成

量子計算の流れ(イメージ)



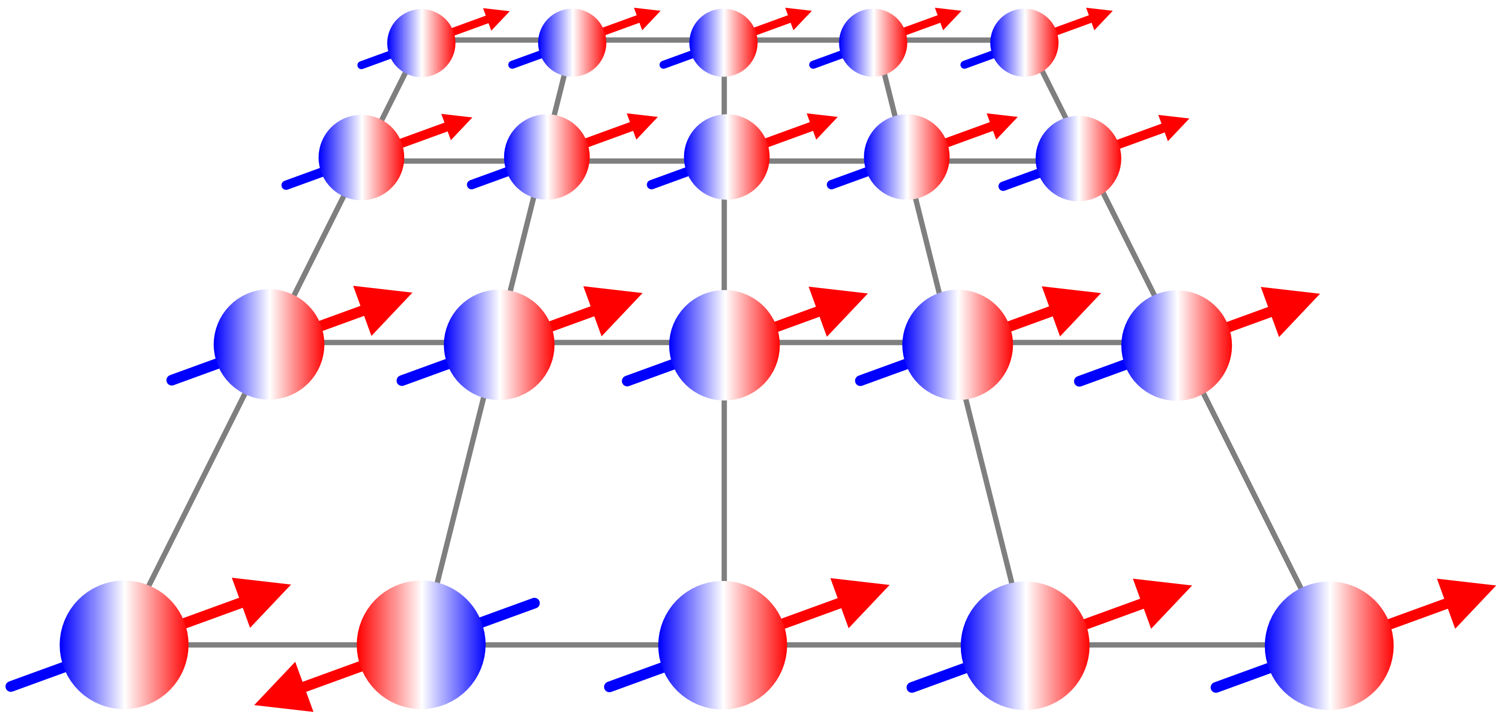
3. 長いコヒーレンス時間

量子計算の流れ(イメージ)



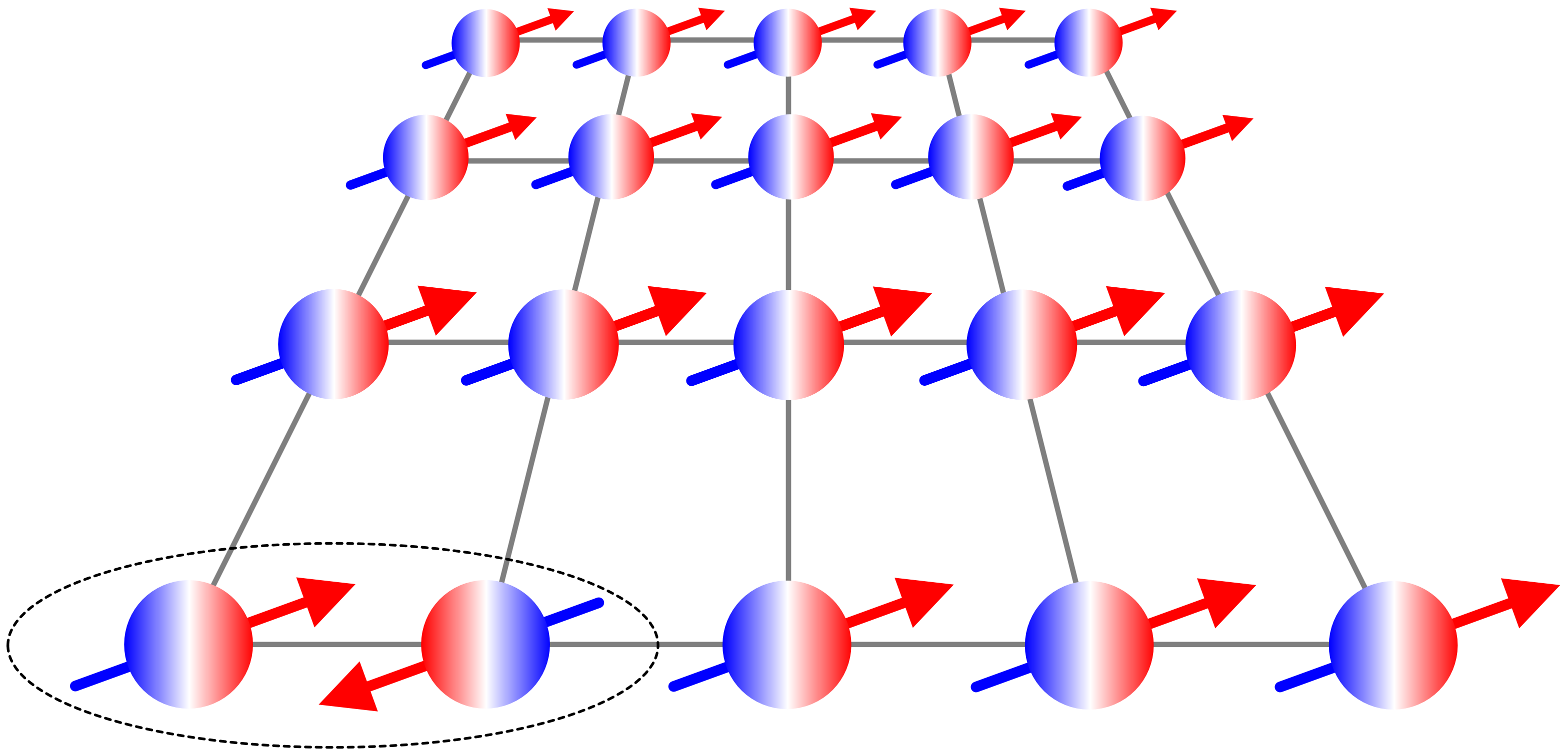
4.2. ユニバーサル量子ゲート(2Q)

量子計算の流れ(イメージ)



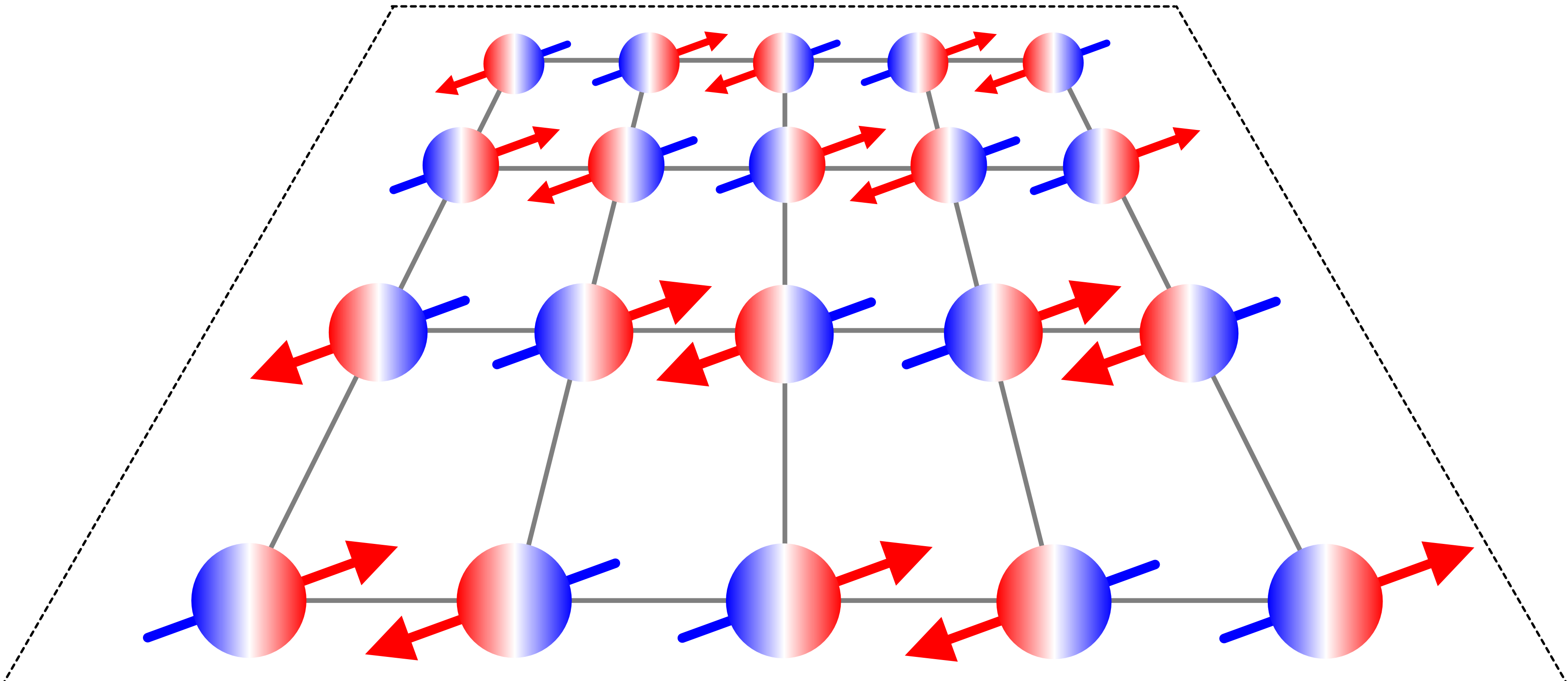
4.2. ユニバーサル量子ゲート(2Q)

