

# 量子輸送現象

阿部 英介

慶應義塾大学 先導研究センター

応用物理情報特別講義A

2016年度春学期後半 金曜4限@14-202

# 講義内容

- **量子輸送の基礎**
  - 2次元電子系
  - ランダウアー公式
  - 量子ポイントコンタクト
- **整数量子ホール効果**
- **量子ホール効果とノーベル賞**

# 参考書

- J. H. Davies (1997)
  - **“The Physics of Low-Dimensional Semiconductors”** (邦訳あり)
- S. Datta (1997)
  - **“Electronic Transport in Mesoscopic Systems”** (邦訳あり)
- 吉岡大二郎 (1998)
  - **“量子ホール効果”** (英訳あり)

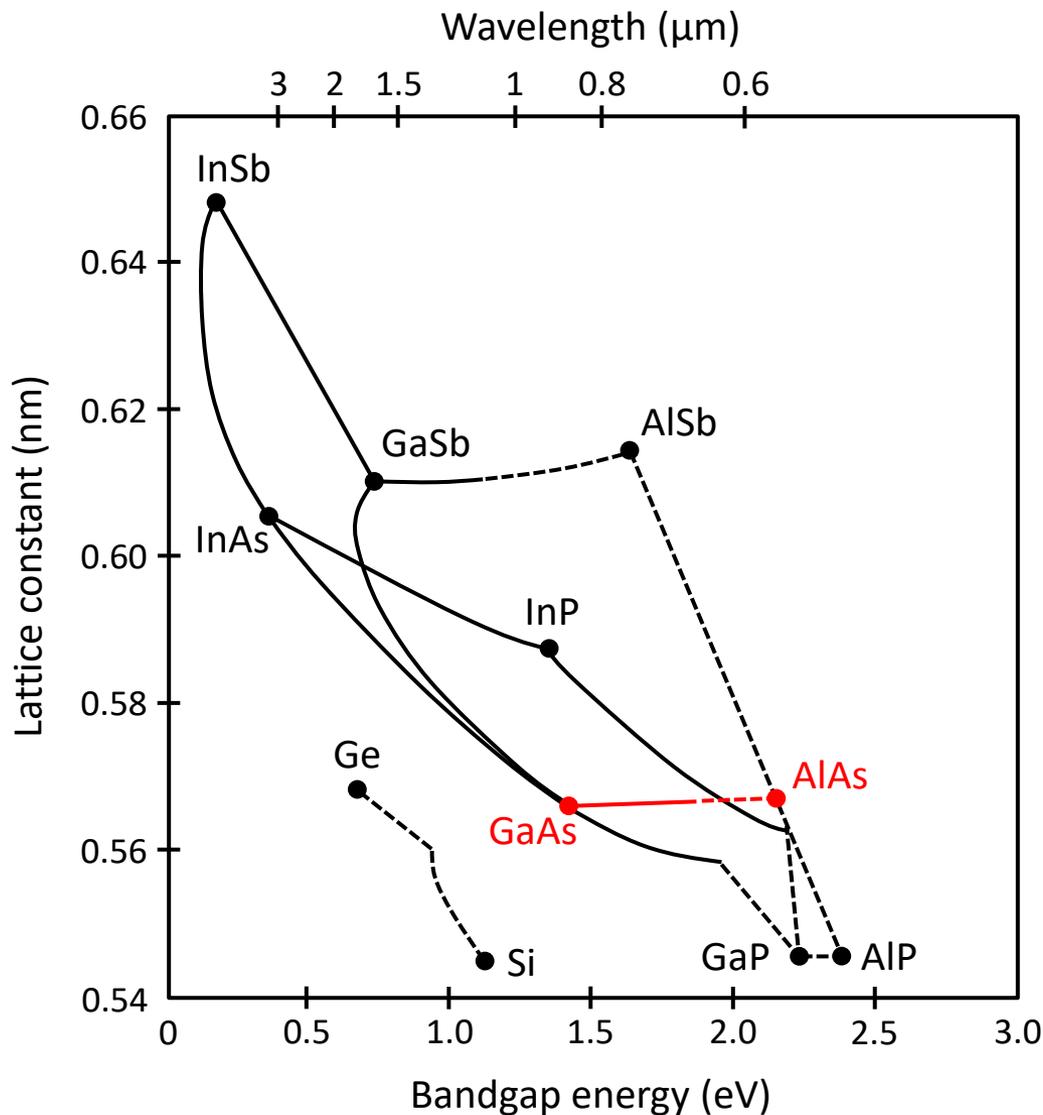
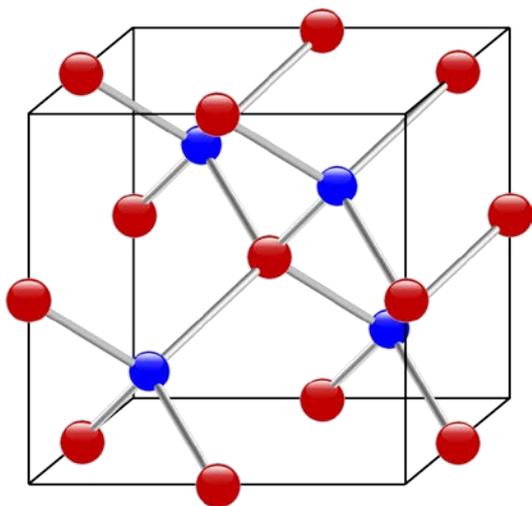
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# GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ ヘテロ構造

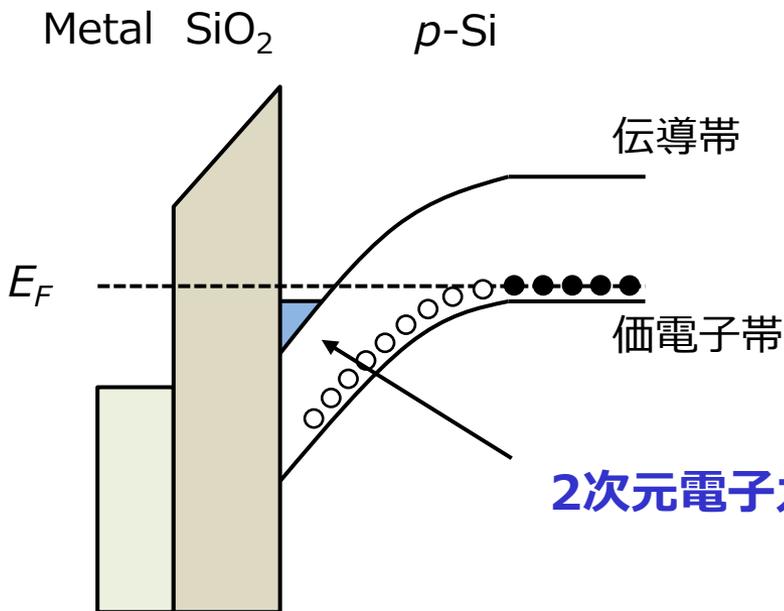
III (13)	IV (14)	V (15)
Al	Si	P
Ga	Ge	As
In	Sn	Sb

閃亜鉛鉱構造

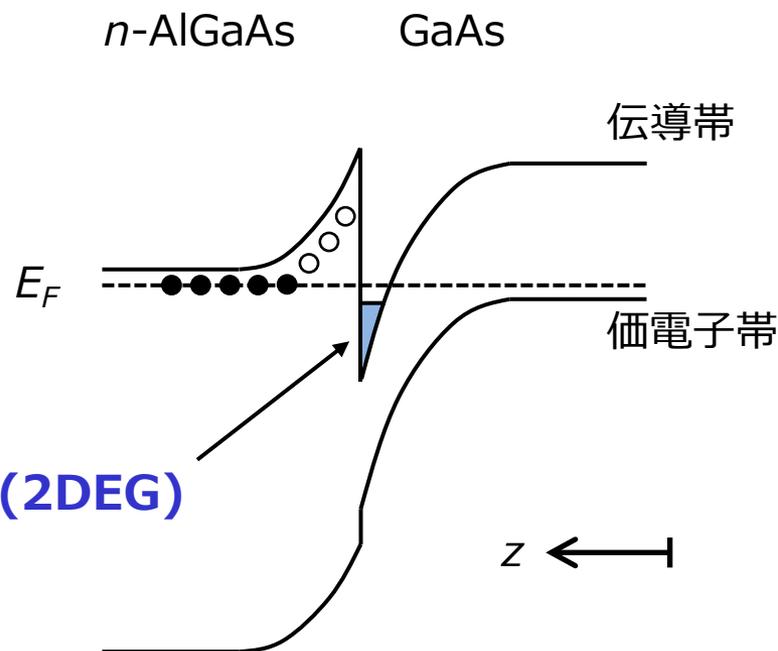


# 2次元電子系

## $p$ -Si MOS反転層



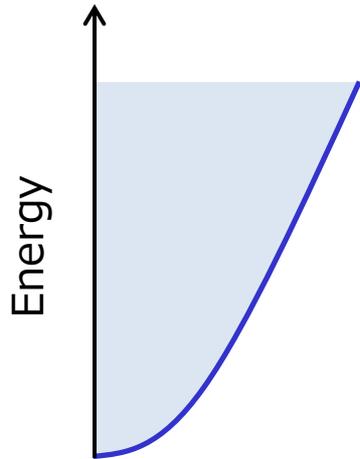
## 変調ドーピングヘテロ界面



フェルミ準位以下の閉じ込め準位(有効質量近似)が1つだけならば  
 $z$ 方向の自由度は凍結されて2次元系と見なせる

# 状態密度(自由電子)の次元性

3次元



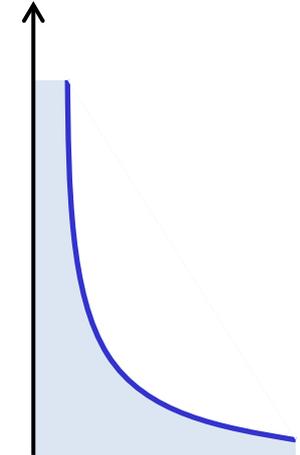
$$D_{3D}(E) = \frac{\sqrt{2m^3 E}}{\pi^2 \hbar^2}$$

2次元



$$D_{2D}(E) = \frac{m}{\pi \hbar^2}$$

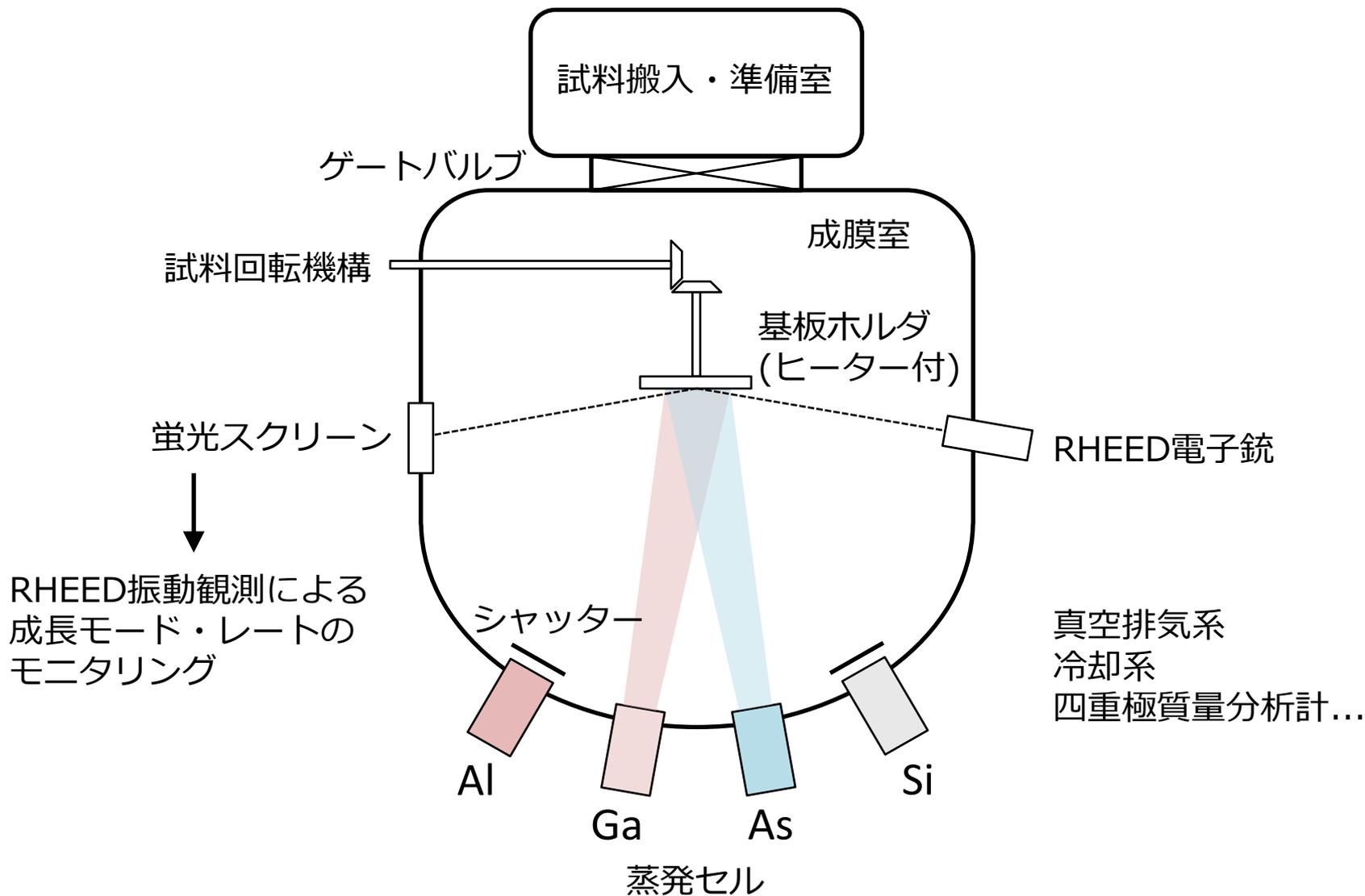
1次元



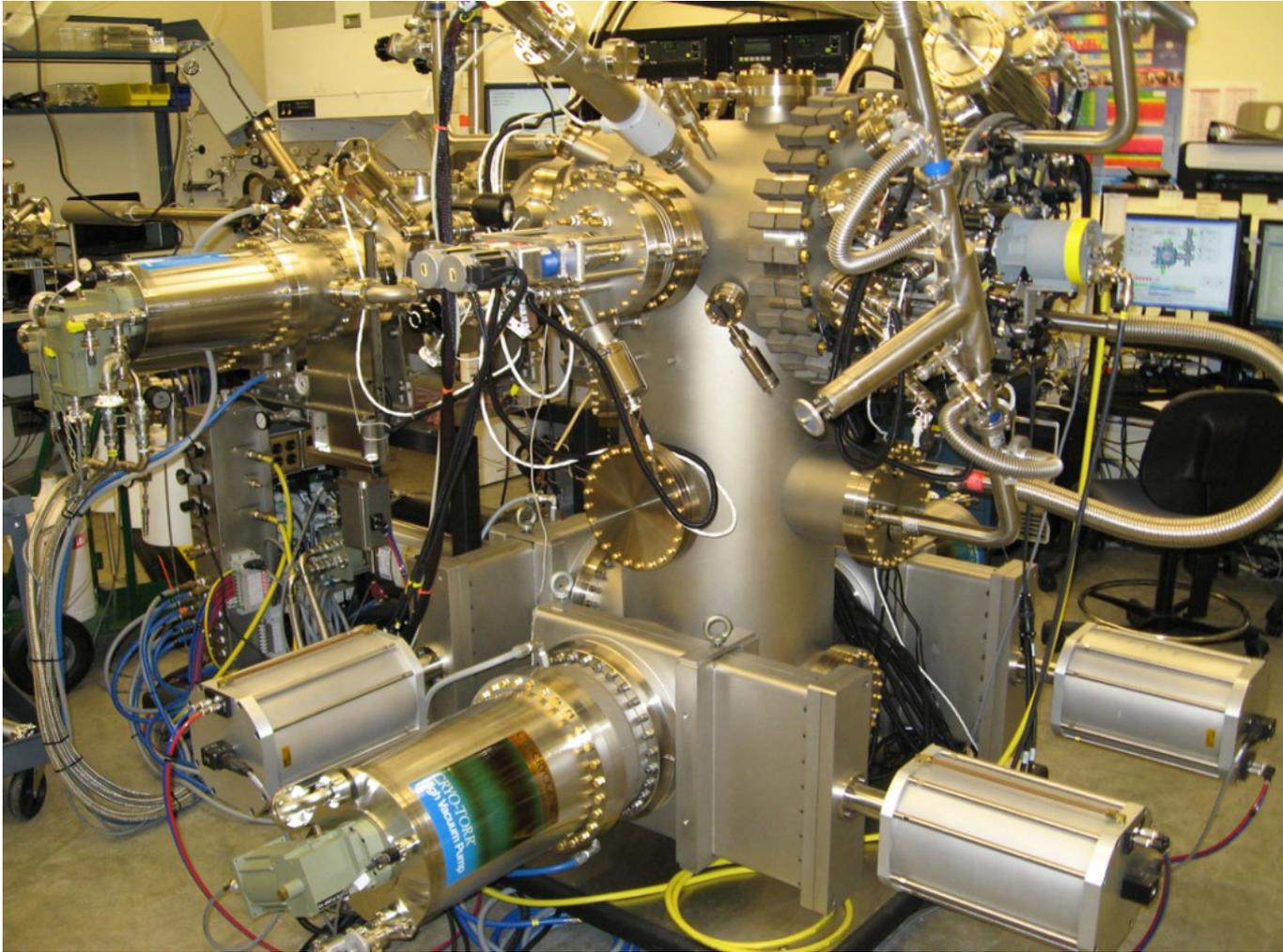
$$D_{1D}(E) = \frac{1}{\pi \hbar} \sqrt{\frac{2m}{E}}$$