量子計算の流れ(イメージ)



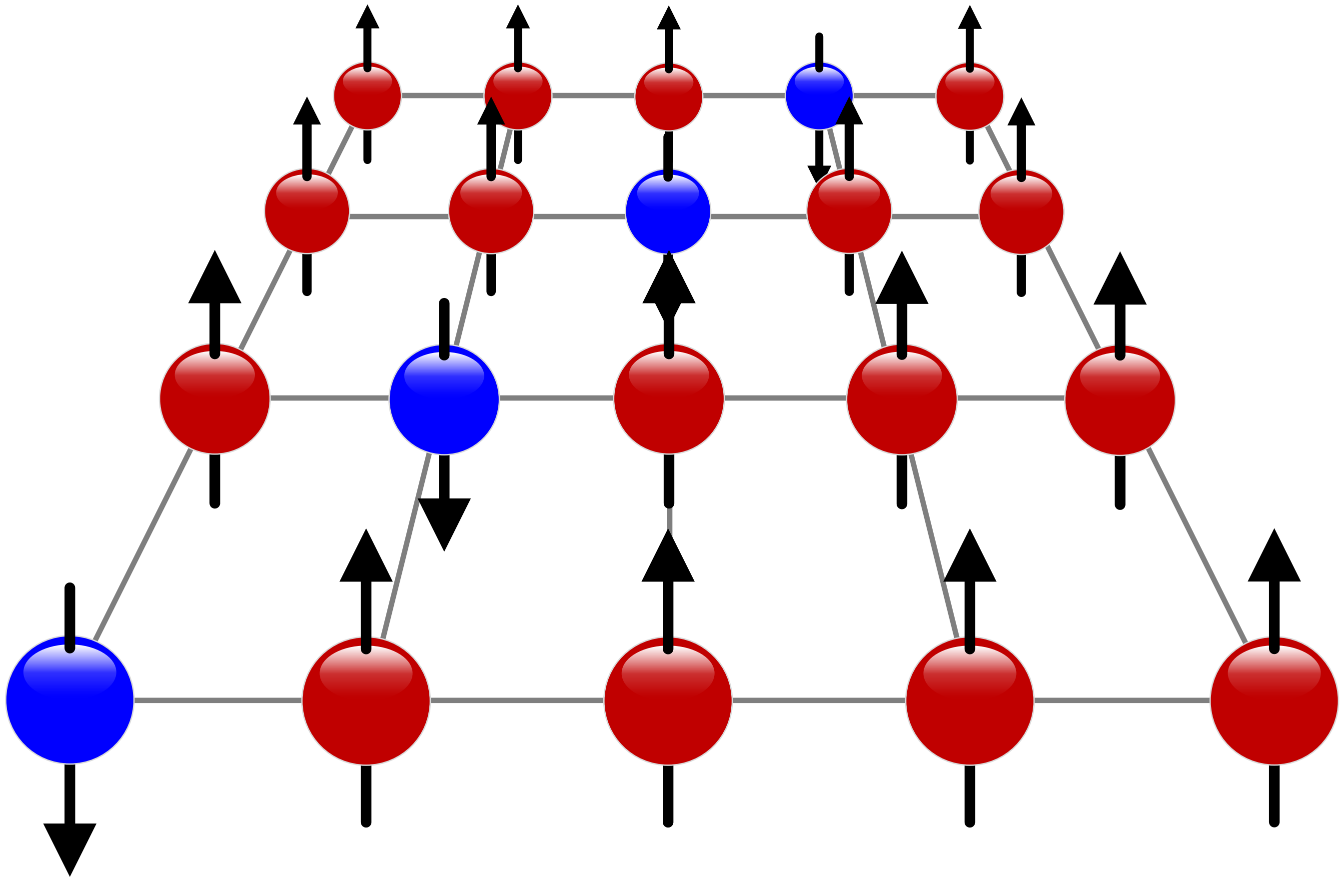
量子もつれ生成

量子計算の流れ(イメージ)



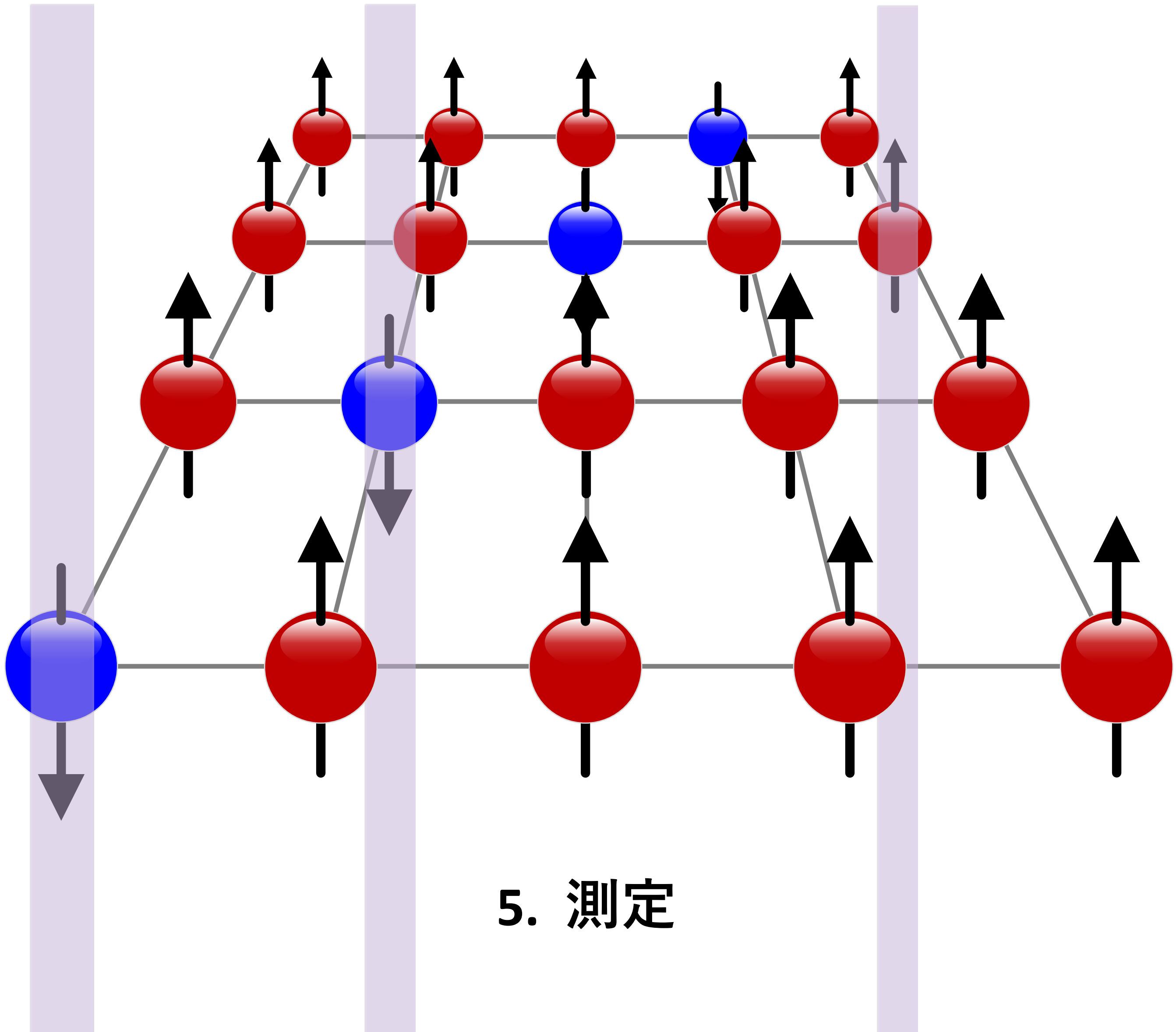
巨大な量子もつれ生成 → 量子アルゴリズム実行

量子計算の流れ(イメージ)



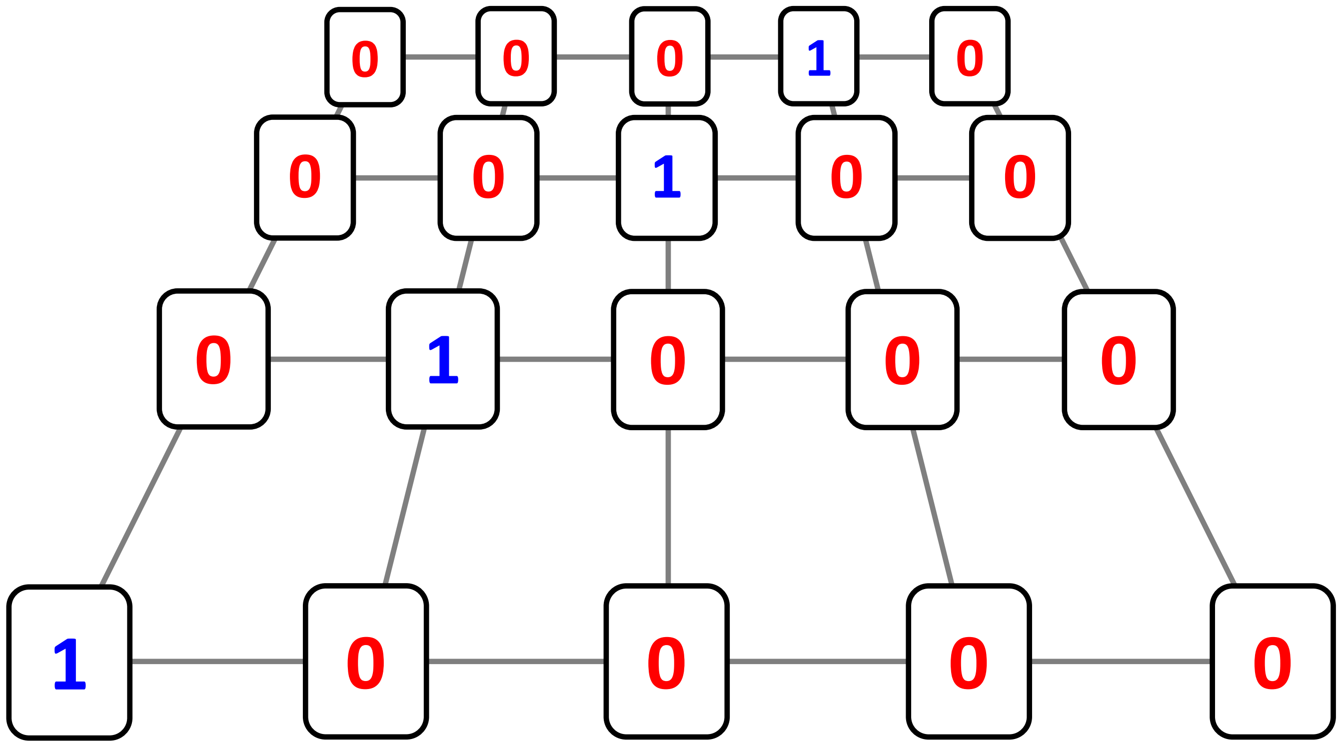
干渉効果

量子計算の流れ(イメージ)



5. 測定

量子計算の流れ(イメージ)

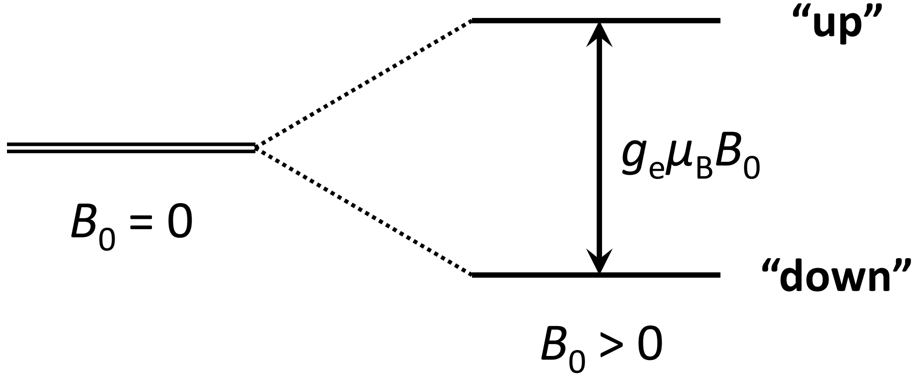


アルゴリズムの出力(解)

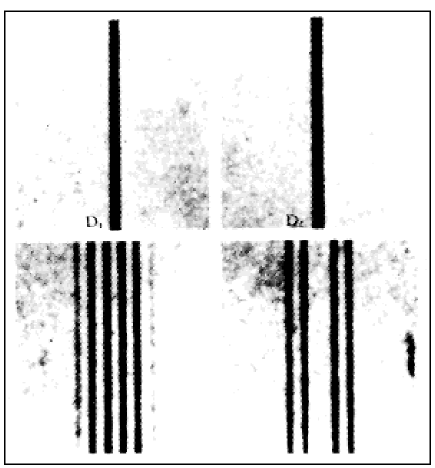
講義内容

- **量子技術の概要**
  - 物理系の例
  - 量子コンピューティングの難しさ
  - ディビンチェンゾの要請
- **スピンと磁気共鳴**
  - 量子ビットとの対応
  - 磁気共鳴によるスピン操作

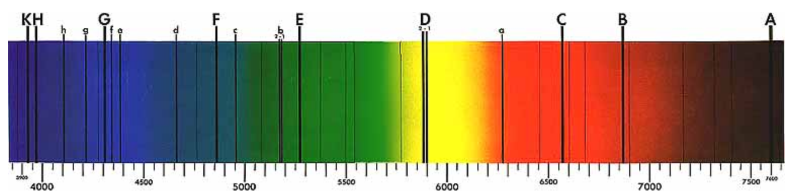
ゼーマン効果



g-因子: $g_e = 2.002319$
ボーア磁子: $\mu_B = 9.274 \times 10^{-24}$ J/T



(from Wikipedia)

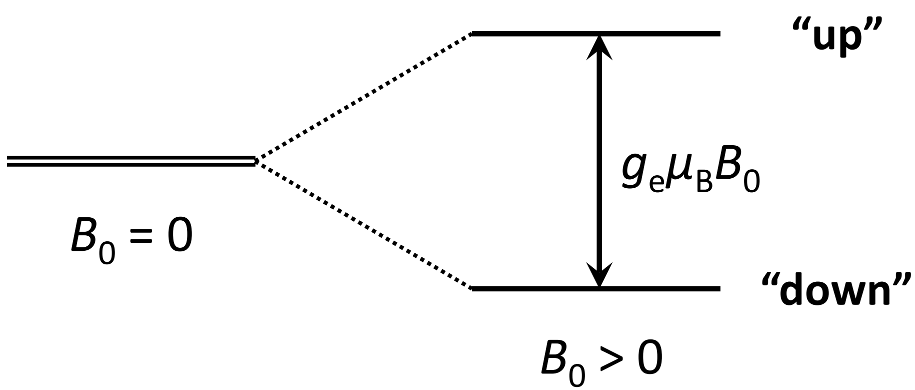


(from Wikipedia)

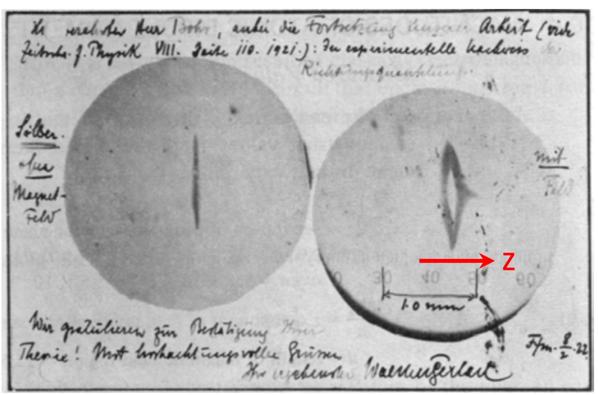
ナトリウムD_1, D_2線のゼーマン分裂 (1896)
→ 実は複雑(スピン軌道相互作用)
→ 電子の発見(1897, J. J. Thomson)よりも前

Pieter Zeeman
(1865–1943)