次元を下げると低エネルギーでの状態数が増える  
低消費電力(低しきい値)デバイスの実現

# 分子線エピタキシ

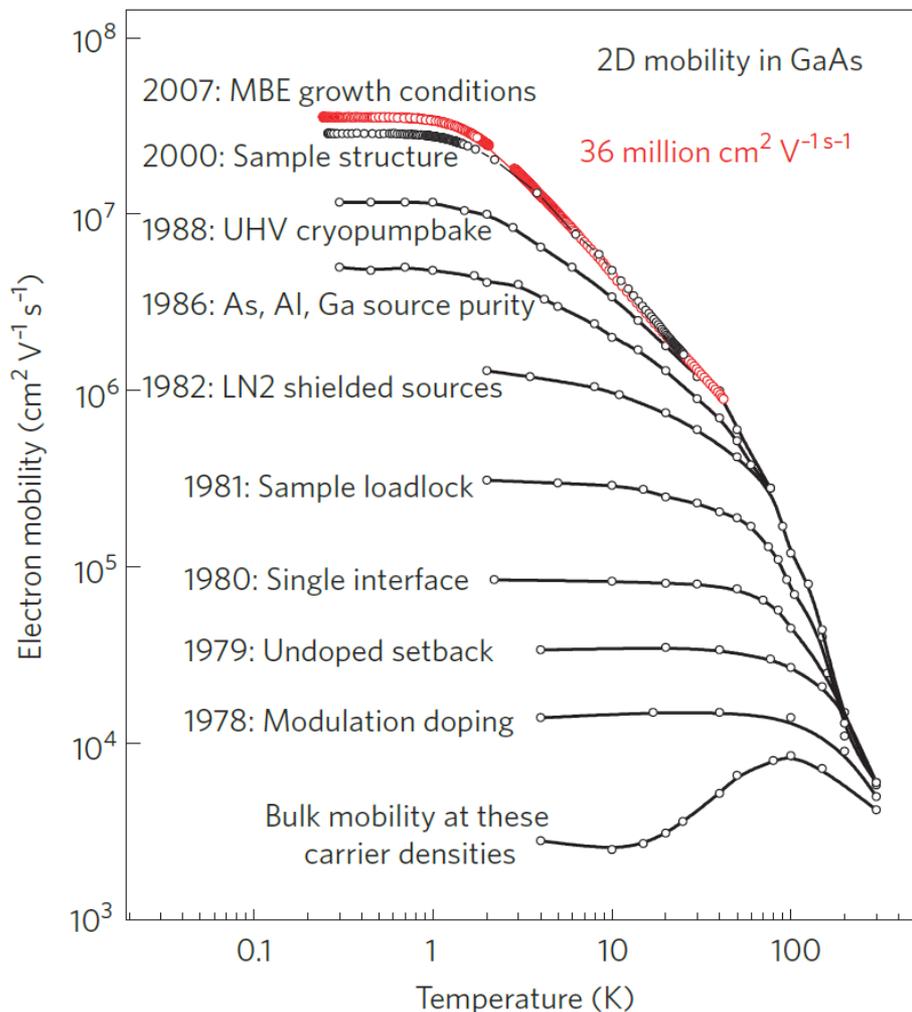


# 分子線エピタキシ



MBE system at Purdue University © Manfra group (<http://manfragroup.org/>)

# 移動度の向上

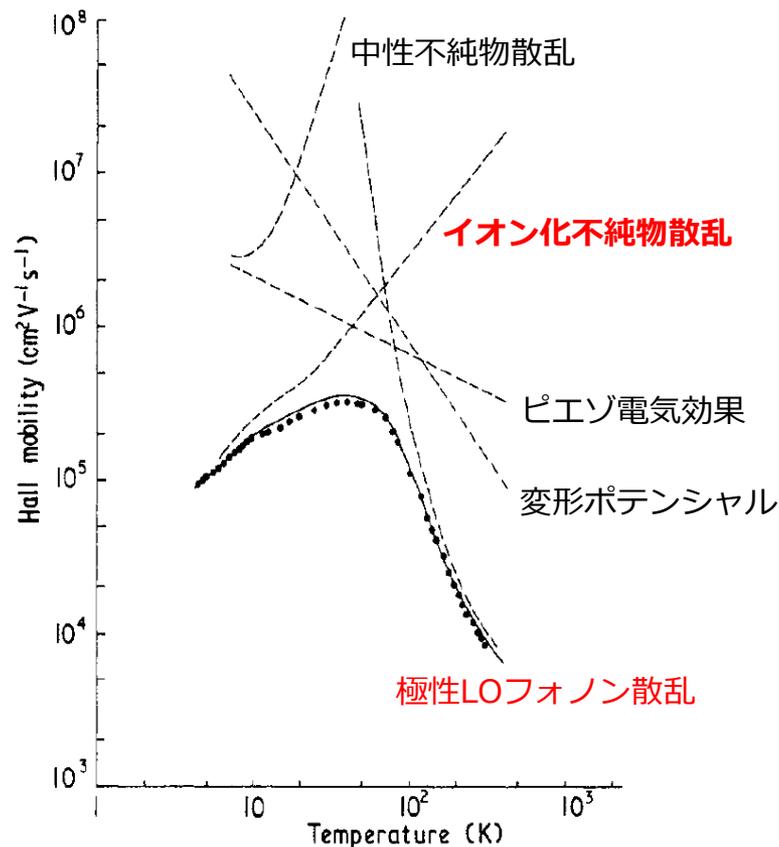


Nature Mat. **9**, 881 (2010) Schlom & Pfeiffer

## バルクGaAsの移動度温度依存性

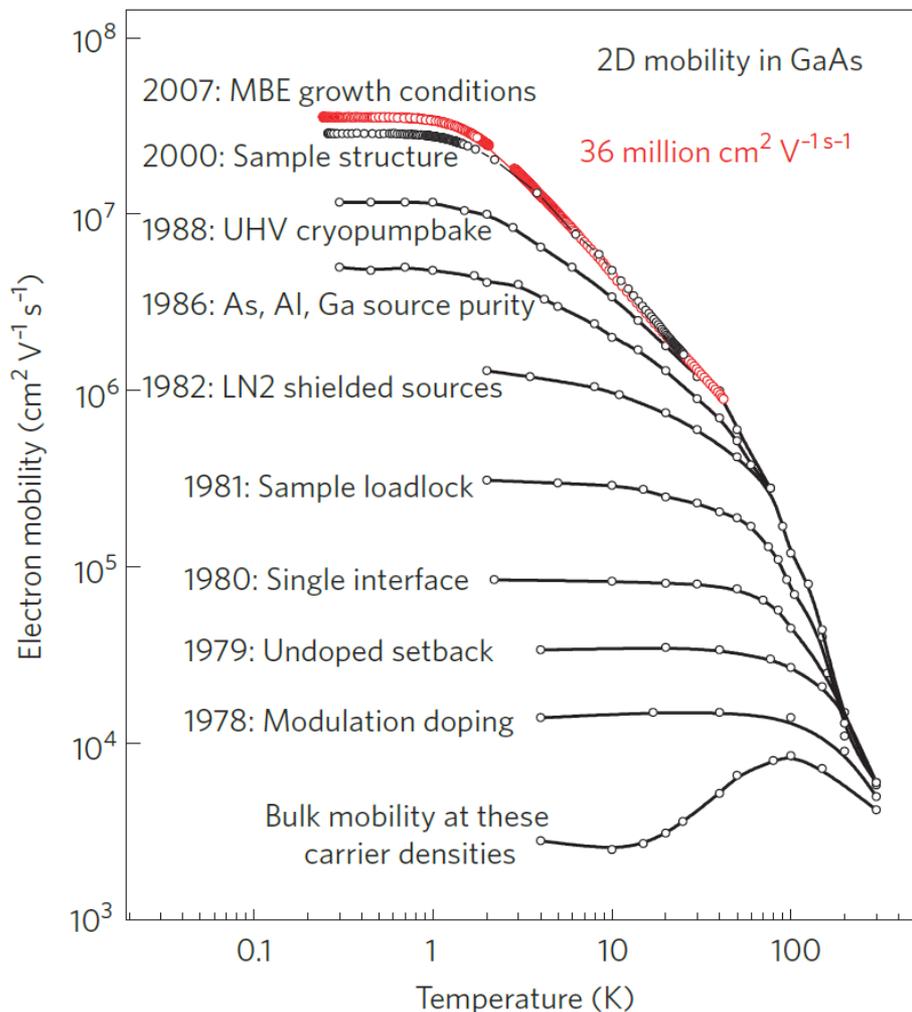
$$N_D = 4.8 \times 10^{13} \text{ cm}^{-3}$$

$$N_A = 2.1 \times 10^{13} \text{ cm}^{-3}$$



J. Phys. C **5**, 212 (1972) Fletcher & Butcher

# 移動度の向上



Nature Mat. **9**, 881 (2010) Schlom & Pfeiffer

$\mu > 3.5 \cdot 10^7 \text{ cm}^2/\text{Vs}$ を達成したグループ

Pfeiffer & West at Princeton (ex-Bell Labs)

Umansky & Heiblum at Weitzman Inst. Sci.  
J. Cryst. Growth **311**, 1658 (2009) Umansky *et al.*

Manfra at Purdue (ex-Bell Labs)  
J. Cryst. Growth **441**, 71 (2016) Gardner *et al.*



Good old days at Bell Labs (1978)  
(左から)Wiegmann, Gossard, Störmer & Dingle  
from Nobel Lecture by Störmer

# HEMT

JAPANESE JOURNAL OF APPLIED PHYSICS  
VOL. 19, No. 5, MAY, 1980 pp. L225-L227

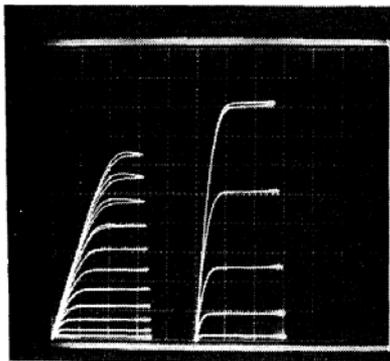
## A New Field-Effect Transistor with Selectively Doped GaAs/n-Al<sub>x</sub>Ga<sub>1-x</sub>As Heterojunctions

Takashi MIMURA, Satoshi HIYAMIZU, Toshio FUJII  
and Kazuo NANBU

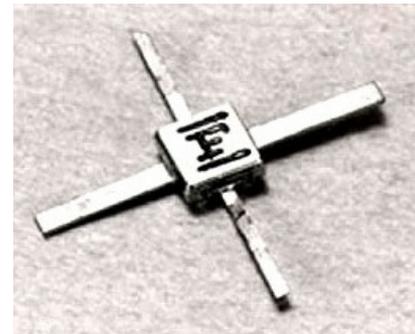
*Fujitsu Laboratories Ltd.,  
1015, Kamikodanaka, Nakahara-ku, Kawasaki 211*

(Received March 24, 1980)

Studies of field-effect control of the high mobility electrons in MBE-grown selectively doped GaAs/n-Al<sub>x</sub>Ga<sub>1-x</sub>As heterojunctions are described. Successful fabrication of a new field-effect transistor, called a high electron mobility transistor (HEMT), with extremely high-speed microwave capabilities is reported.



300 K 77 K



© Fujitsu

# バリステイック伝導

ざっくり言うと、

電子が試料中で不純物による散乱を受けずに流れる輸送現象

**フェルミ波長**  $\lambda_F = \frac{2\pi}{k_F} = \frac{2\pi}{\sqrt{2\pi n_e}} \sim 25 \text{ nm}$  ( $n_e \sim 10^{12} \text{ cm}^{-2}$ )

**平均自由行程**  $l_m = v_F \tau_m = \frac{\hbar k_F}{m^*} \tau_m \sim 30 \text{ }\mu\text{m}$  ( $\tau_m \sim 100 \text{ ps}$ )

**位相緩和長**  $l_\phi = v_F \tau_\phi \sim l_m$  (高移動度試料で  $\tau_\phi \sim \tau_m$ )

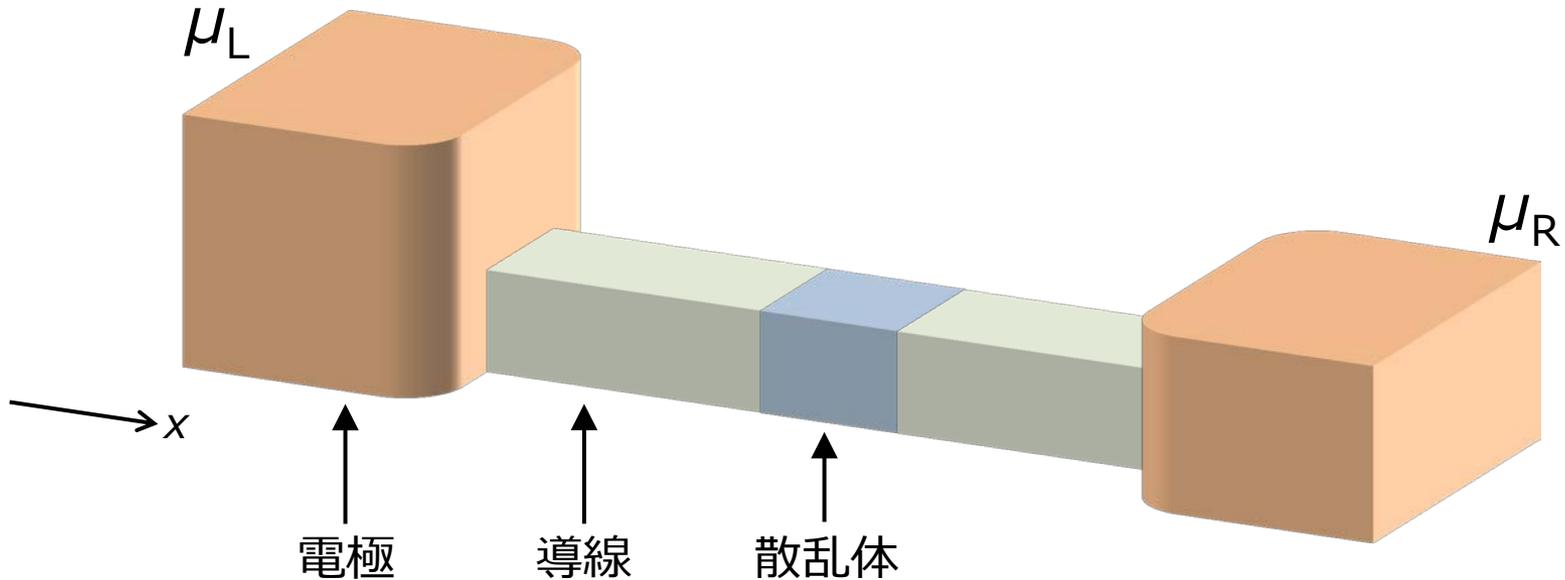
  $L \ll l_m, l_\phi$

低温下の高移動度2DEGで実現

# 講義内容

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# ランダウアー公式

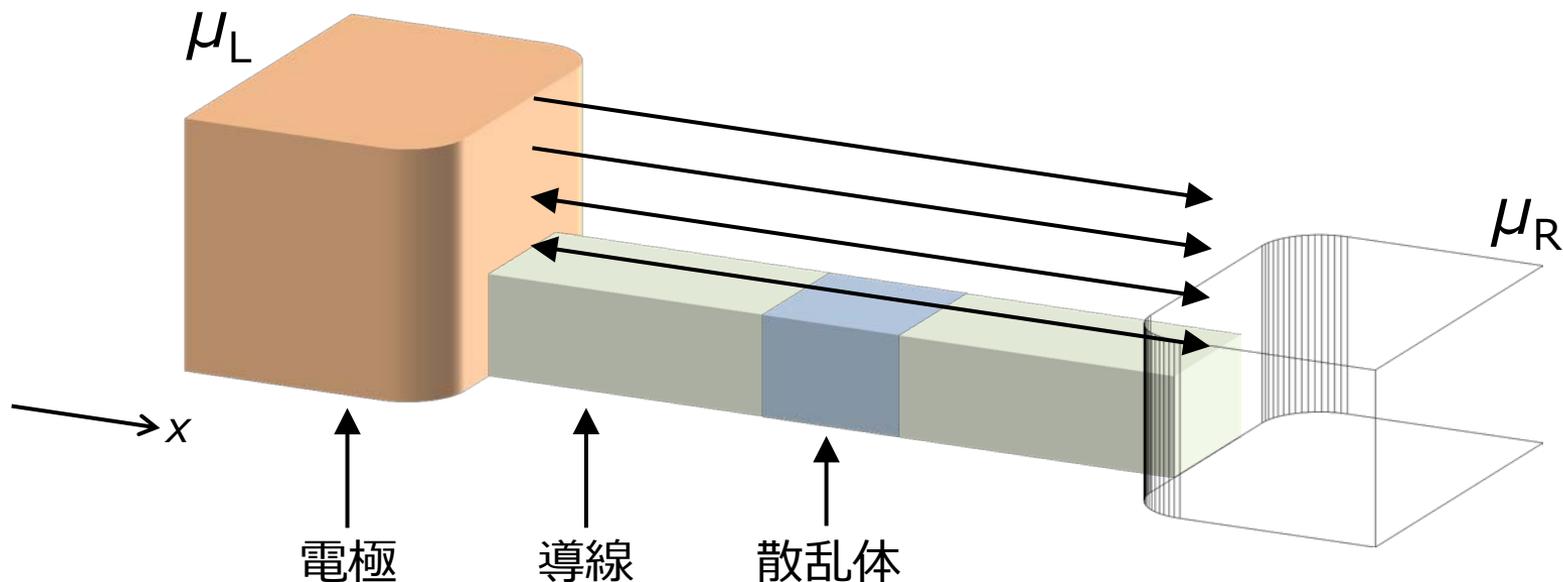


**理想電極(電子溜め):** 導線への電子の供給と吸収を無尽蔵に(熱平衡を保ちながら)行う

**理想導線:** 内部で散乱は起きず、電極から散乱体、散乱体から電極へ電子を受け渡しする

**散乱体:** 確率  $T$  で電子を透過、 $R(=1-T)$  で反射

# ランダウアー公式

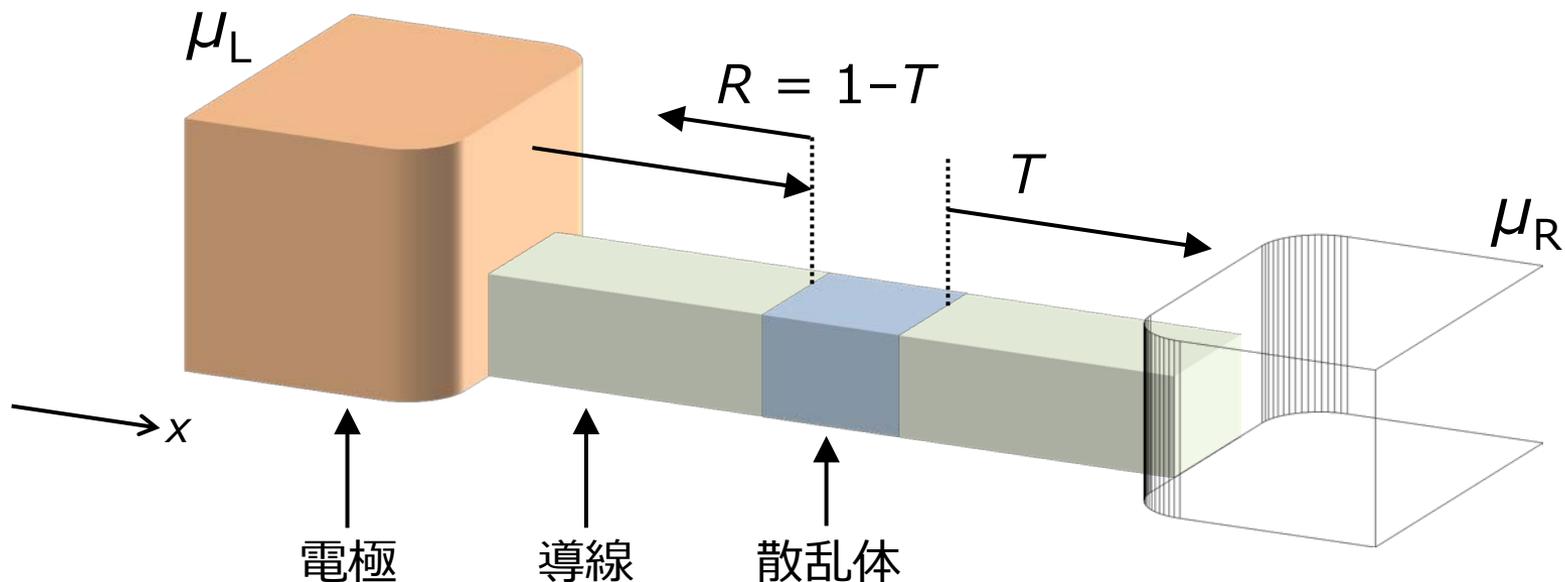


$T = 1$ 、単一モードの場合

$$I = \sum_k i(k) = \int_{k_R}^{k_L} i(k) \frac{L}{2\pi} dk = -\frac{e}{h} \int_{\mu_R}^{\mu_L} dE = -\frac{e}{h} (\mu_L - \mu_R) = \frac{e^2}{h} V$$

$$i(k) = -\frac{e}{L} v_g = -\frac{e}{L} \frac{dE(k)}{\hbar dk} \quad \Delta k = \frac{2\pi}{L}$$

# ランダウアー公式



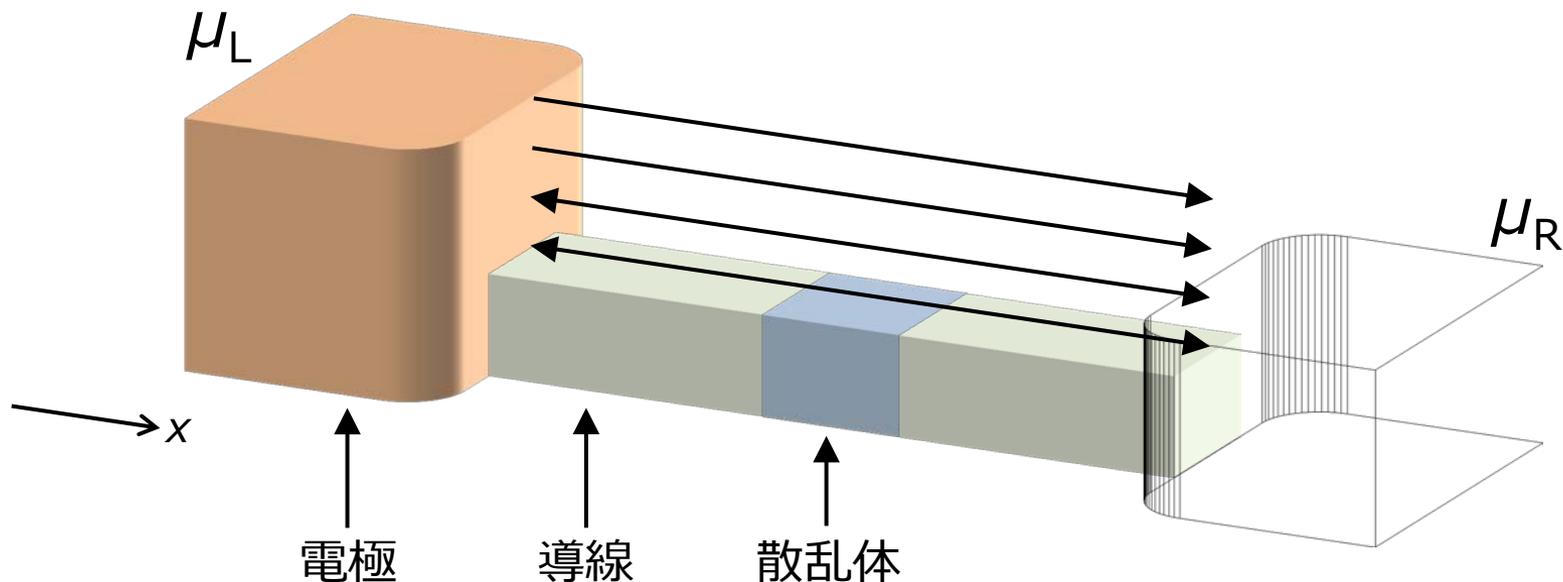
スピンを考慮

透過  $T$ 、モード数  $M$  の場合

$$G = \frac{I}{V} = \frac{2e^2}{h} \quad \longrightarrow \quad G = \frac{2e^2}{h} MT$$

コンダクタンス量子  $g_0 = \frac{2e^2}{h} = 7.7480917346(25) \times 10^{-5} \text{S}$

# ランダウアー公式

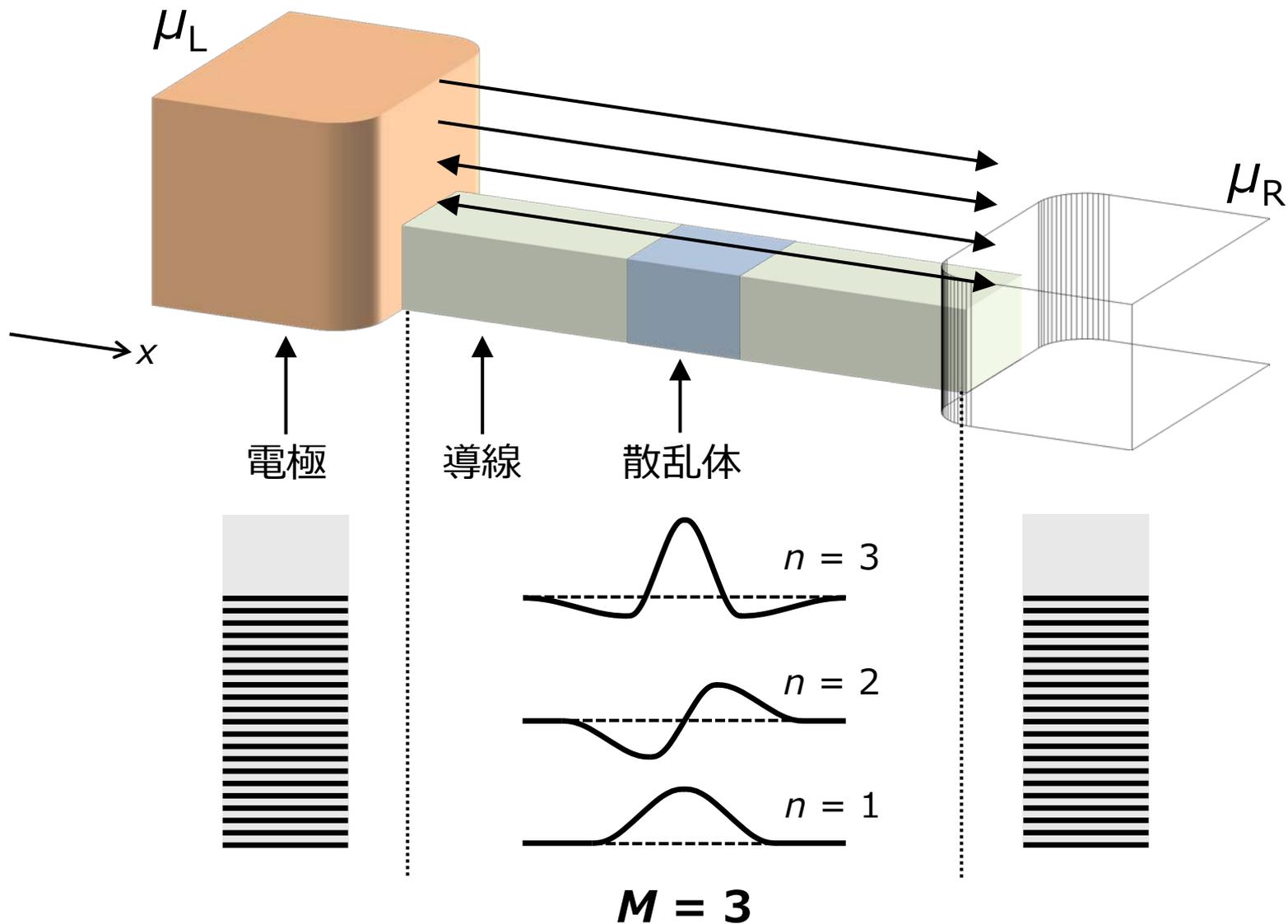


$T = 1$ の場合には散乱がないのに抵抗が存在する

$$G = \frac{2e^2}{h} M \quad \longrightarrow \quad R = \frac{h}{2e^2} \frac{1}{M} = \frac{12.906}{M} \text{ k}\Omega$$

**電極と導線の間”接触抵抗”**

# ランダウアー公式



# 色々な疑問点

- **オームの法則との関係?**
  - 抵抗の起源
  - 散乱体での散逸
  - 散乱体が複数あるケース
- **温度やバイアスの効果?**
- **複数の端子がある場合?**
  - ランダウアー・ビュティカー公式
  - 4端子測定における電圧端子の役割
  - 電子の統計性(パウリの排他律)

などなど…

# ランダウアー公式の初出?

IBM J. Res. Devel. **1**, 223 (1957) Landauer

“Spatial variation of currents and fields due to localized scatterers in metallic conduction”

引用件数3000回以上 (Google Scholar)

J. Math. Phys. **37**, 5259 (1996)に再録時のLandauerによるコメント

The paper ... is not all that easily located in 1996. As a result **the frequent citations** to it often assign content to that paper which does **not agree with reality**. ... My 1957 paper is most often cited in connection with the **now widely used expressions** ... That result is, however, **not contained in the 1957 paper**. ... It took me several more years to understand that the relation between conductance and transmission is general ... It took about another decade after that to get the material accepted by a journal.

cf. Philos. Mag. **21**, 863 (1970) Landauer

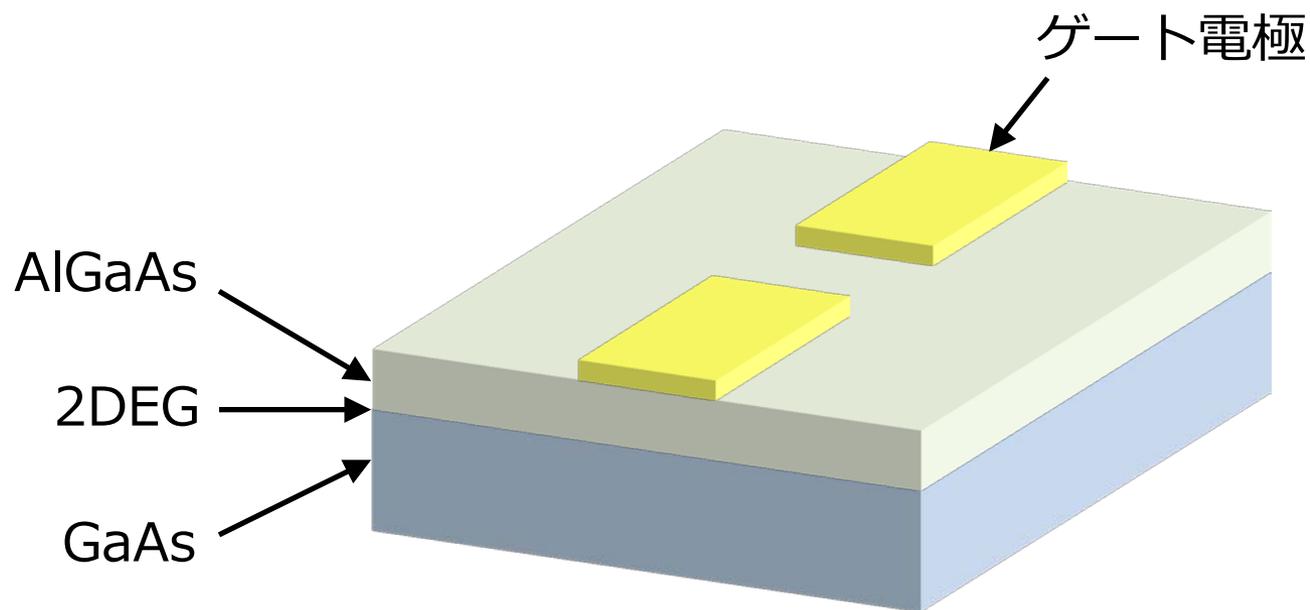
Phys. Rev. B **22**, 3519 (1980) Anderson *et al.*