シュテルン–ゲルラッハの実験



g-因子: $g_e = 2.002319$
ボーア磁子: $\mu_B = 9.274 \times 10^{-24}$ J/T

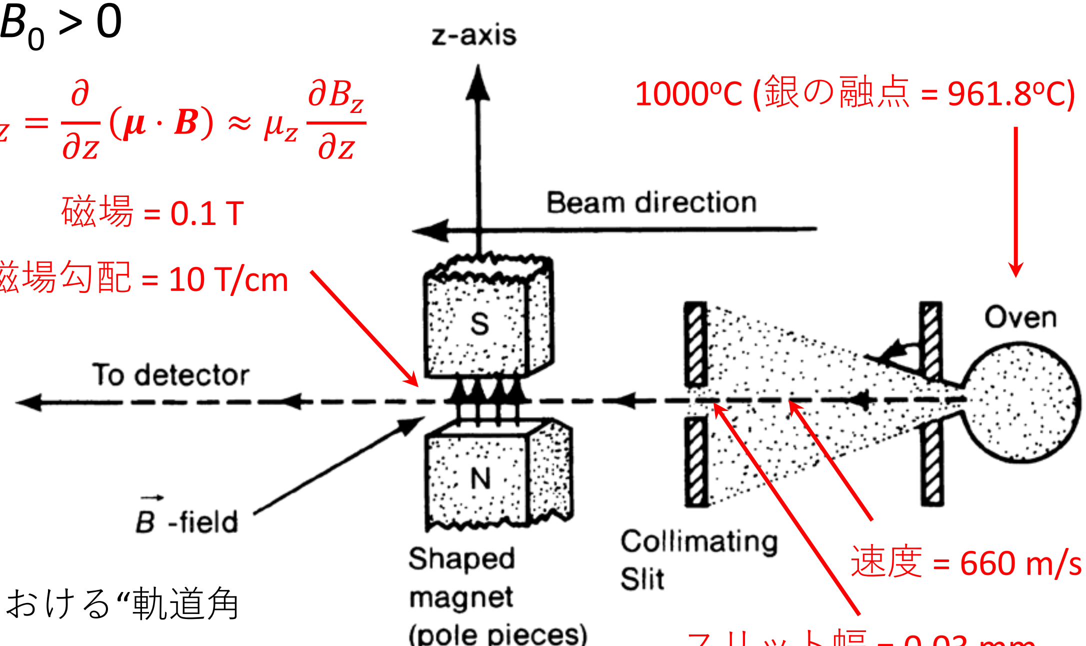


©AIP Emilio Segrè Visual Archives

$$F_z = \frac{\partial}{\partial z}(\boldsymbol{\mu} \cdot \boldsymbol{B}) \approx \mu_z \frac{\partial B_z}{\partial z}$$

磁場 = 0.1 T

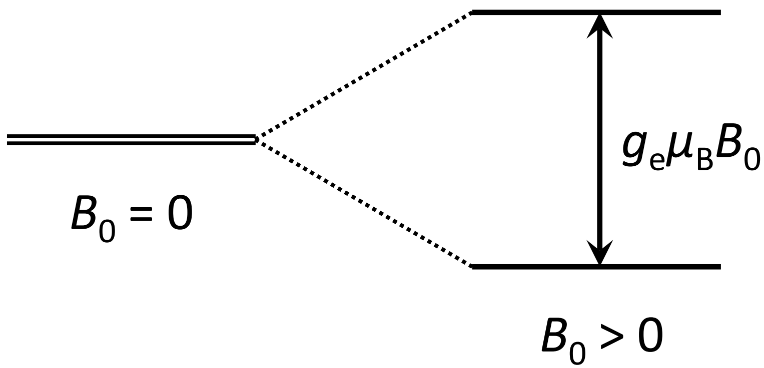
磁場勾配 = 10 T/cm



Modern Quantum Mechanics (Rev. Ed.) Sakurai

実質的にスピンを観測した実験(1922)
→ そもそもの目的は、ボーアモデル(1913)における"軌道角運動量の量子化"の検証 (分裂幅 ∝ $2\mu_B$)
→ スピンの提案 (1925, G. E. Uhlenbeck & S. Goudsmit)よりも前

電子スピン



$$H_z|\uparrow\rangle = \frac{g\mu_B B_0}{2}|\uparrow\rangle$$

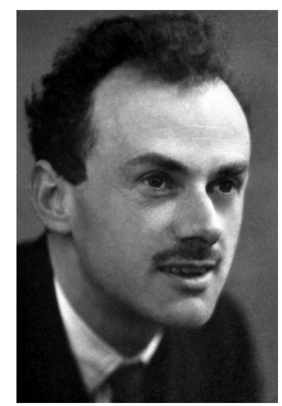
$$H_z|\downarrow\rangle = -\frac{g\mu_B B_0}{2}|\downarrow\rangle$$

電子の持つ量子力学的角運動量(内部自由度) S = 1/2 (m_s = ±1/2)

上を満たすハミルトニアンと状態ベクトルは?

$$H_Z = \frac{g\mu_B B_0}{2}\sigma_z$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Paul Dirac
(1902–1984)

→ ディラック方程式(1928)

電子スピン

スピンのS_z成分

$$|\uparrow\rangle = |S_z;+\rangle$$

$$|\downarrow\rangle = |S_z;-\rangle$$

$$S_z = \frac{\hbar}{2}\sigma_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

⇓

$$S_z|\uparrow\rangle = \frac{\hbar}{2}|\uparrow\rangle$$

$$S_z|\downarrow\rangle = -\frac{\hbar}{2}|\downarrow\rangle$$

S_x成分をどう表すか…?

$$S_x|S_x;+\rangle = \frac{\hbar}{2}|S_x;+\rangle$$

$$S_x|S_x;-\rangle = -\frac{\hbar}{2}|S_x;-\rangle$$

となって欲しい

⇓

$$|S_x;+\rangle = |\rightarrow\rangle = \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$|S_x;-\rangle = |\leftarrow\rangle = \frac{|\uparrow\rangle - |\downarrow\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix}$$

$$S_x = \frac{\hbar}{2}\sigma_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

パウリ行列

固有ベクトル

(固有値 = 1) (固有値 = −1)

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad |{\rightarrow}\rangle = \frac{|{\uparrow}\rangle + |{\downarrow}\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad |{\leftarrow}\rangle = \frac{|{\uparrow}\rangle - |{\downarrow}\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad |{\rightarrow_y}\rangle = \frac{|{\uparrow}\rangle + i|{\downarrow}\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} \qquad |{\leftarrow_y}\rangle = \frac{|{\uparrow}\rangle - i|{\downarrow}\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad |{\uparrow}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |{\downarrow}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

交換関係等

$$[\sigma_i, \sigma_{i+1}] = \sigma_i\sigma_{i+1} - \sigma_{i+1}\sigma_i = 2i\sigma_{i+2}$$

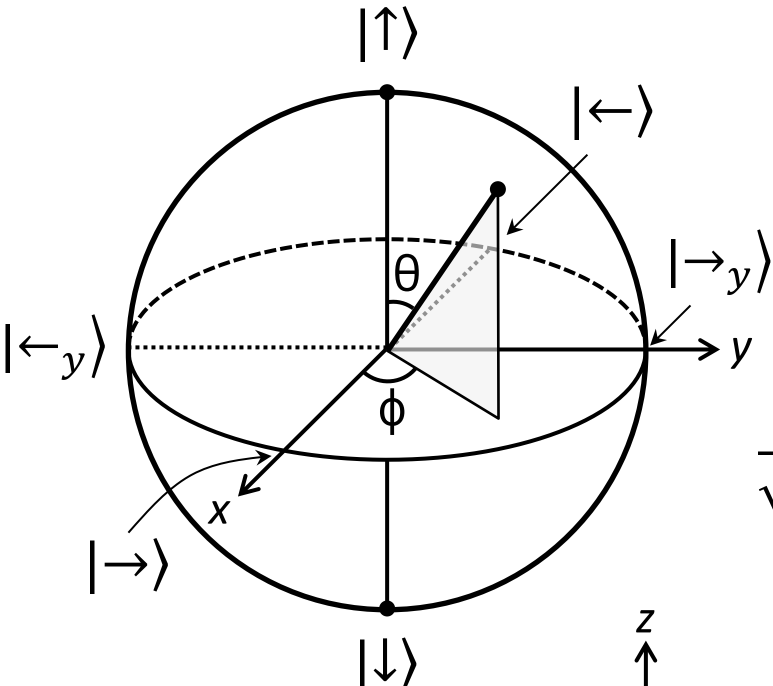
$$\{\sigma_i, \sigma_{i+1}\} = \sigma_i\sigma_{i+1} + \sigma_{i+1}\sigma_i = 0$$

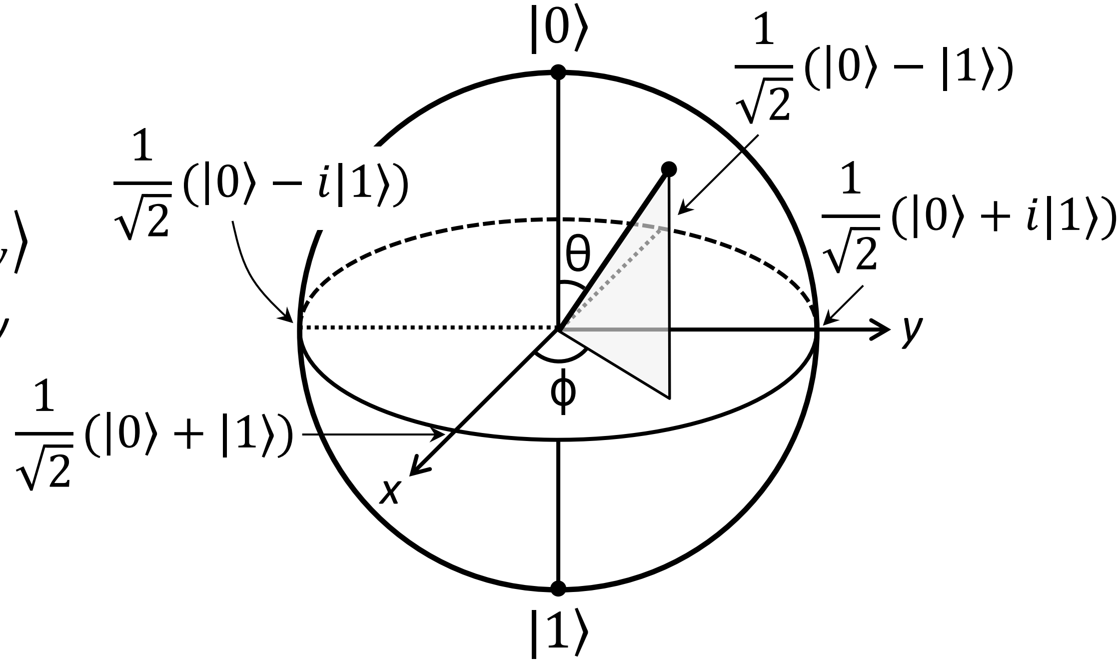
$$\sigma_i^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = I$$