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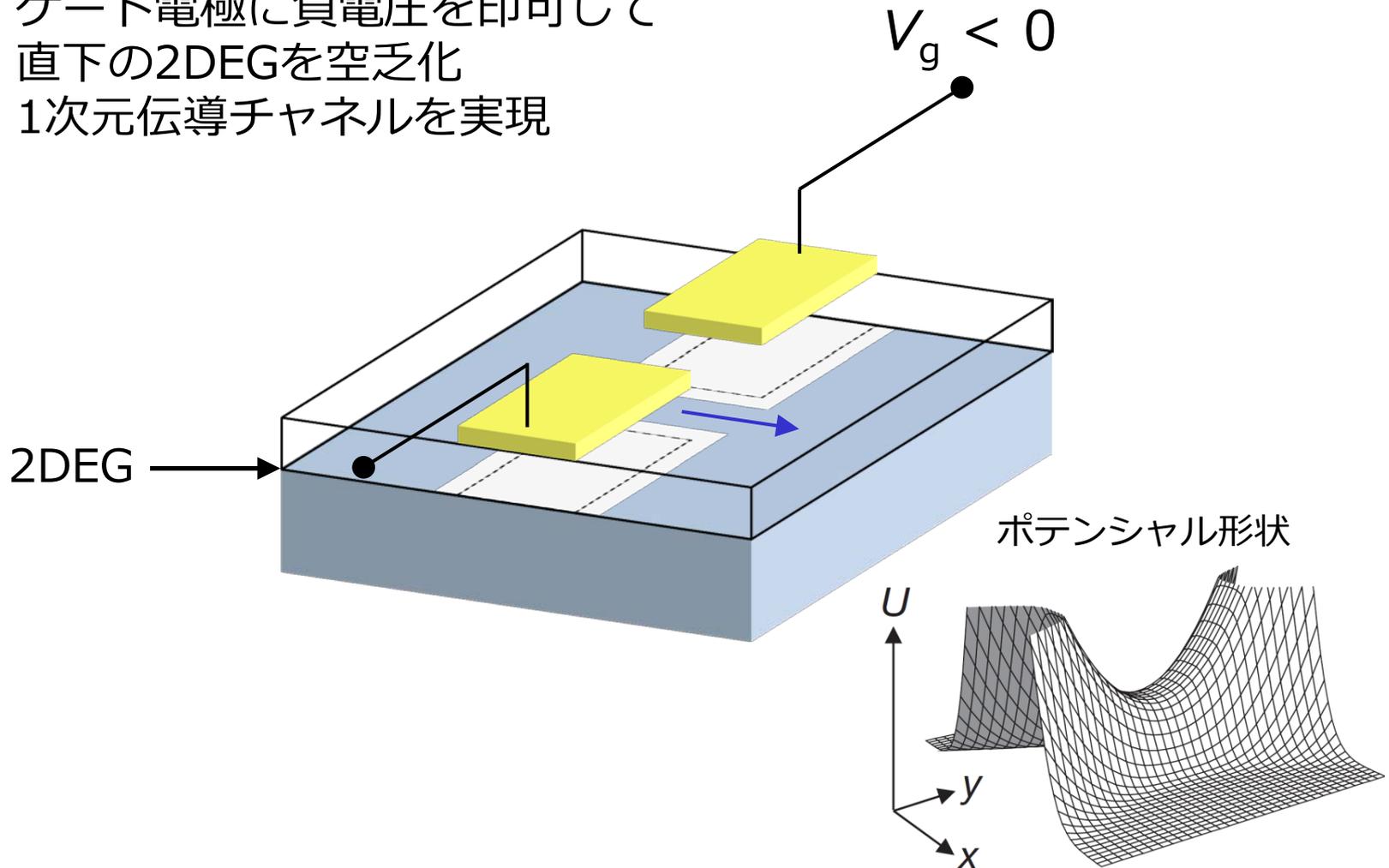
# 量子ポイントコンタクト

Quantum Point Contact = QPC



# 量子ポイントコンタクト

- ゲート電極に負電圧を印可して直下の2DEGを空乏化
- 1次元伝導チャネルを実現



# 典型的なナノ構造作製手順

## 1. メサ構造作製

- フォトリソグラフィ
- 電流の流れを欲しくない場所を化学エッチングで取り除いて空乏化する

## 2. オーミック電極形成

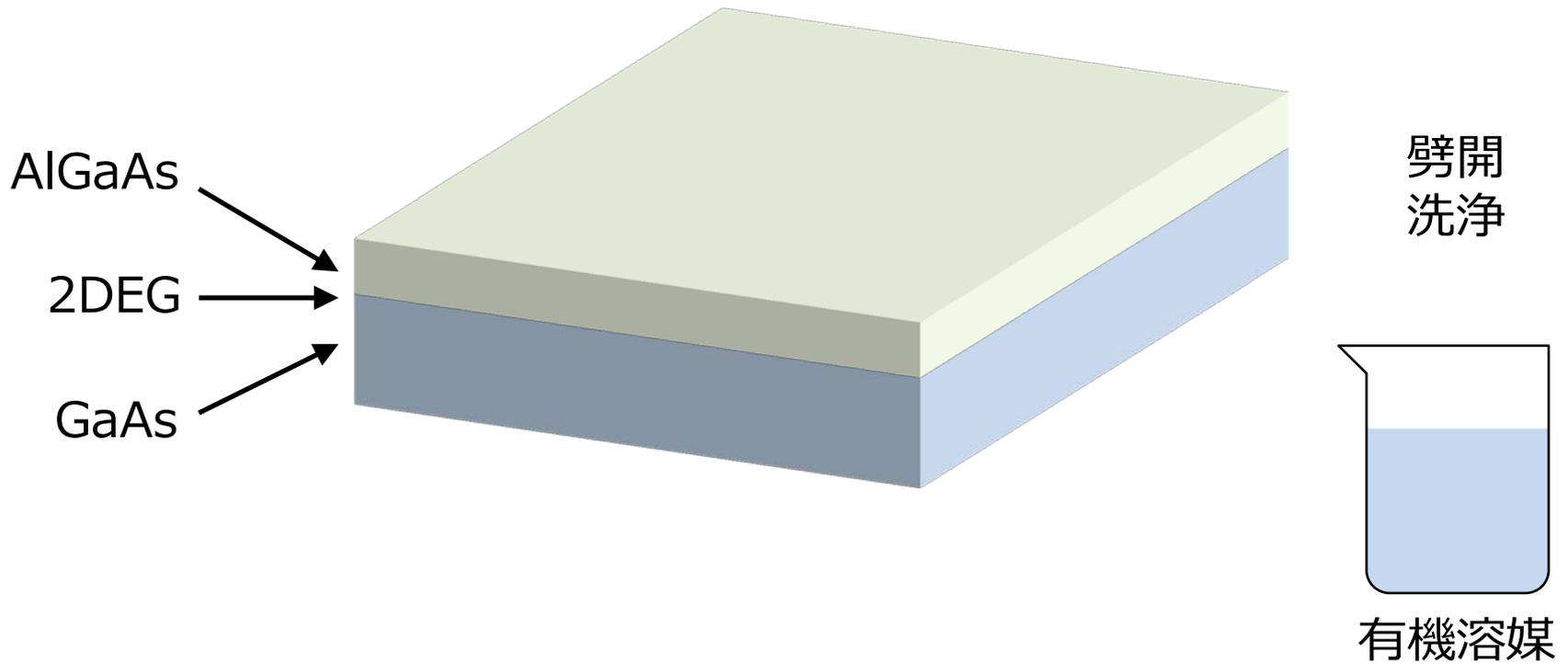
- フォトリソグラフィ
- 基板表面と2次元電子のいるヘテロ界面を繋ぐ

## 3. ショットキー電極作製

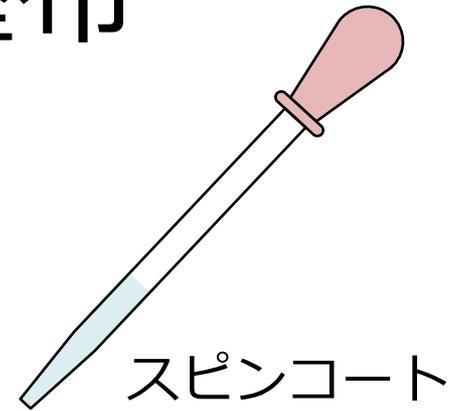
- 電子線リソグラフィ
- 基板表面に配置するゲート電極によりナノ構造形成

細かいファブレシビは多岐に渡る

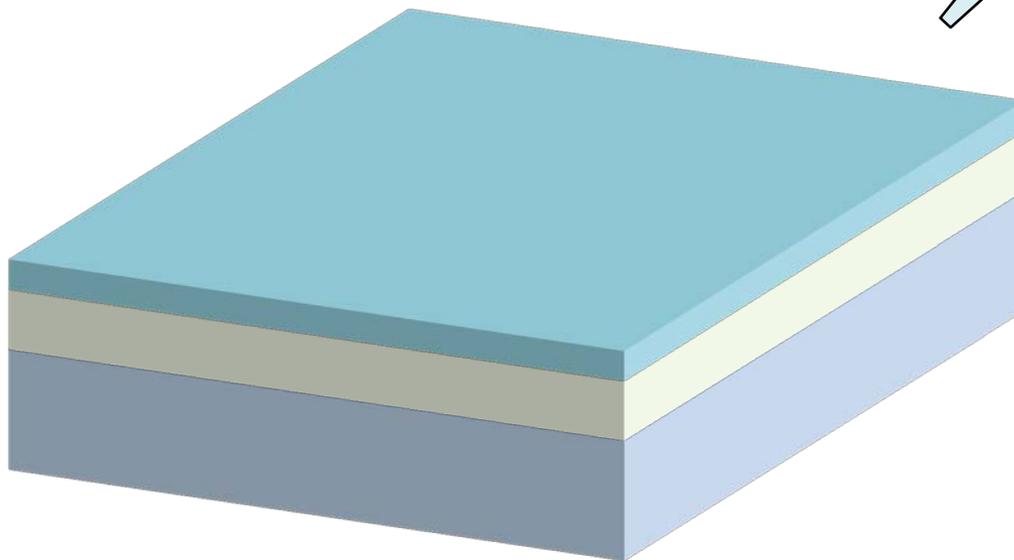
# ヘテロ構造基板



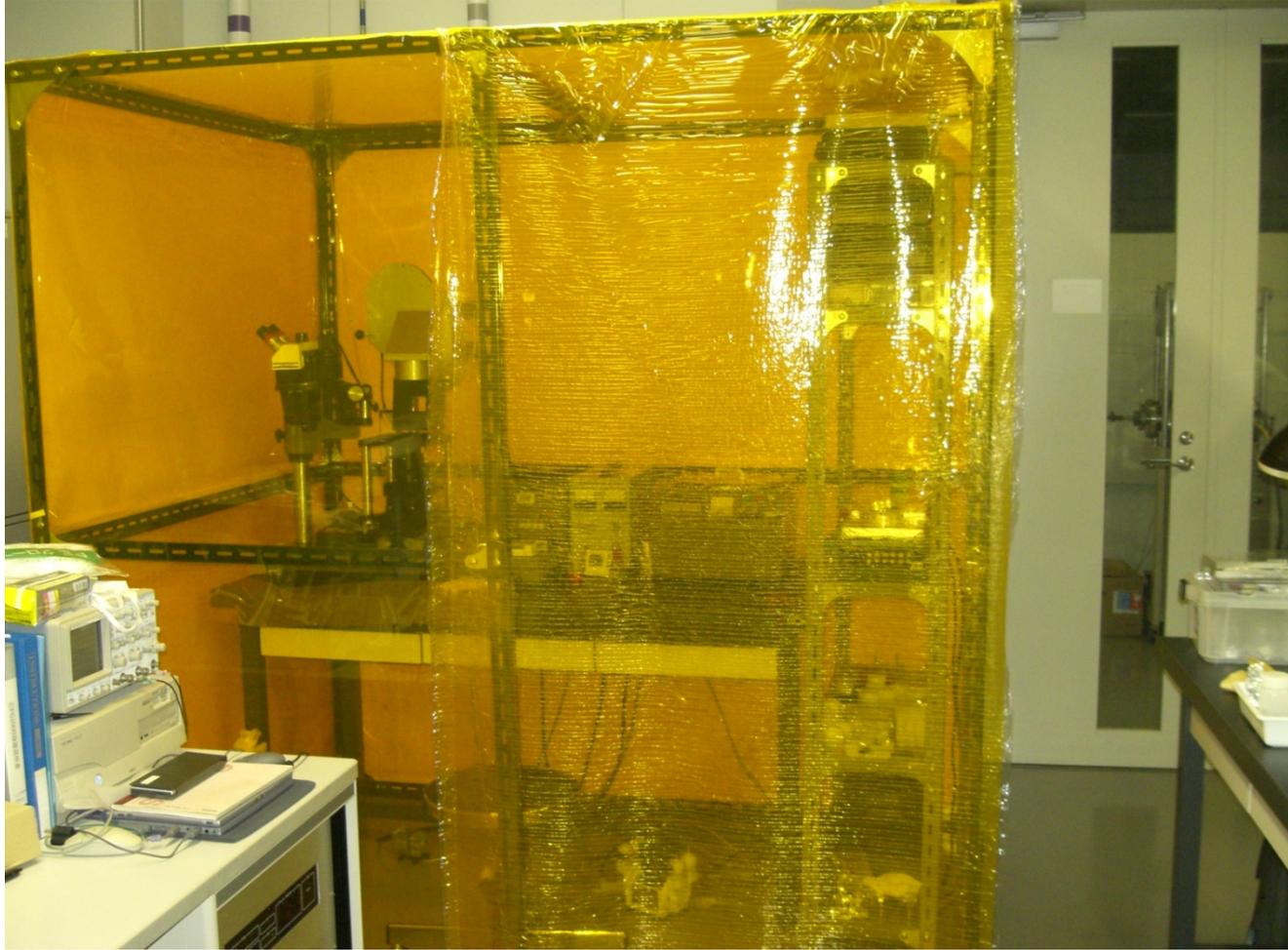
# フォトレジスト塗布



レジスト(ポジ)



# フォトリソグラフィ

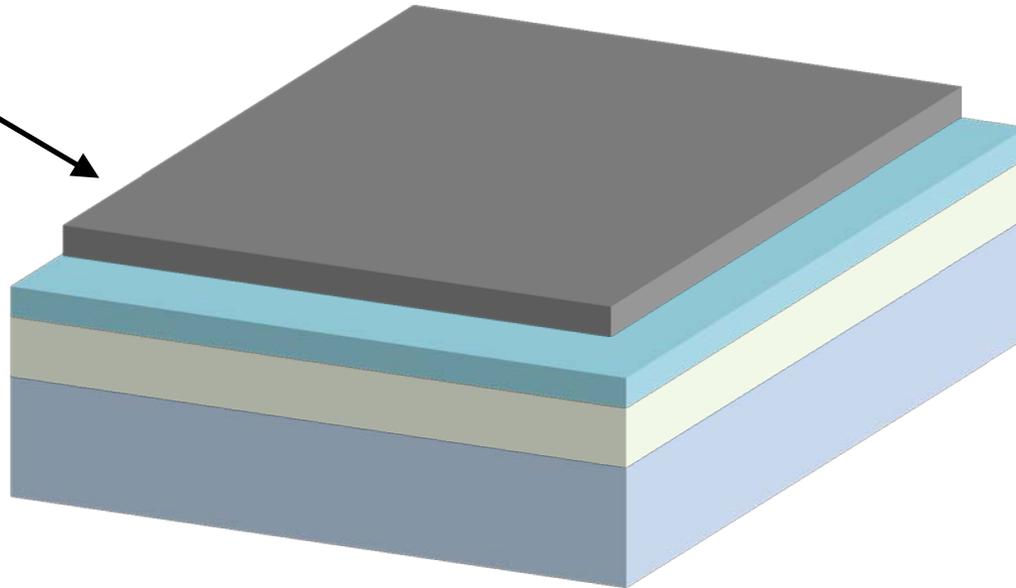
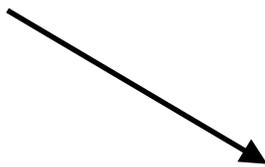


東京大学 物性研究所 家・勝本研究室 (2008)

# メサ構造用マスクして露光

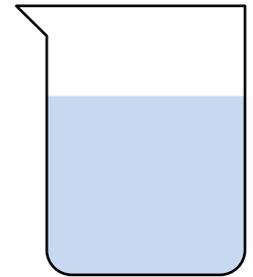
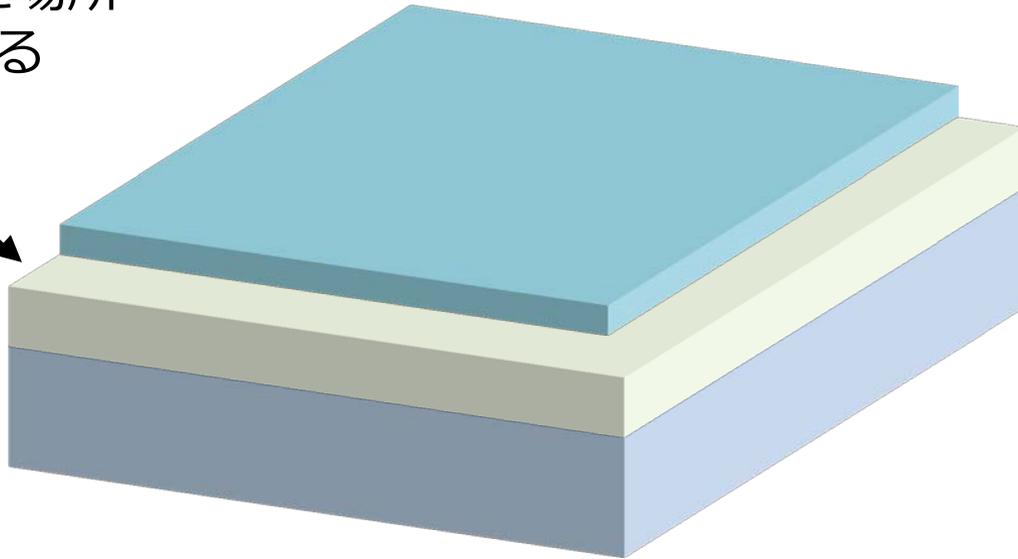
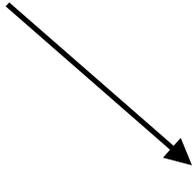


マスク



# 現像

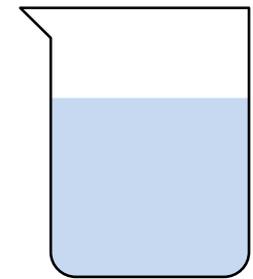
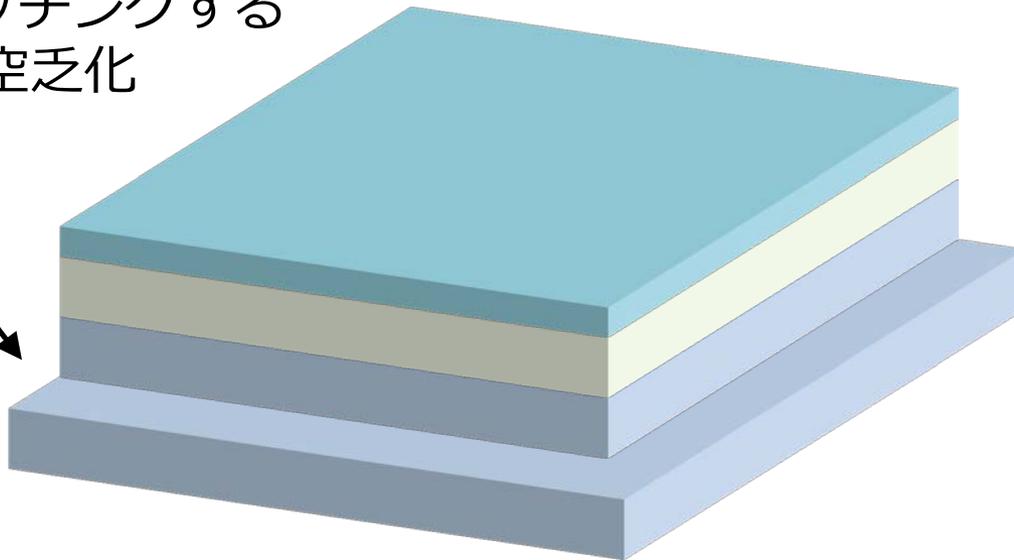
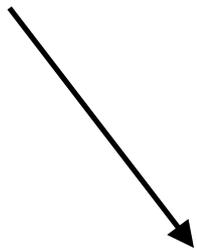
UVの照射された場所  
のみ取り除かれる



現像液

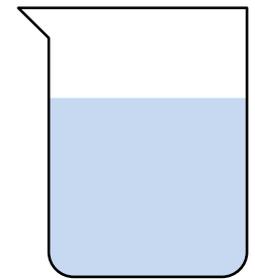
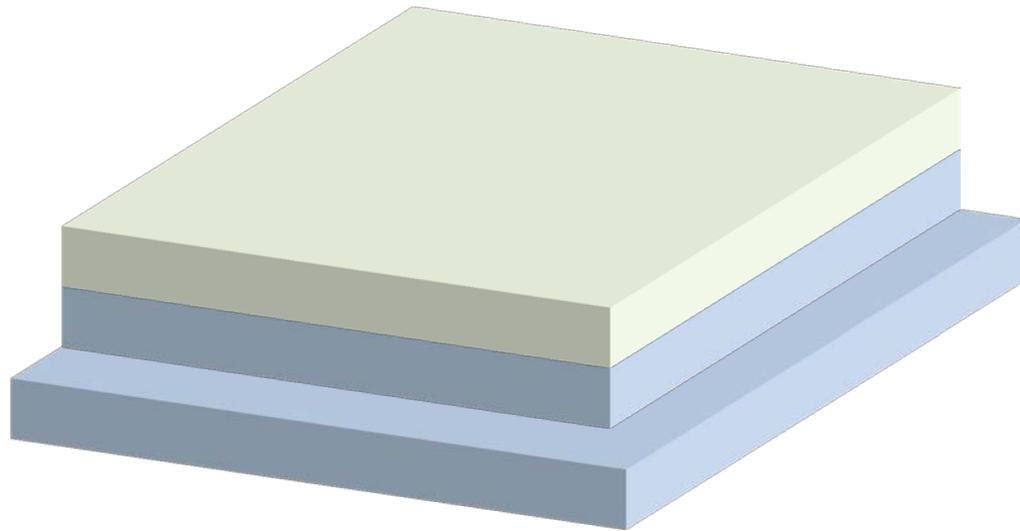
# 化学エッチング

GaAs層までエッチングすること  
で2DEGを空乏化



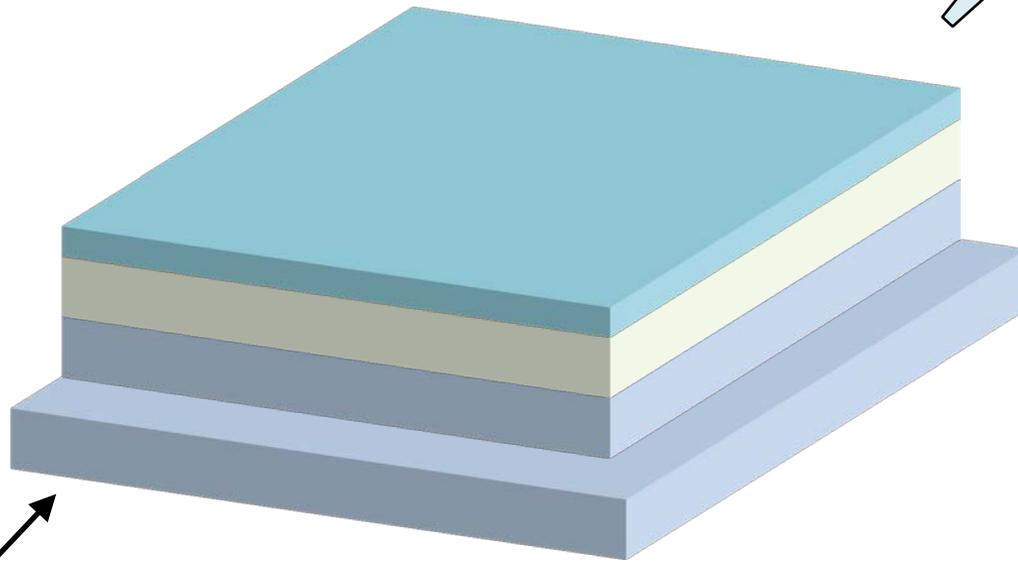
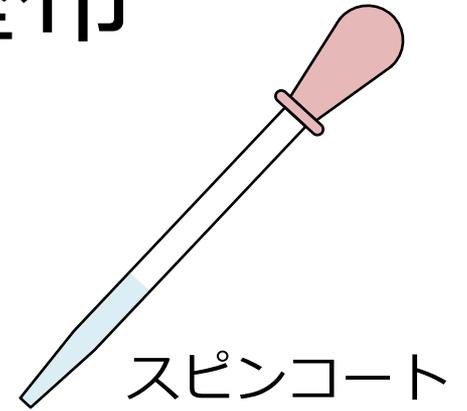
酸混合液

# リフトオフ



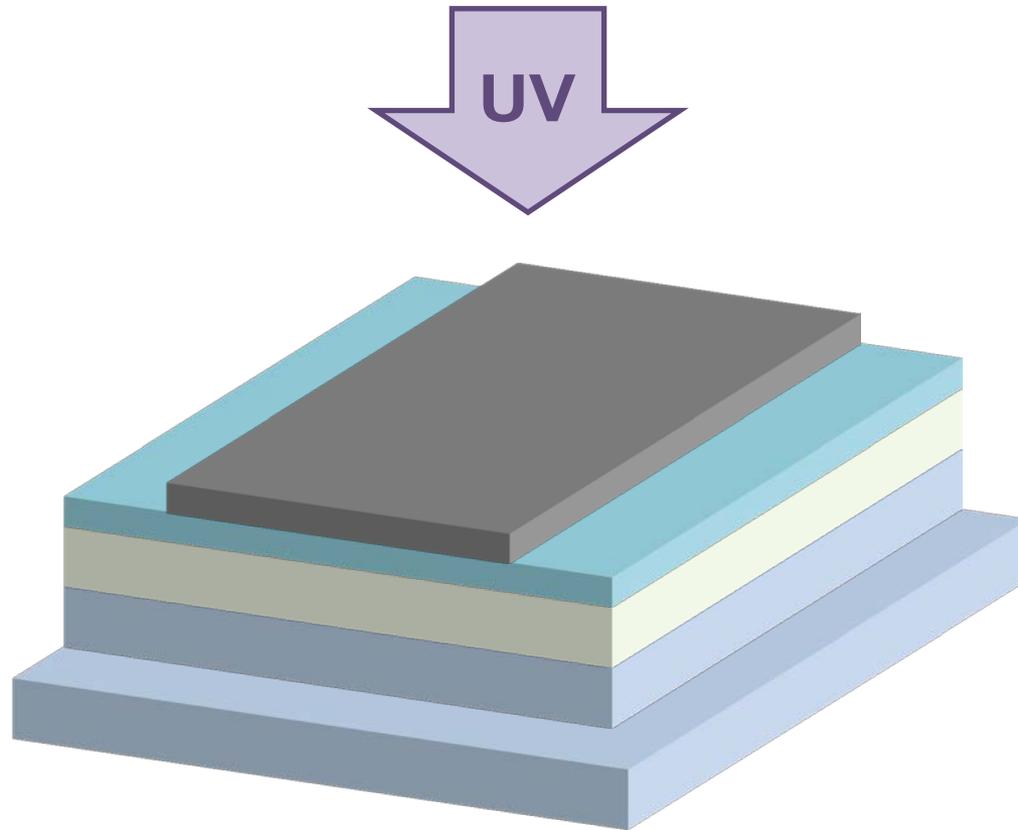
リムーバ

# フォトレジスト塗布

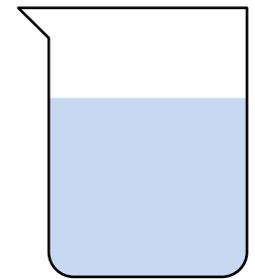
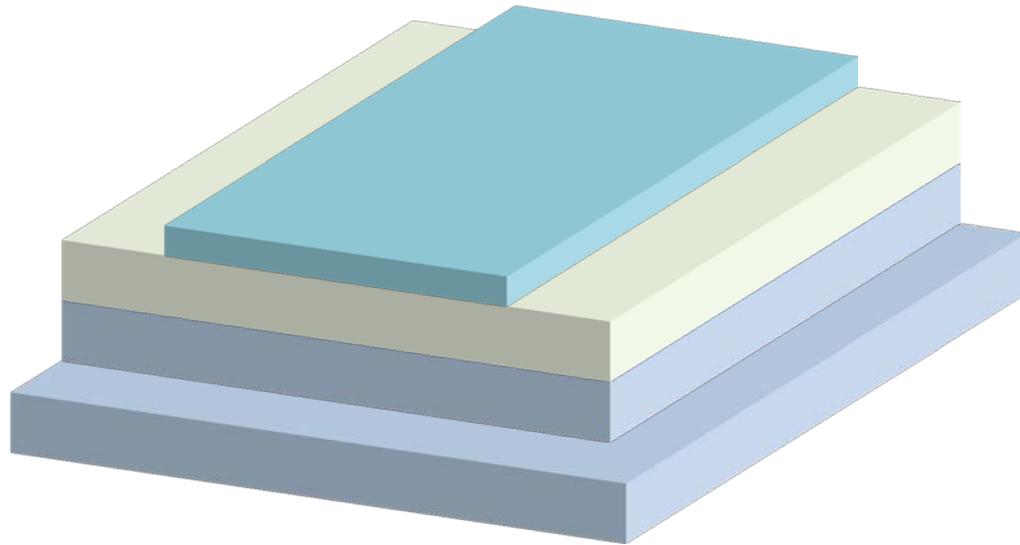


(実際には空乏化した場所にもレジストが塗布される)

# 電極構造用マスクして露光



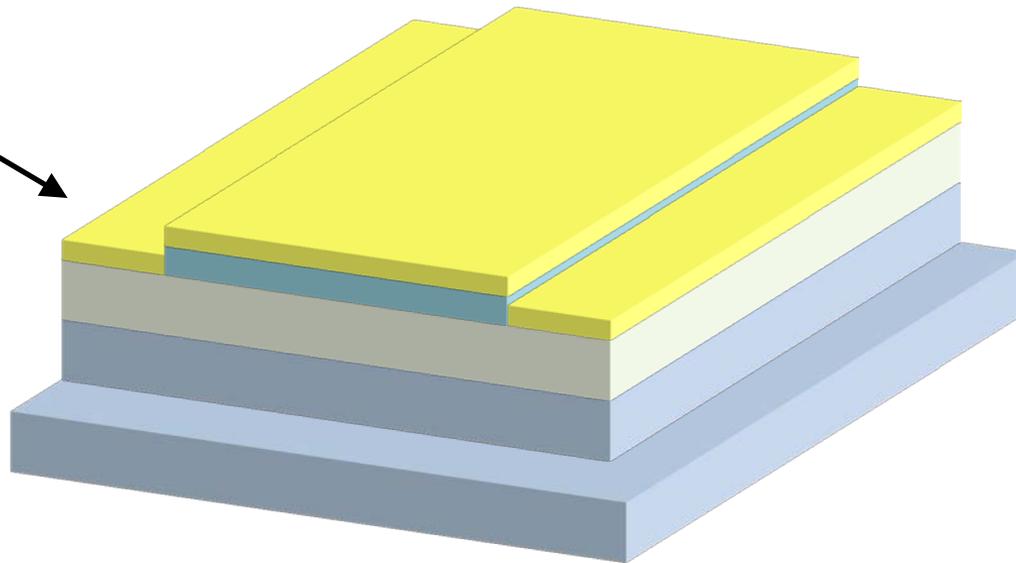
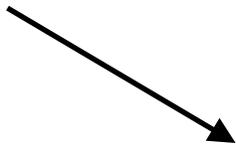
# 現像



現像液

# オーミック電極用金属を蒸着

AuGe/Ni

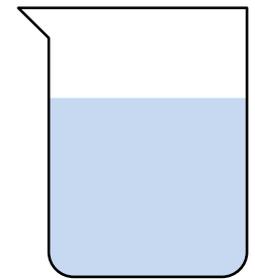
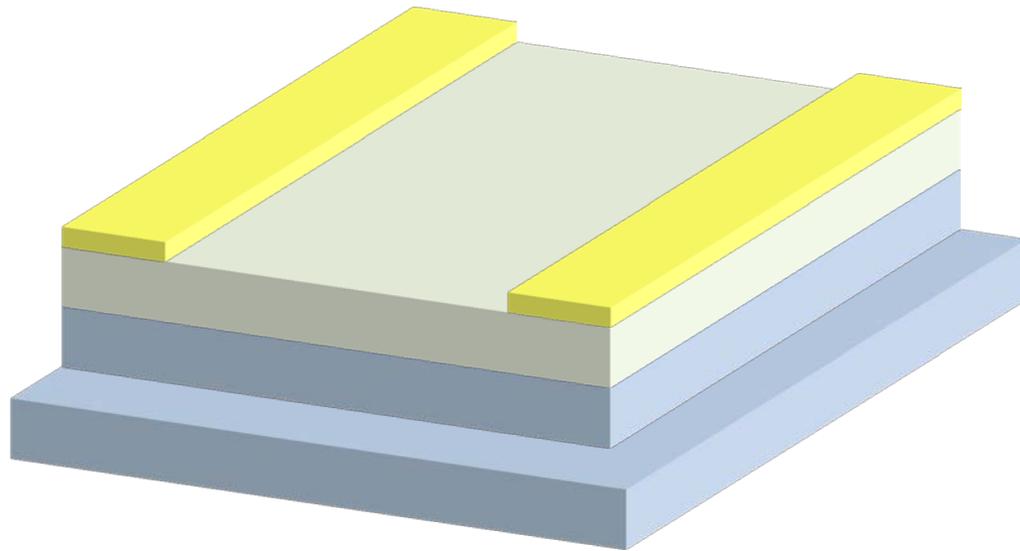


# イオンビームスパッタ



東京大学 物性研究所 家・勝本研究室 (2008)