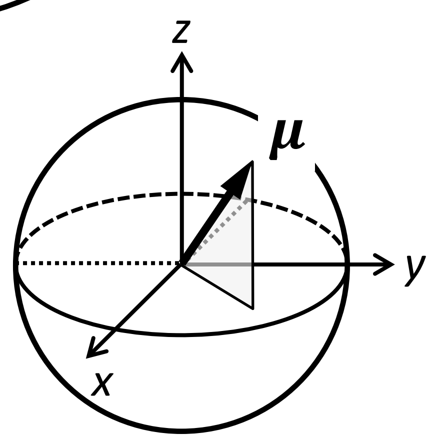
Wolfgang Pauli
(1900–1958)

対応関係

スピン1/2 ⟺ **量子ビット**







とはいえ、$|\Psi\rangle$を実空間の(古典的な)
磁気モーメントと考えてよいことも多い

講義内容

- **量子技術の概要**
  - 物理系の例
  - 量子コンピューティングの難しさ
  - ディビンチェンゾの要請
- **スピンと磁気共鳴**
  - 量子ビットとの対応
  - 磁気共鳴によるスピン操作

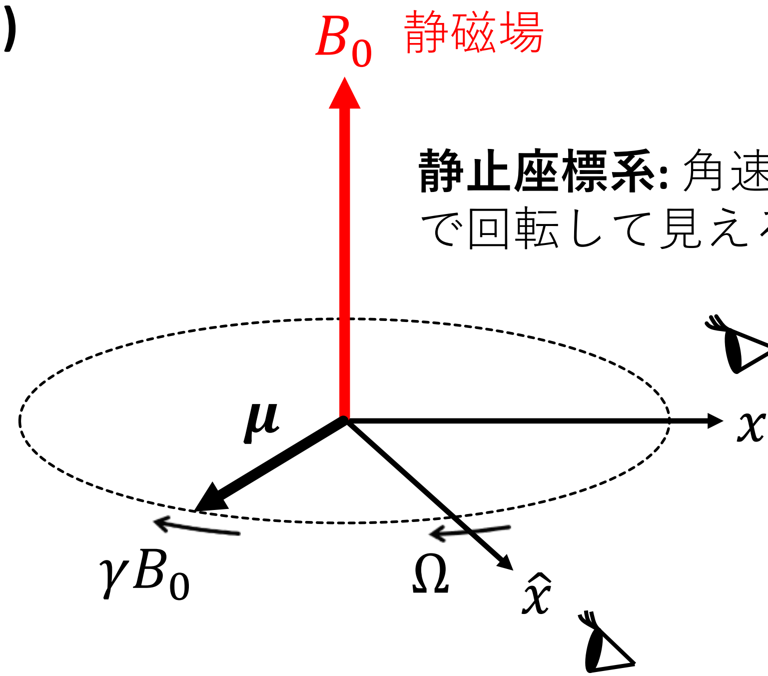
磁気共鳴

トルク方程式(ラーモア歳差運動)

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\mu} \times \gamma \boldsymbol{B}_0$$

磁気回転比$g_{\mathrm{e}}\mu_{\mathrm{B}}$

磁気モーメント: $\boldsymbol{\mu} = \gamma \boldsymbol{J}$



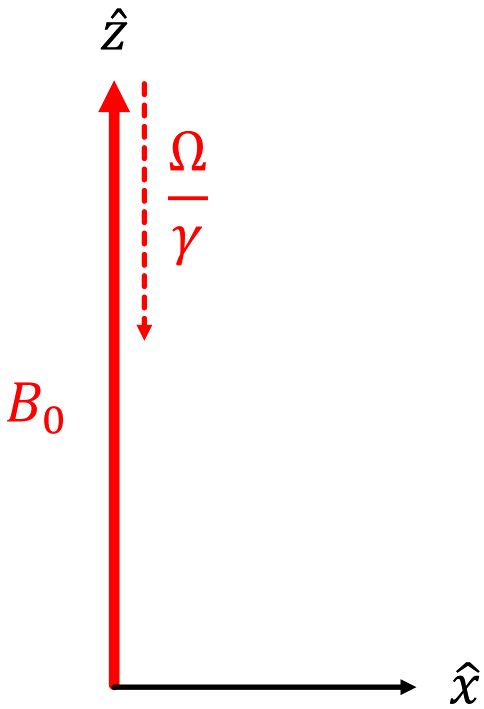
静止座標系: 角速度γB_0で回転して見える

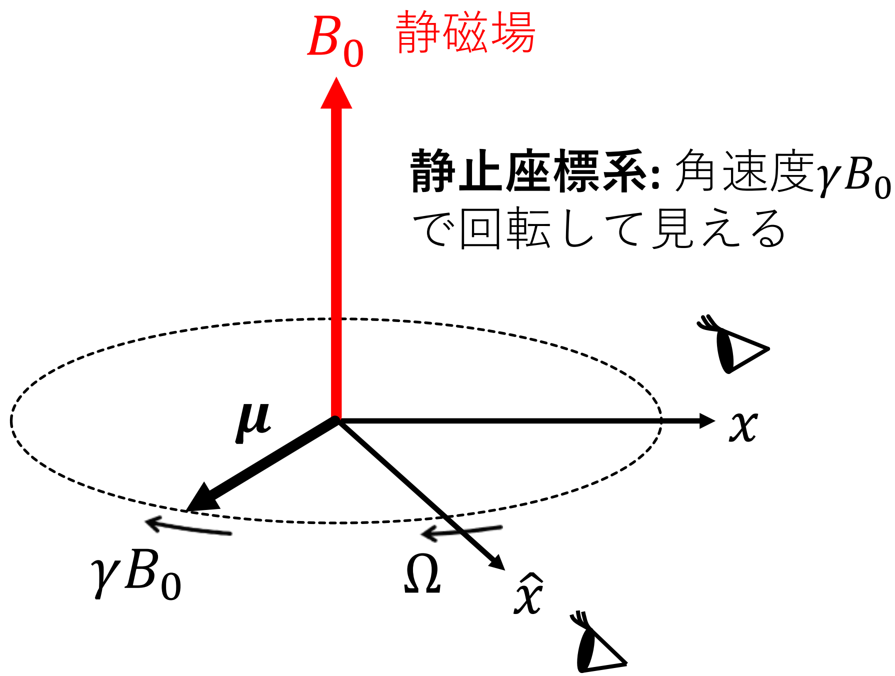
角速度Ωで回転する座標系: 遅くなって見える...なぜ??

Joseph Larmor (1857–1942)

(from Wikipedia)