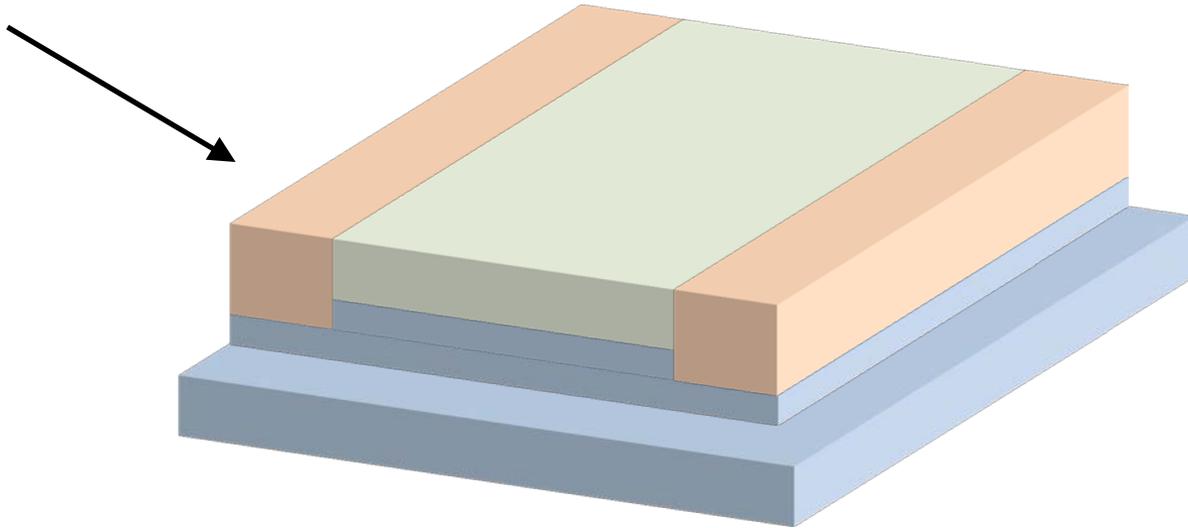
# リフトオフ



リムーバ

# アニール(オーミック電極形成)

2DEGまで拡散して合金化

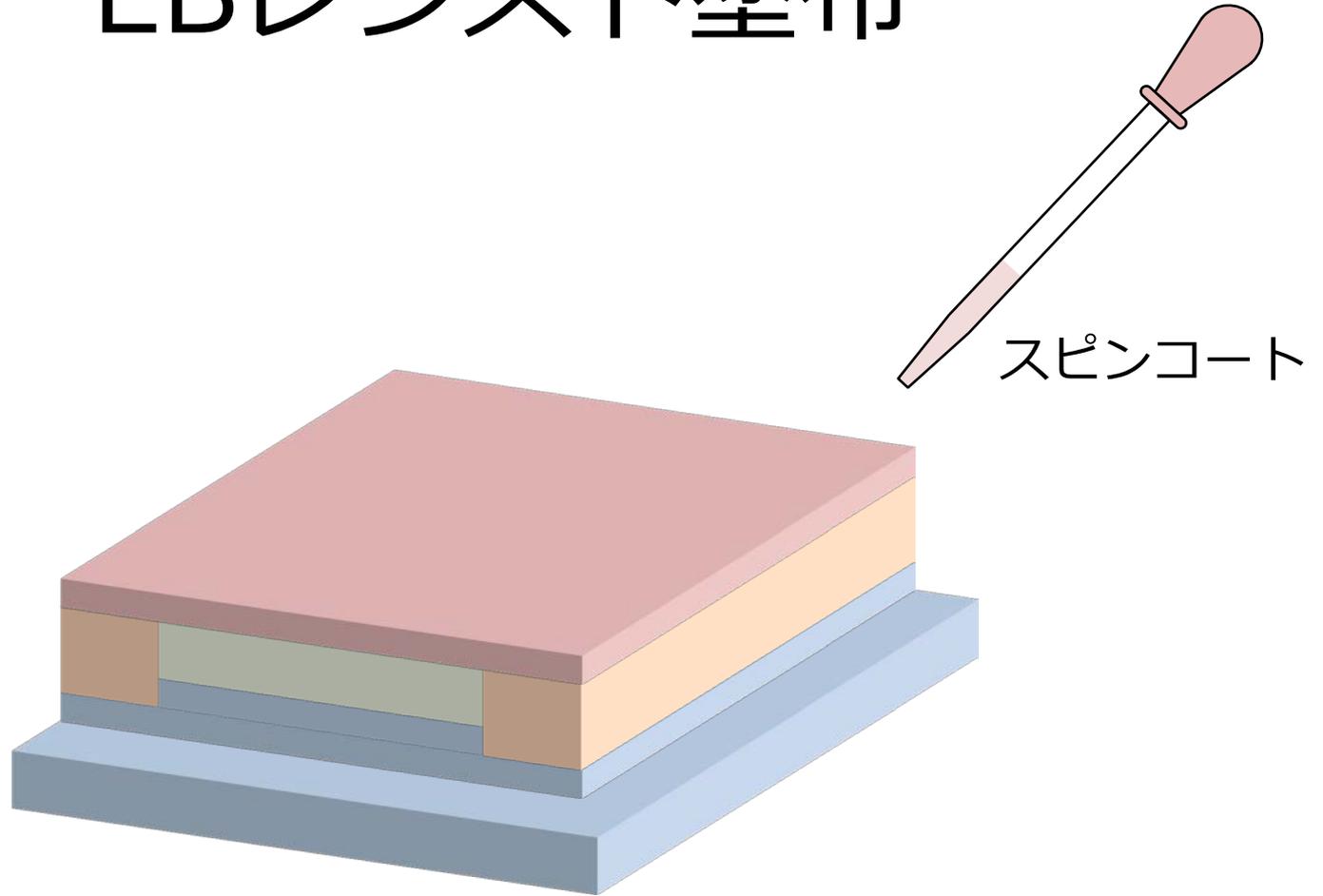


# アニール炉

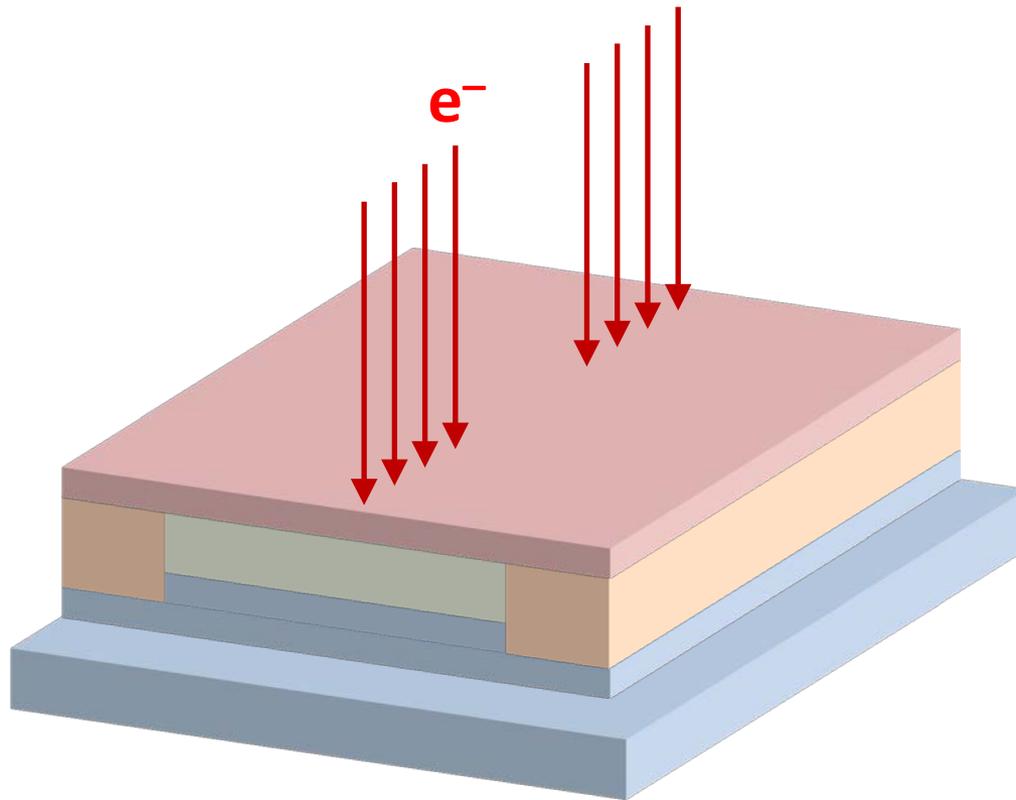


東京大学 物性研究所 家・勝本研究室 (2008)

# EBレジスト塗布



# ナノ構造を電子線描画

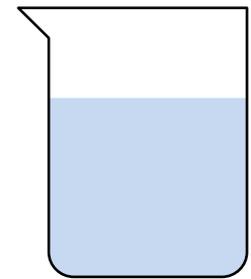
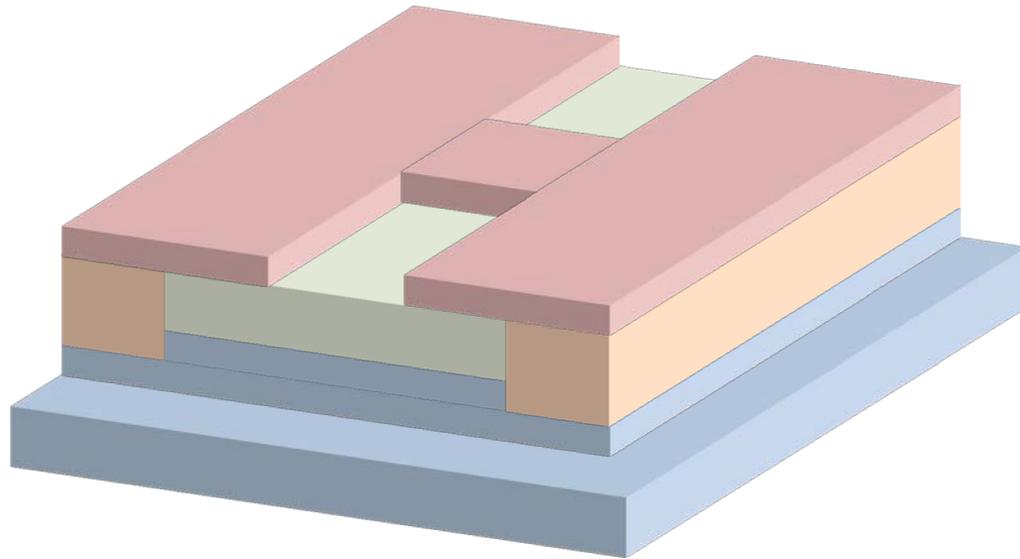


# 電子線リソグラフィ装置



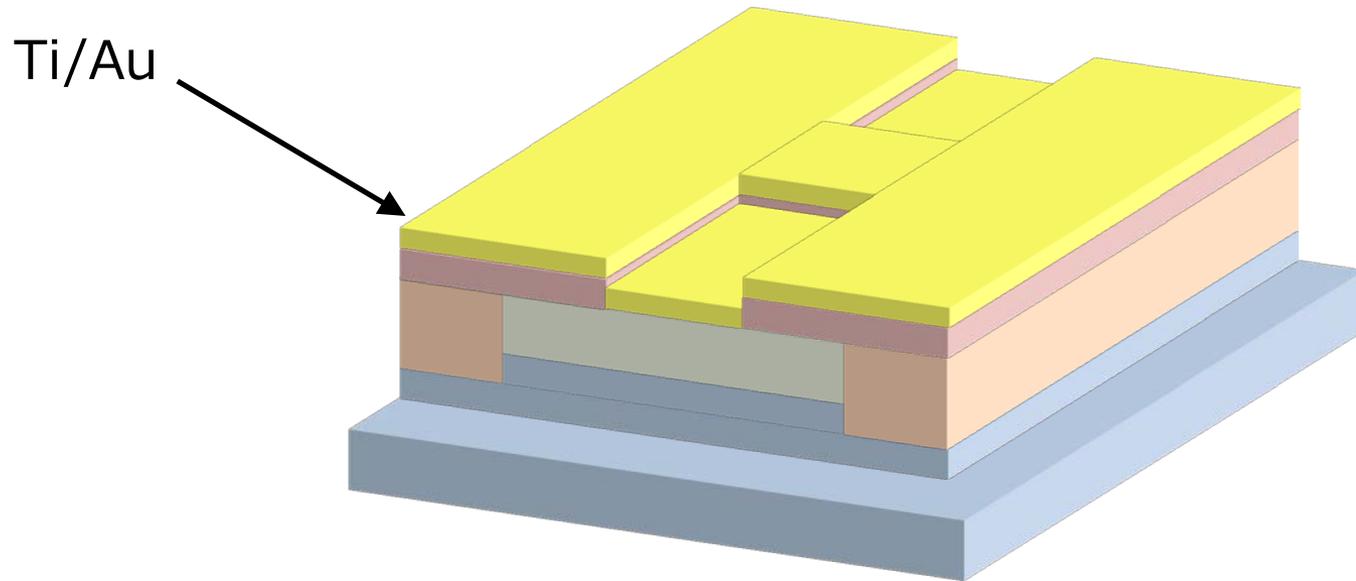
東京大学 物性研究所 家・勝本研究室 (2008)

# 現像



現像液

# 金属蒸着

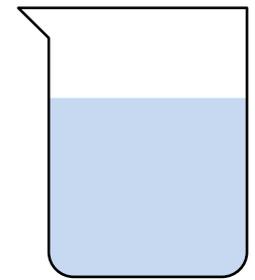
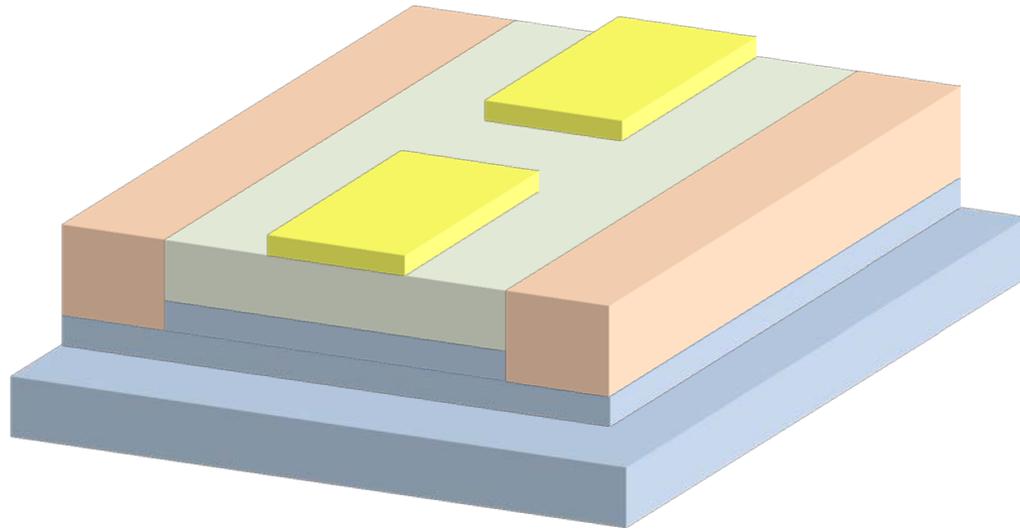


# 電子線蒸着装置



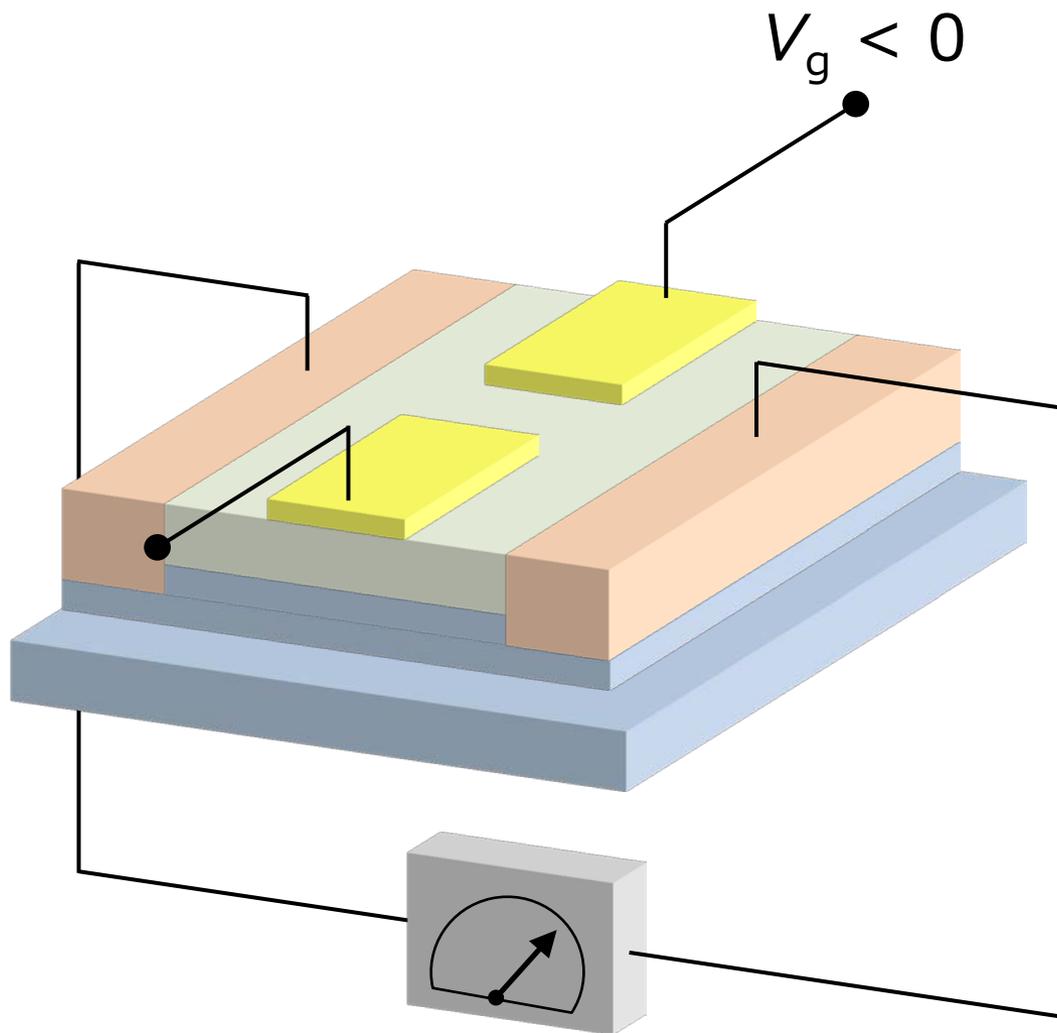
東京大学 物性研究所 家・勝本研究室 (2008)