磁気共鳴

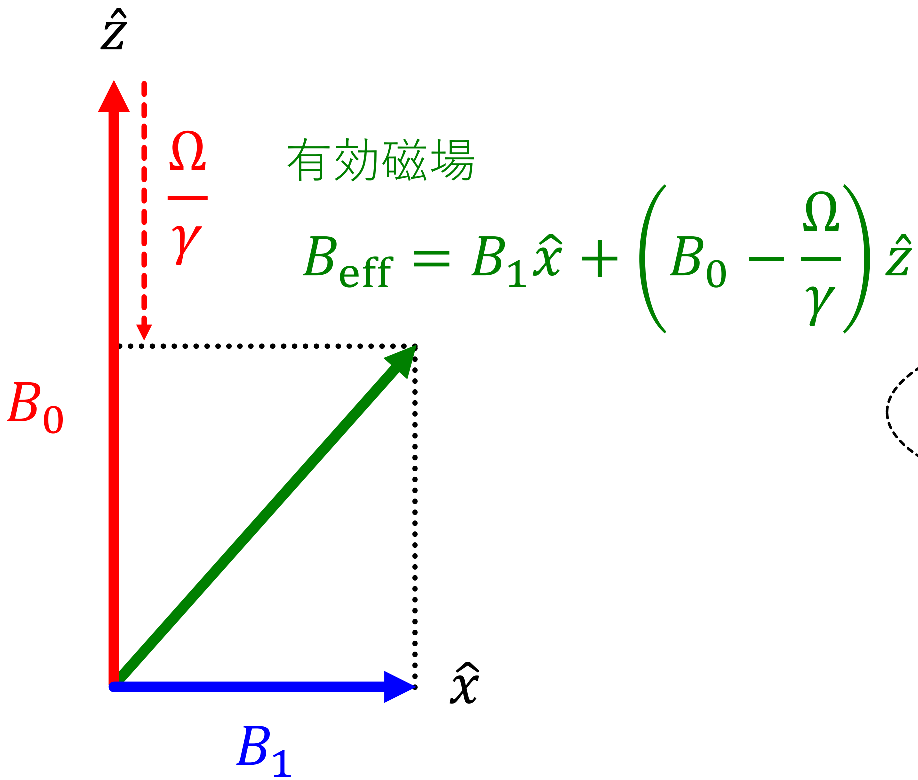




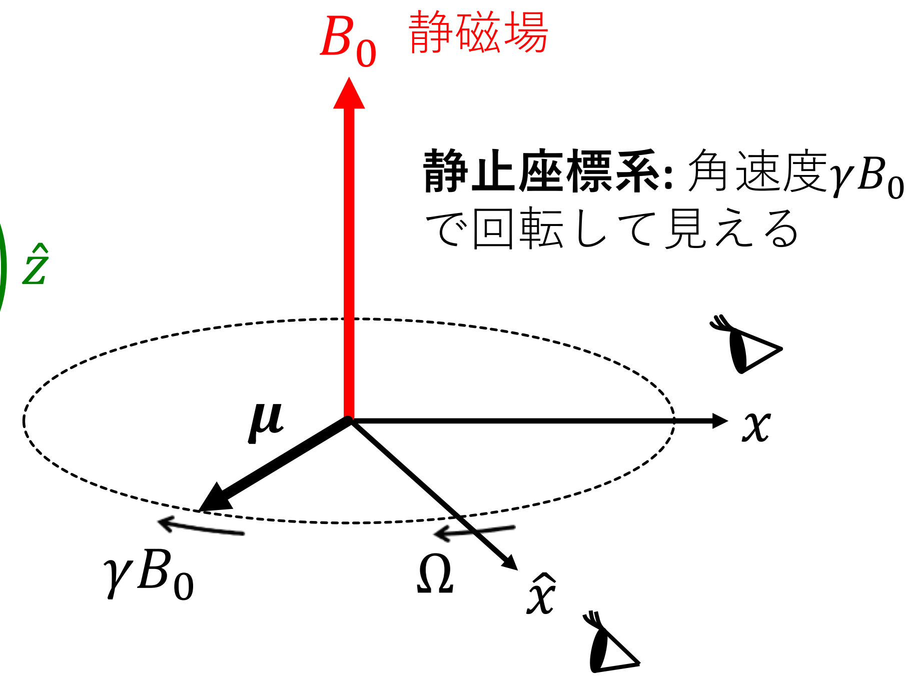
角速度Ωで回転する座標系:
遅くなって見える...なぜ??

z方向の磁場が弱くなったから

磁気共鳴



xy平面を角速度Ωで回転する交流磁場



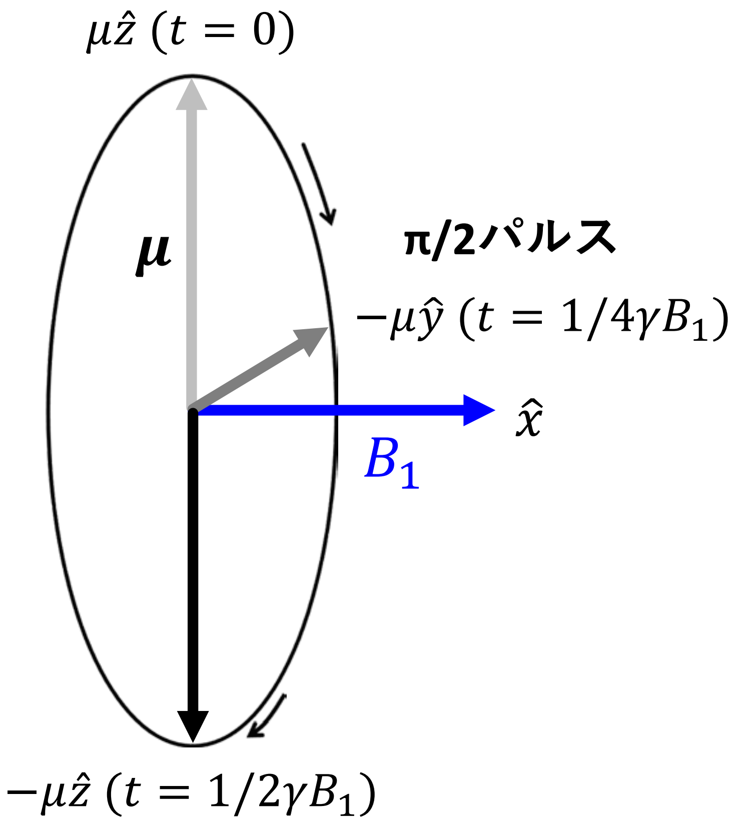
角速度Ωで回転する座標系:
遅くなって見える…なぜ??

$\Downarrow$

z方向の磁場が弱くなったから

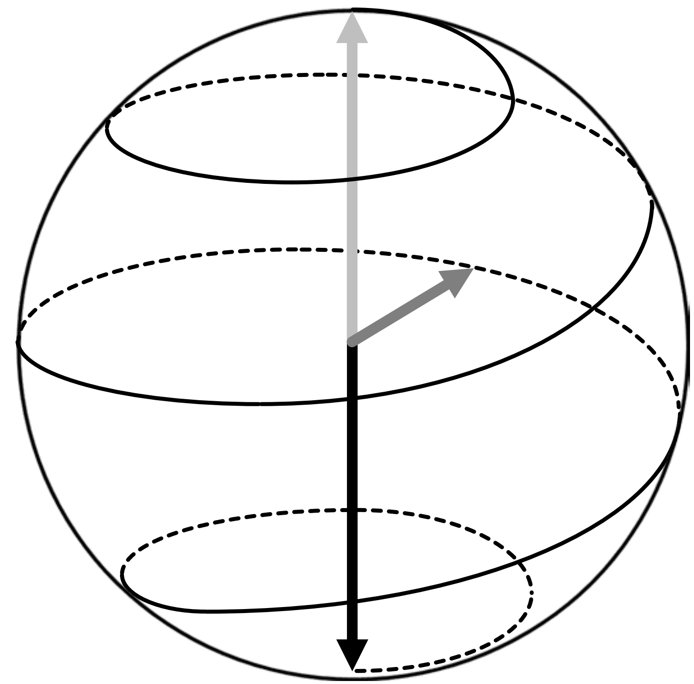
磁気共鳴

$\Omega = \gamma B_0$で回転する座標系



πパルス

静止座標系



- 交流磁場の位相を調整すれば$\pm\hat{x}, \pm\hat{y}$軸周りの回転が実現
- 静止座標系では$\hat{z}$軸周りの回転が加わる

電子スピン共鳴(ESR)装置

$\gamma_e/2\pi$ = 28 GHz/T → 10 GHz (X-band) @B_0 = 360 mT

マイクロ波導波管

アイリス

試料差し込み口

マイクロ波共振器

TE_{102}モード@9.75 GHz

cf.

c = 30 GHz cm

λ = 3 cm

Bruker E580@筑波大学(ca.2005)

https://www.bruker.com/fileadmin/user_upload/8-PDF-Docs/MagneticResonance/EPR_brochures/EPR_accessories.pdf

電子スピン共鳴(ESR)装置

$\gamma_e/2\pi$ = 28 GHz/T → 10 GHz (X-band) @B_0 = 360 mT

マイクロ波導波管

アイリス

試料差し込み口

cf.

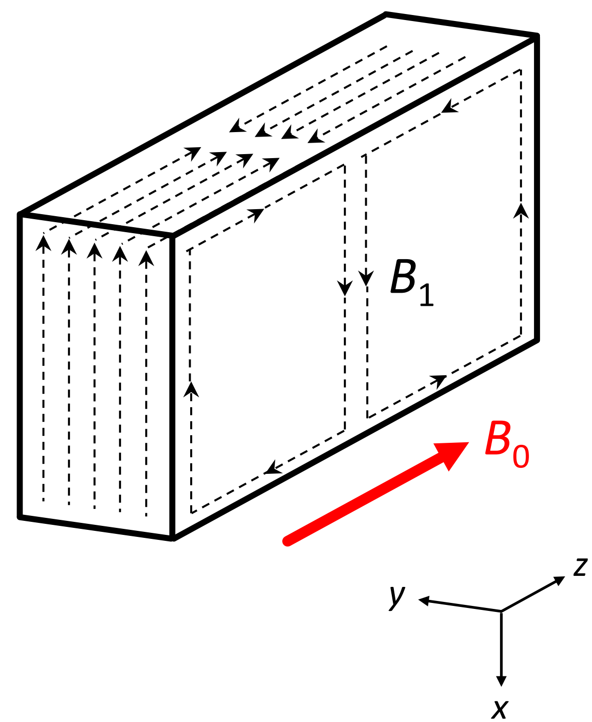
c = 30 GHz cm

λ = 3 cm

マイクロ波共振器

TE$_{102}$モード@9.75 GHz

TE$_{102}$モードの磁場分布



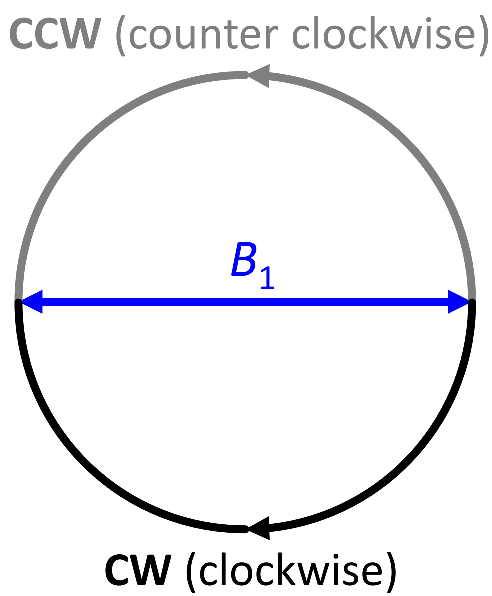
https://www.bruker.com/fileadmin/user_upload/8-PDF-Docs/MagneticResonance/EPR_brochures/EPR_accessories.pdf

実験における回転座標系とは?