# リフトオフ



リムーバ

# ワイヤボンディングして測定



# ワイヤボンダー



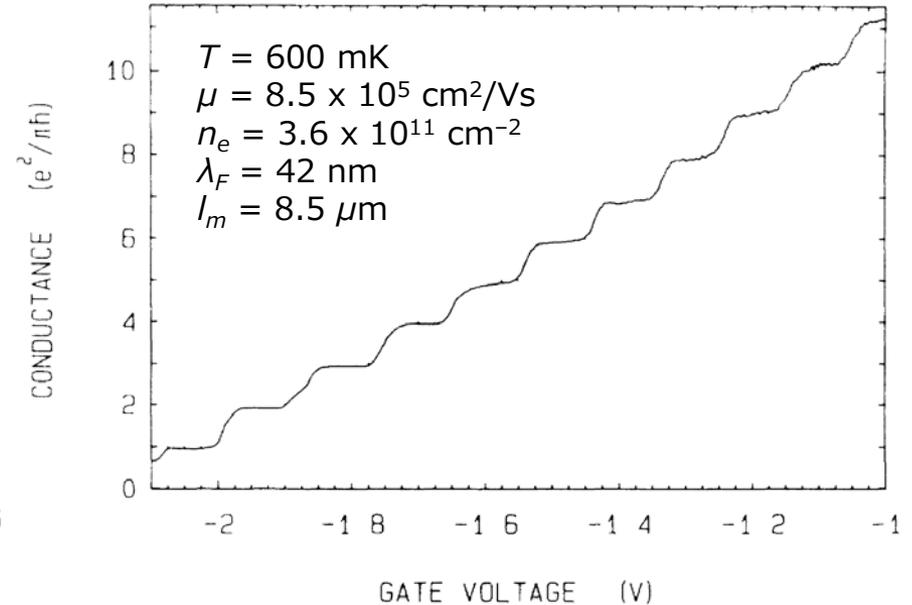
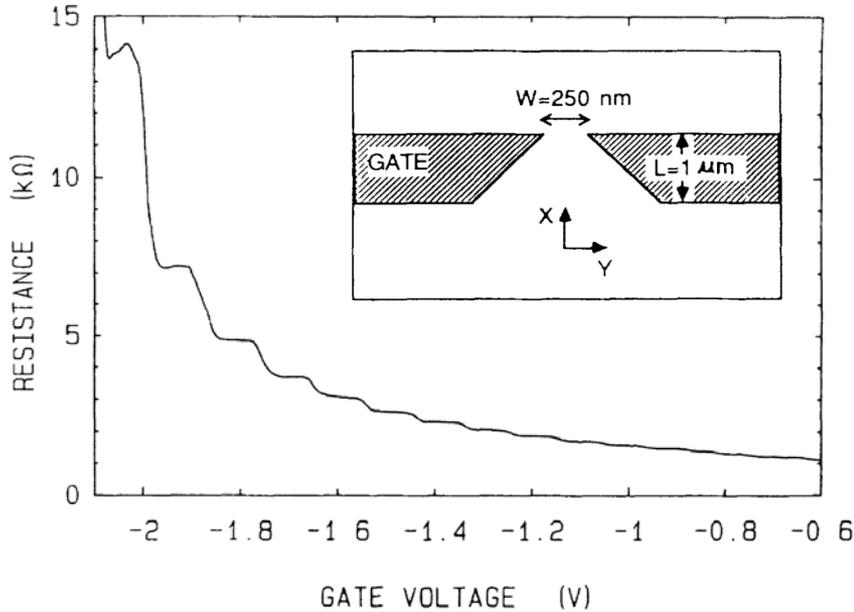
東京大学 物性研究所 家・勝本研究室 (2008)

# 低温強磁場測定系



東京大学 物性研究所 家・勝本研究室 (2008)

# コンダクタンスの量子化



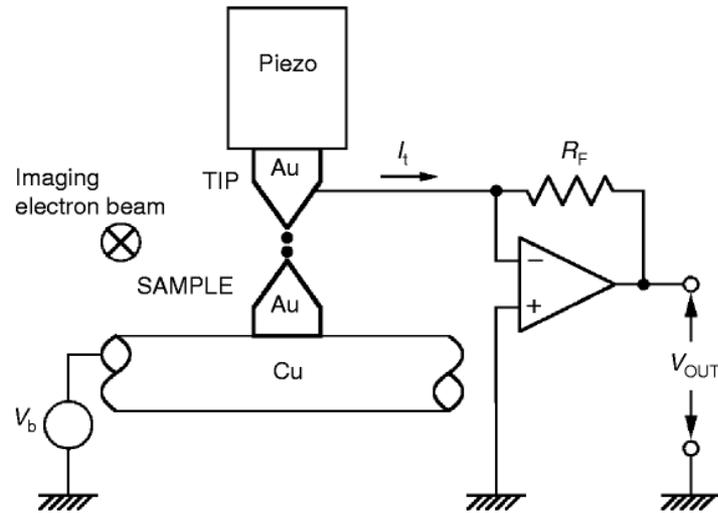
$$\frac{e^2}{\pi\hbar} M = \frac{2e^2}{h} M = g_0 M \text{ でプラトー}$$

**ランダウアー公式そのもの ( $T = 1$ )**

*"Unexpectedly, plateaus are found in the resistance."*

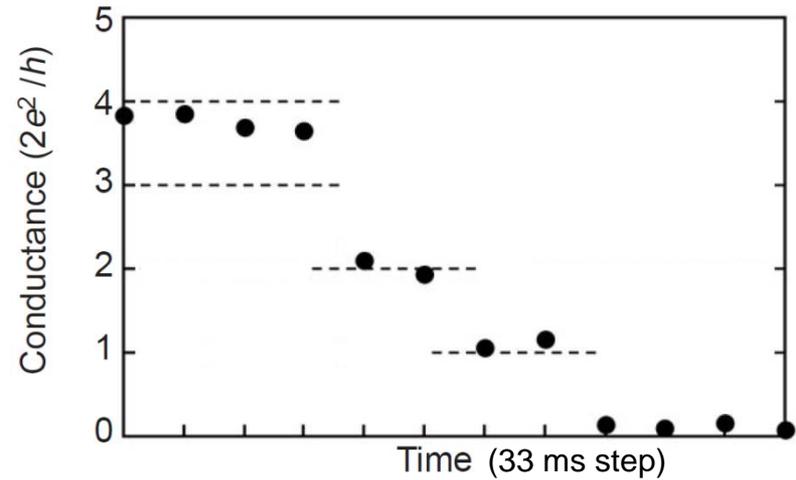
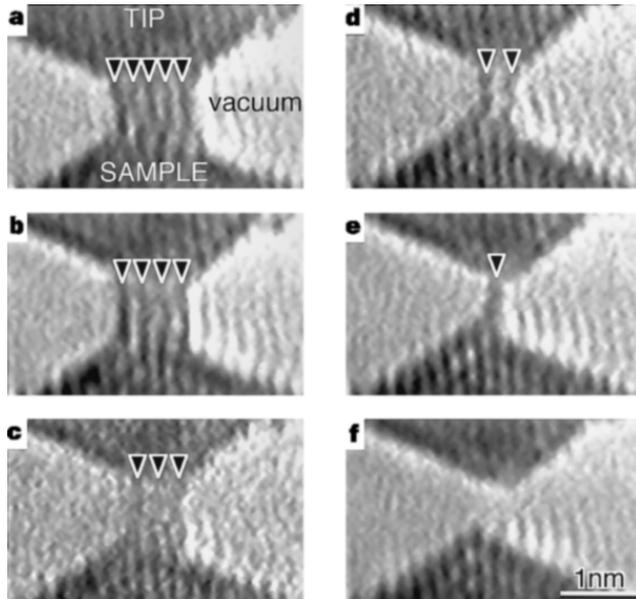
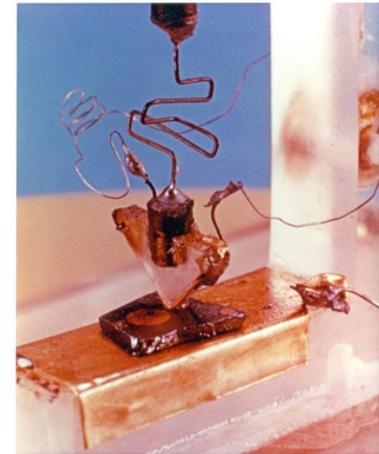
*"The findings ... may imply that we have realized an experimental system which closely approximates the behavior of idealized mesoscopic systems."*

# cf. 金属の場合



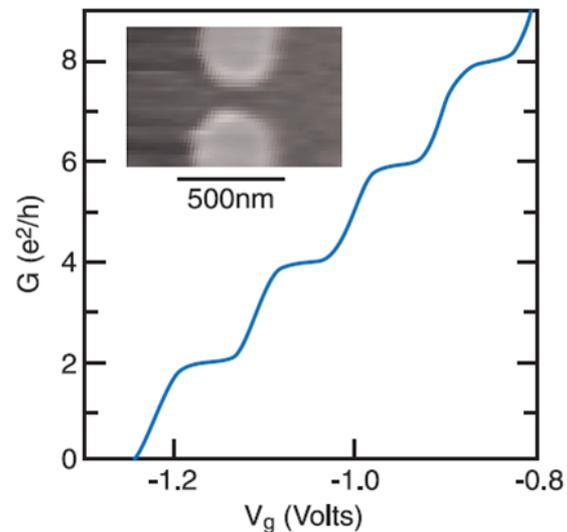
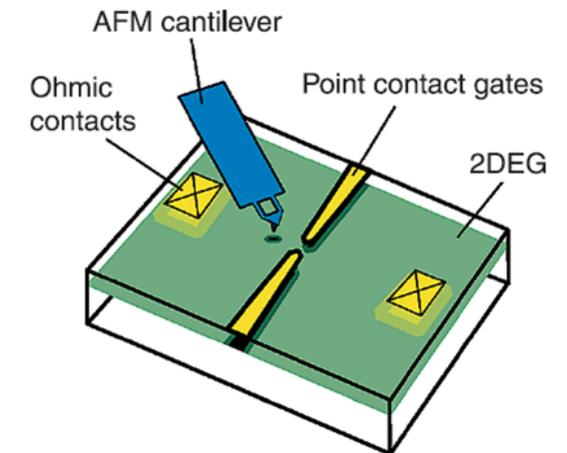
“点接触”  
 $\longleftrightarrow$

© Alcatel-Lucent

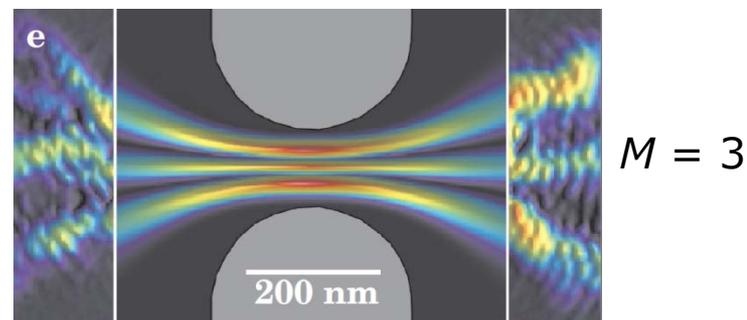
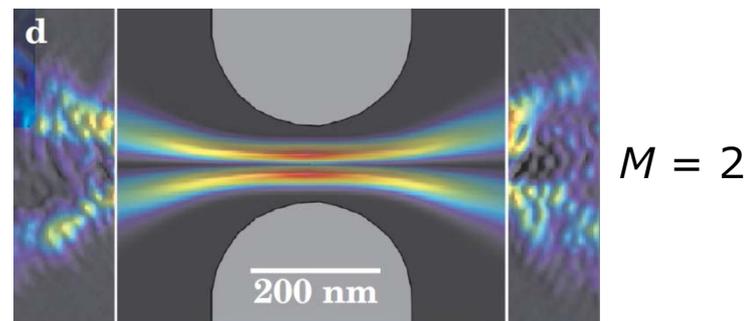
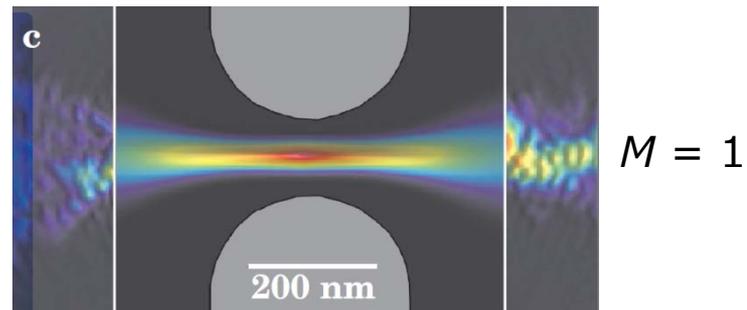