xy平面を角速度Ωで回転する円偏光交流磁場を生成するのは(可能だが)面倒

⇩

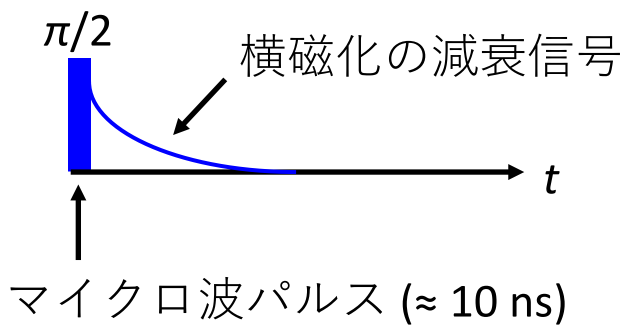
通常は**周波数$\Omega/2\pi$でx方向に振動する直線偏光交流磁場**を生成する



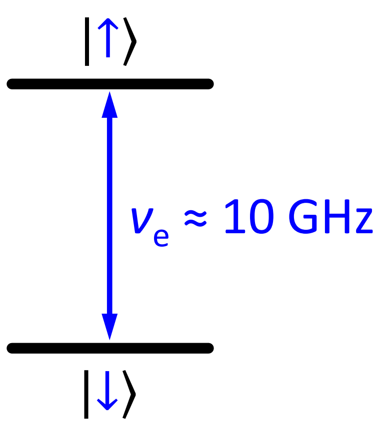
- 直線偏光B_1のCW成分はスピンと同方向に回転して、磁気共鳴に寄与
- CCW成分は非共鳴なので無視できる (回転波近似, RWA)
- 検出系を発振器の周波数に同期することで、回転座標系でスピンを“見る”ことになる

T_2の測定

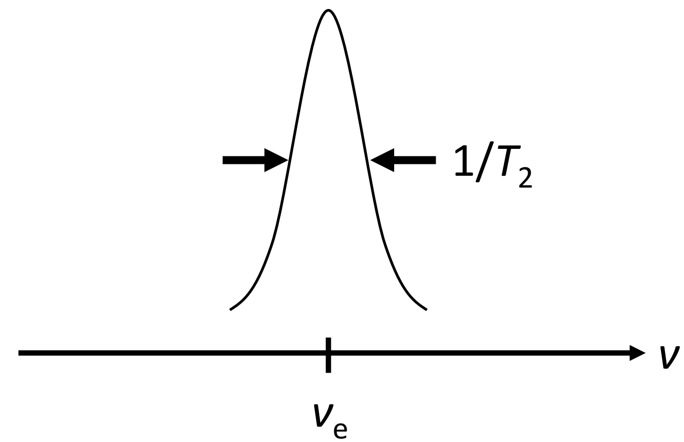
時間領域 → 指数減衰時定数T_2



直感的には、スピンを横に向けて
信号の減衰を見ればよさそう

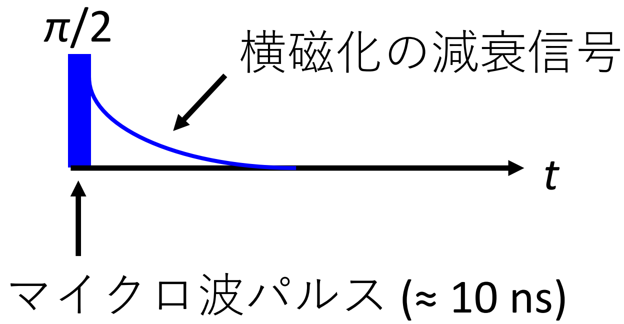


周波数領域 → ローレンツ線幅$1/T_2$

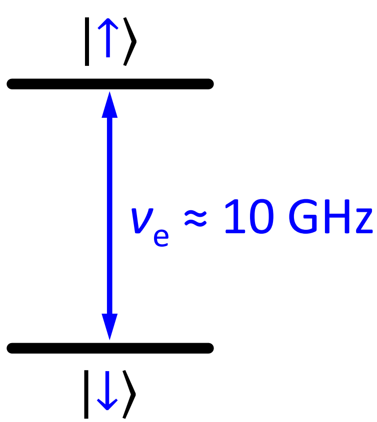


T_2の測定

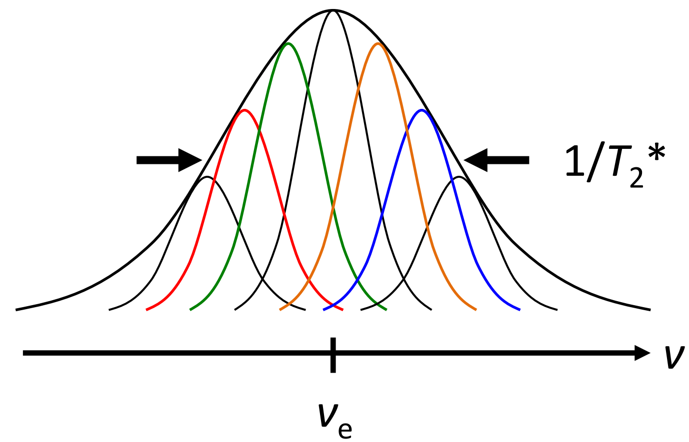
時間領域 → 指数減衰時定数T_2



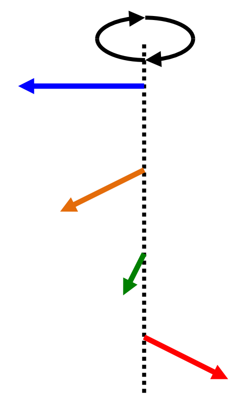
直感的には、スピンを横に向けて
信号の減衰を見ればよさそう



周波数領域 → ローレンツ線幅$1/T_2$

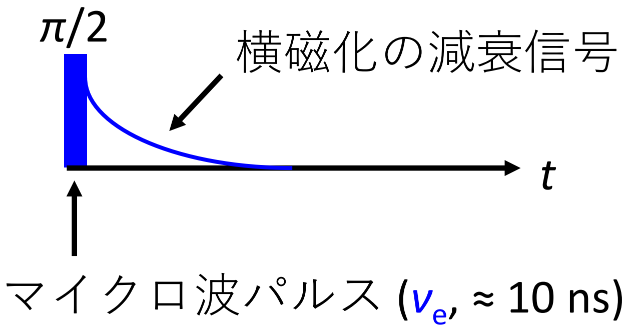


磁場の不均一性による線幅増大

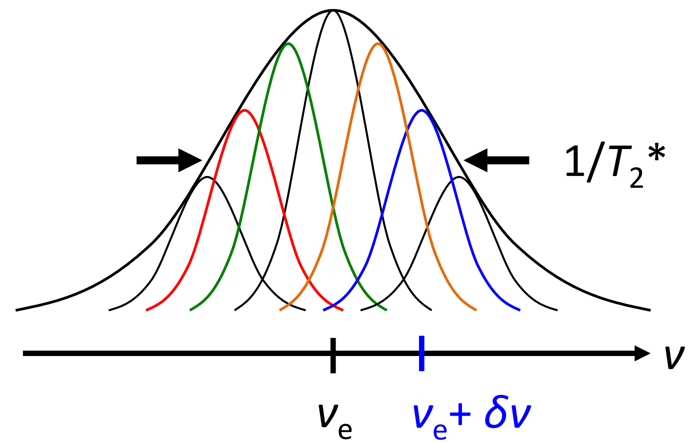


T_2の測定

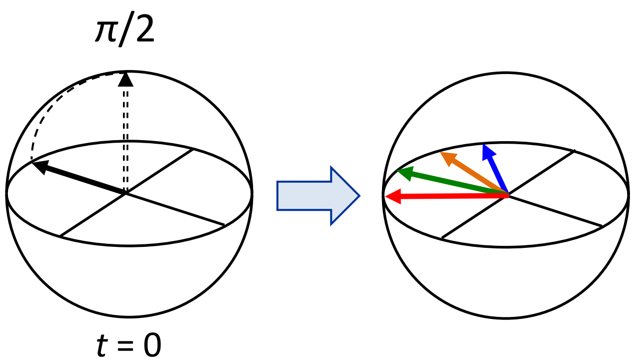
時間領域 → 指数減衰時定数T_2



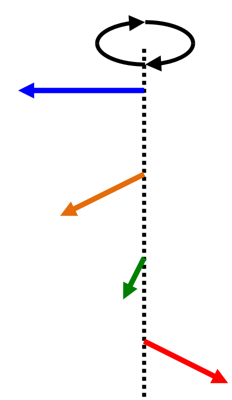
周波数領域 → ローレンツ線幅$1/T_2$



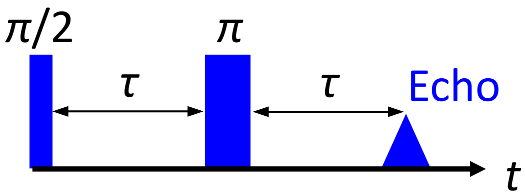
回転座標系では$\delta\nu$で回転するように見える

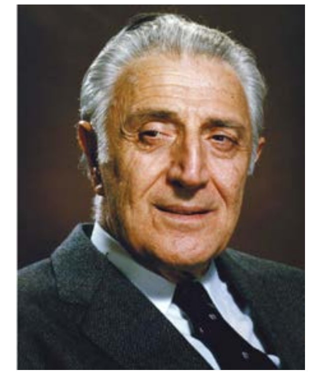


磁場の不均一性による線幅増大



T_2の測定: スピンエコー法

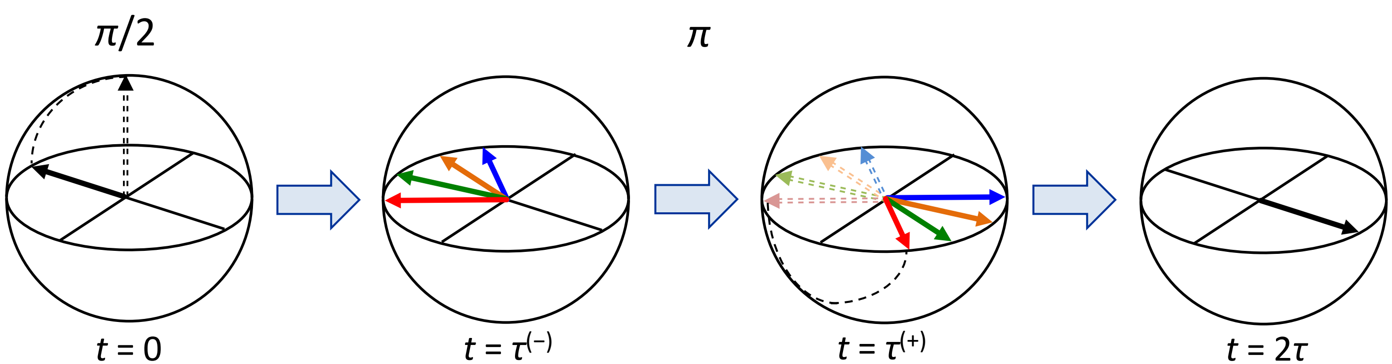




E. Hahn
(1921–2016)

τを変えて測定を繰り返すと不均一性を取り除いた横磁化の減衰信号が得られる(しばしば$T_2^* \ll T_2$)

“リフォーカス”

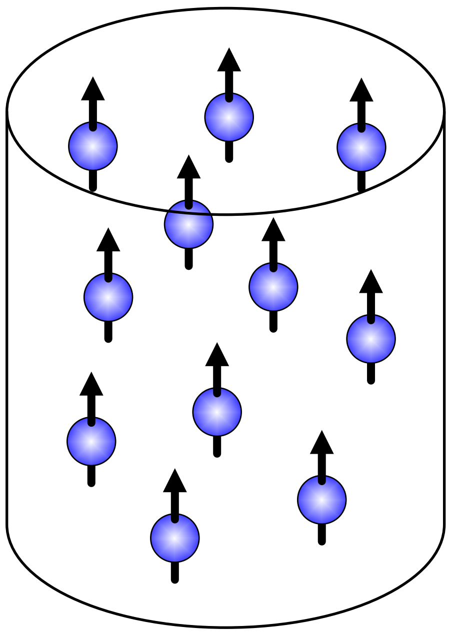


Phys. Rev. **80**, 580 (1950) Hahn

アンサンブル測定と時間平均測定

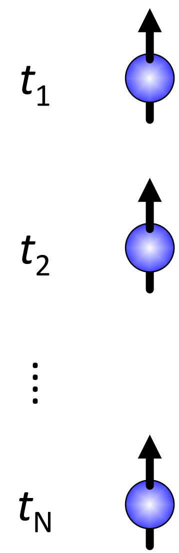
アンサンブル測定

同種スピン*N*個を一度に測定



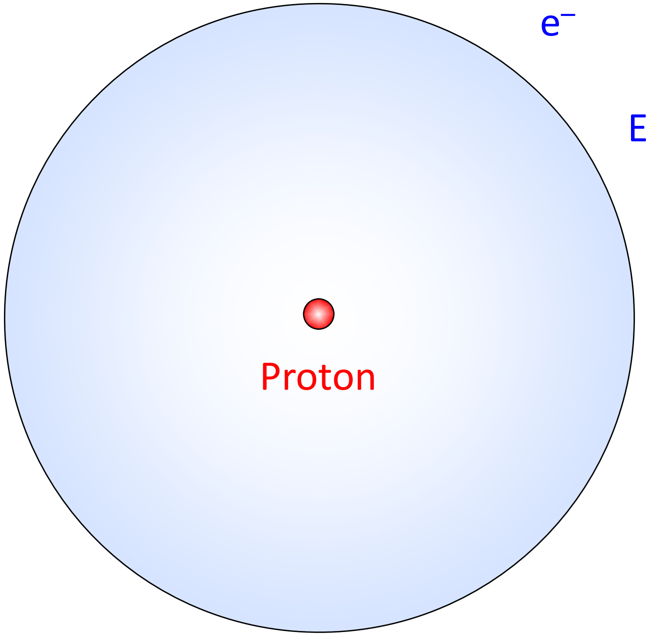
時間平均測定

単一スピンを*N*回測定



- 十分大きい*N*では両者は統計的に同じ
- 不均一性の原因は違い得る(空間的、時間的)
- 射影測定 → シングルショット非破壊測定

電子スピンと核スピン

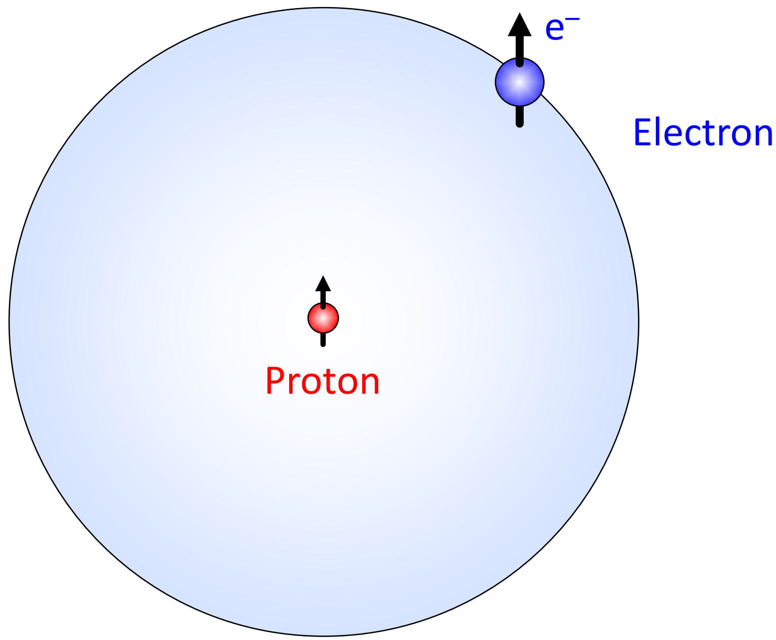


水素原子(^1H)

電子

陽子

電子スピンと核スピン



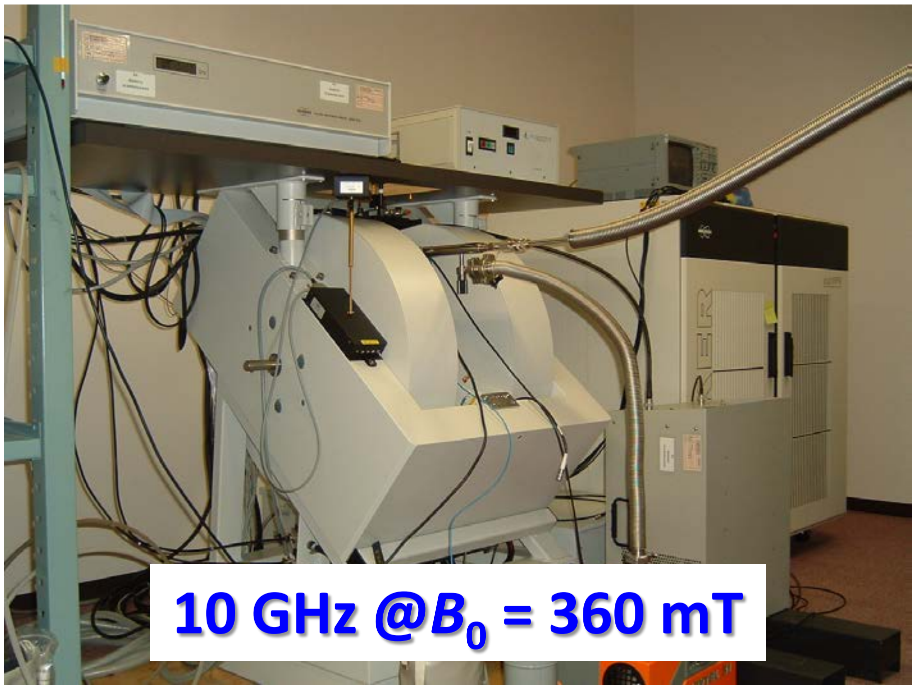
水素原子(^{1}H)

電子スピン(S = ½): $\gamma_e/2\pi$ = 28 GHz/T

核スピン(I = ½): $\gamma_H/2\pi$ = 42.58 MHz/T

電子スピンと核スピン

ESR: Bruker E580@筑波大学(ca.2005)



NMR: Chemagnetics CMX@慶應大学(ca.2003)

電子スピン(S = ½): $\gamma_e/2\pi$ = 28 GHz/T

核スピン(I = ½): $\gamma_H/2\pi$ = 42.58 MHz/T

レポート課題 1 (10点)

トルク方程式

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\mu} \times \gamma \boldsymbol{B}_0 = \begin{pmatrix} \mu_x(t) \\ \mu_y(t) \\ \mu_z(t) \end{pmatrix} \times \gamma \begin{pmatrix} 0 \\ 0 \\ B_0 \end{pmatrix}$$

を初期条件

$$\boldsymbol{\mu}(\mathrm{t} = 0) = \begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}$$

のもとで解け。