Nature **395**, 780 (1998) Ohnishi *et al.*

# 伝導チャネルの観測



$\Delta G$ :  $0 e^2/h$   $-1.7e^2/h$



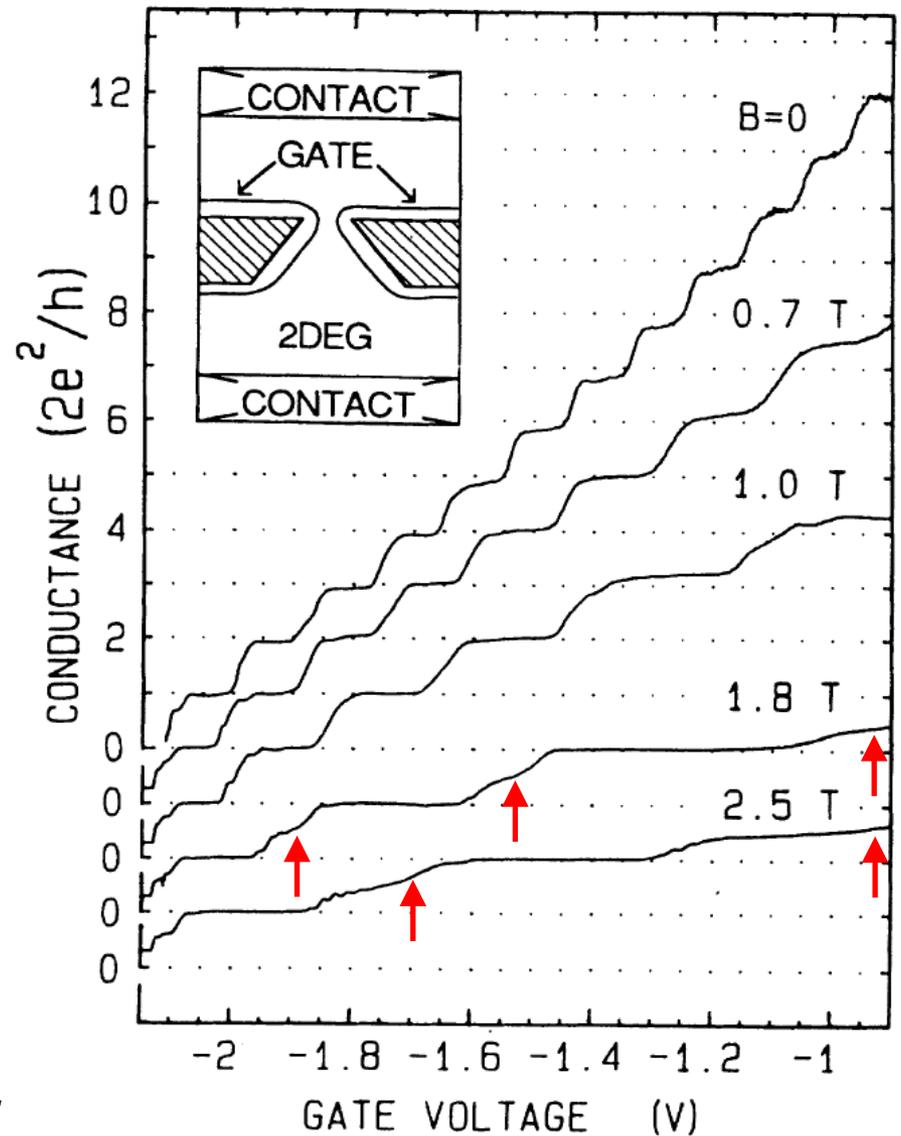
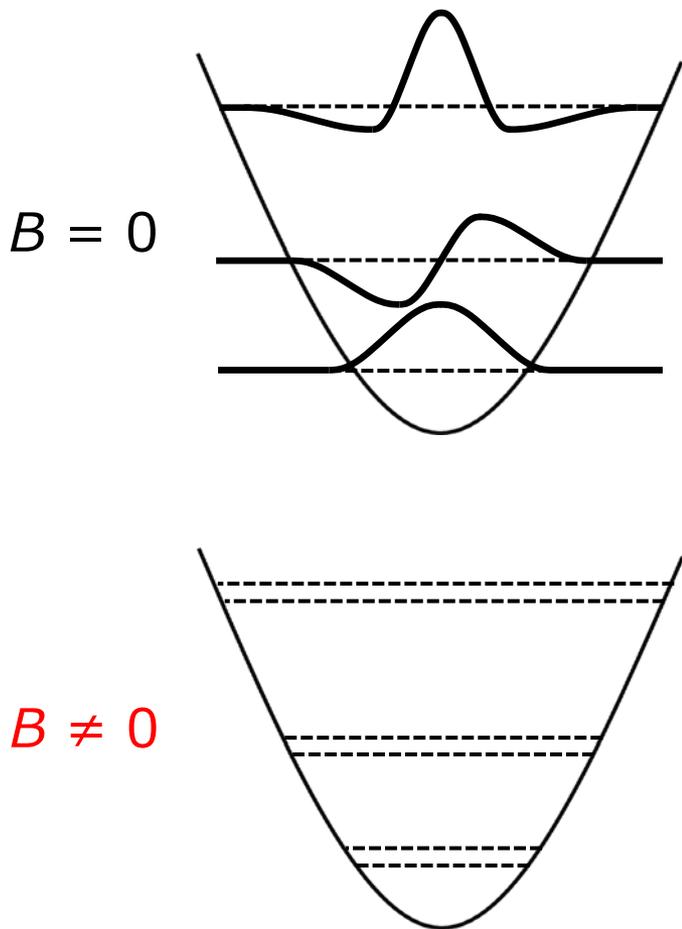
実験

理論計算

実験

Science **289**, 2323 (2000) Topinka *et al.*  
Phys. Today **56**, (12) 47 (2003) Topinka *et al.*

# 磁場によるチャネルの分離





# 0.7異常

スピン偏極や近藤効果との関連を示唆するデータが多数報告されている  
最近でも...

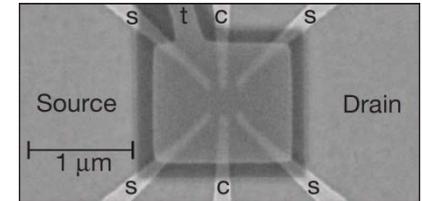
## LETTER

doi:10.1038/nature12421

### Microscopic origin of the '0.7-anomaly' in quantum point contacts

Florian Bauer<sup>1,2\*</sup>, Jan Heyder<sup>1,2\*</sup>, Enrico Schubert<sup>1</sup>, David Borowsky<sup>1</sup>, Daniela Taubert<sup>1</sup>, Benedikt Bruognolo<sup>1,2</sup>, Dieter Schuh<sup>3</sup>, Werner Wegscheider<sup>4</sup>, Jan von Delft<sup>1,2</sup> & Stefan Ludwig<sup>1</sup>

Nature **501**, 73 (2013) Bauer *et al.*



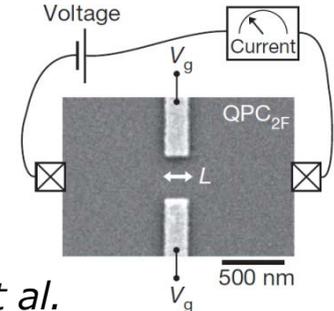
## LETTER

doi:10.1038/nature12491

### Odd and even Kondo effects from emergent localization in quantum point contacts

M. J. Iqbal<sup>1</sup>, Roi Levy<sup>2</sup>, E. J. Koop<sup>1</sup>, J. B. Dekker<sup>1</sup>, J. P. de Jong<sup>1</sup>, J. H. M. van der Velde<sup>1</sup>, D. Reuter<sup>3</sup>, A. D. Wieck<sup>3</sup>, Ramón Aguado<sup>4</sup>, Yigal Meir<sup>2,5</sup> & C. H. van der Wal<sup>1</sup>

Nature **501**, 79 (2013) Iqbal *et al.*



PRL **116**, 136801 (2016)

PHYSICAL REVIEW LETTERS

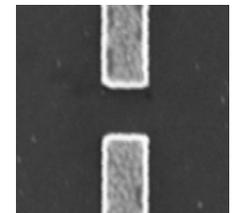
week ending  
1 APRIL 2016



### Electron Phase Shift at the Zero-Bias Anomaly of Quantum Point Contacts

B. Brun,<sup>1,2</sup> F. Martins,<sup>3</sup> S. Faniel,<sup>3</sup> B. Hackens,<sup>3</sup> A. Cavanna,<sup>4</sup> C. Ulysse,<sup>4</sup> A. Ouerghi,<sup>4</sup> U. Gennser,<sup>4</sup> D. Mailly,<sup>4</sup> P. Simon,<sup>5</sup> S. Huant,<sup>1,2</sup> V. Bayot,<sup>1,3</sup> M. Sanquer,<sup>1,6</sup> and H. Sellier<sup>1,2,\*</sup>

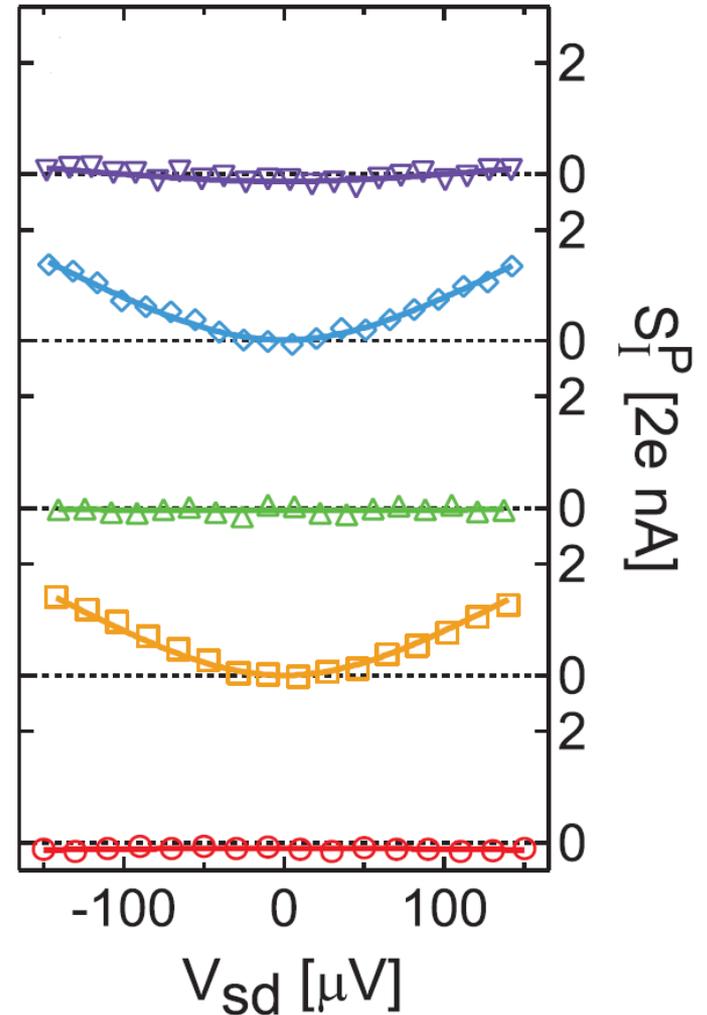
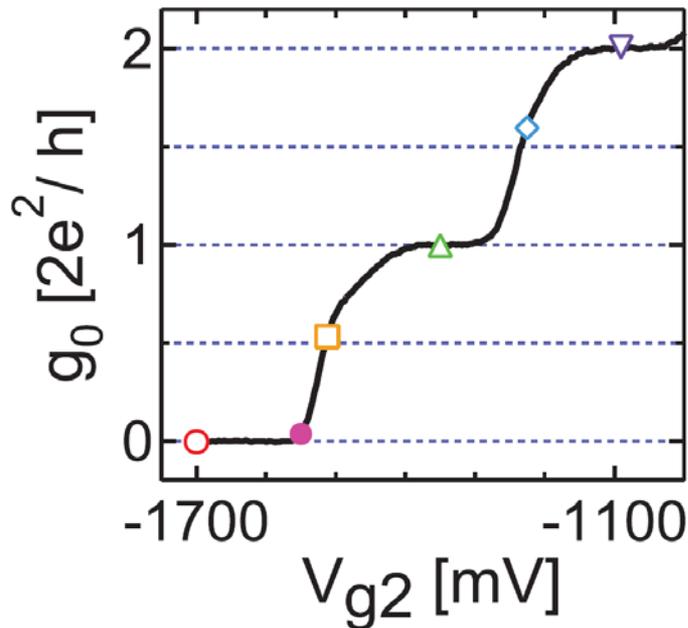
Phys. Rev. Lett. **116**, 136801 (2015) Brun *et al.*



# ノイズ測定

## ショットノイズ

- 素電荷 $e$ の離散性を反映
- **プラトーでノイズレス(完全透過)**
- 0.7異常の振る舞いや、量子ホール系・近藤効果の準粒子電荷を調べるのに有効



# 講義内容

- 量子輸送の基礎
  - 2次元電子系
  - ランダウアー公式
  - 量子ポイントコンタクト
- **整数量子ホール効果**
- 量子ホール効果とノーベル賞

# 整数量子ホール効果

ホール抵抗率

$$\rho_{xy} = \frac{h}{ie^2} \quad (i = 1, 2, 3 \dots)$$

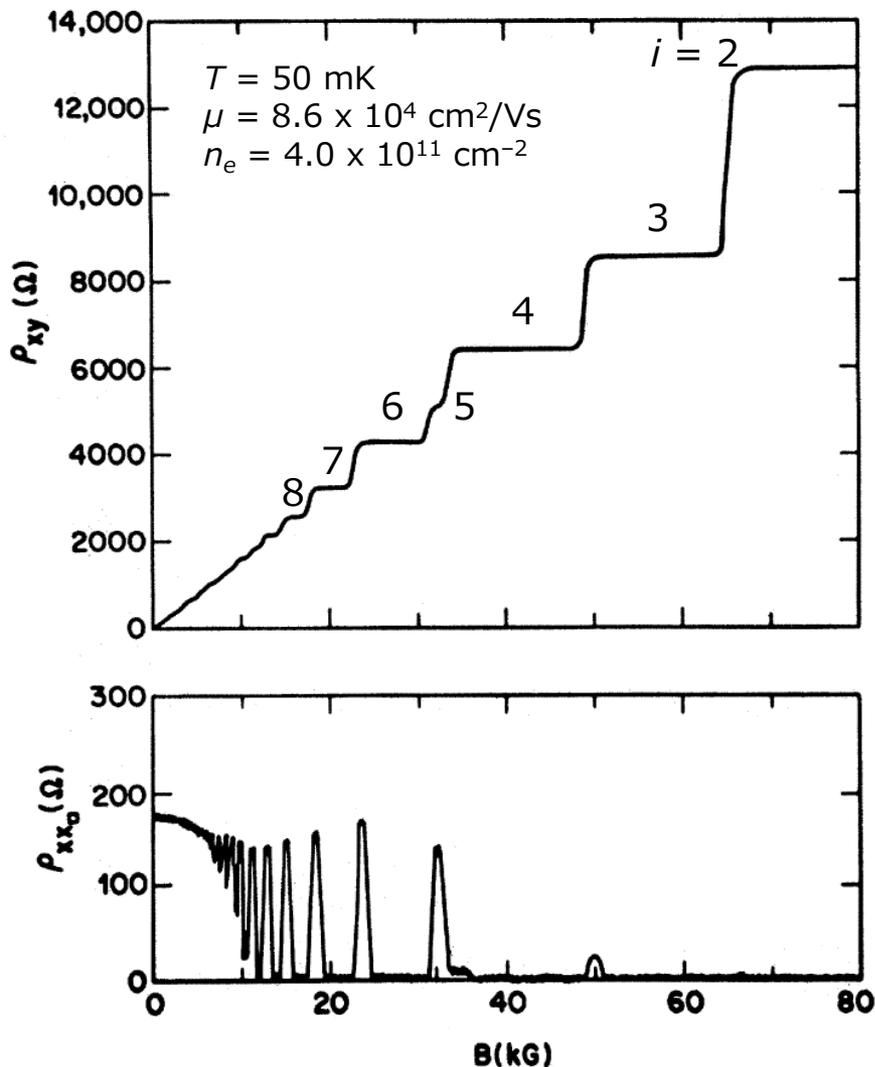
にプラトー(コンダクタンスの量子化よりも遥かに正確・頑強)

同時に縦抵抗率

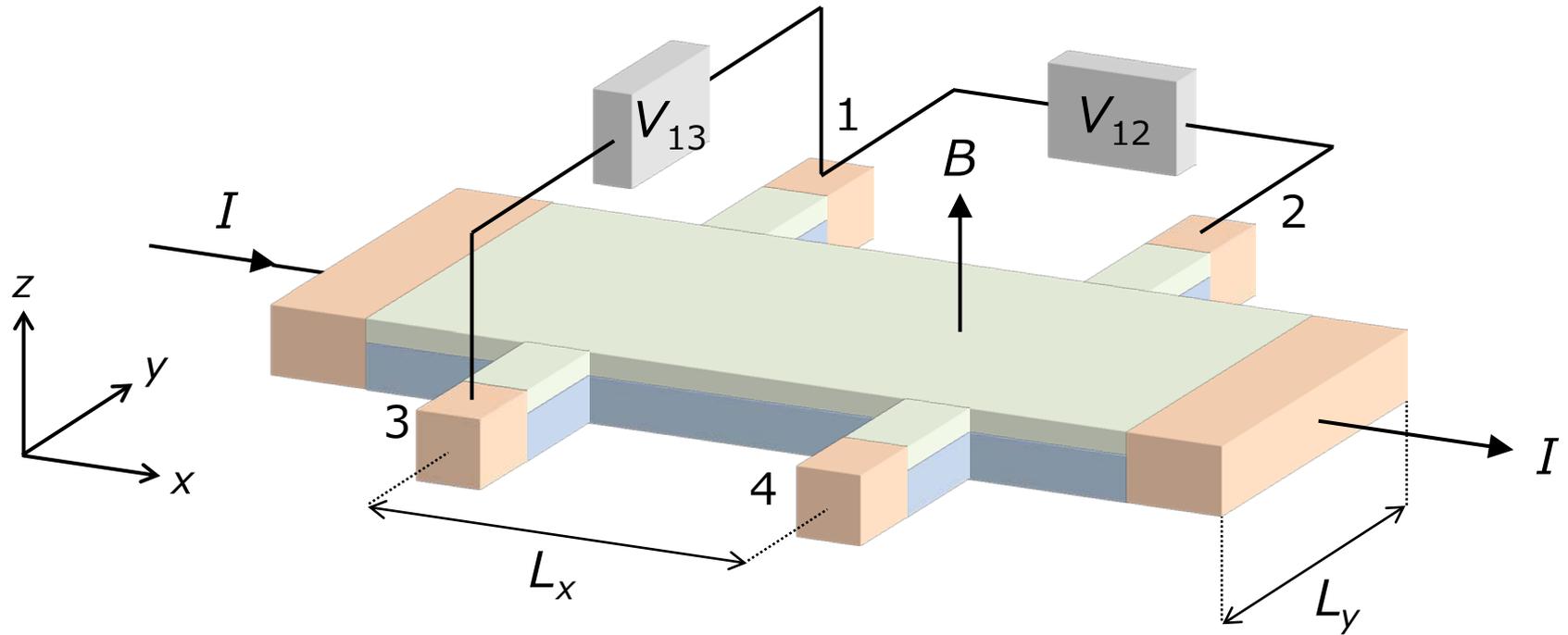
$$\rho_{xx} = 0$$

von Klitzing定数(抵抗標準)

$$R_K = \frac{h}{e^2} = 25812.8074555(59)\Omega$$



# ホールレバー



**電流密度**

$$i_x = \frac{I}{L_y}$$

$$i_y = 0$$

**電場**

$$E_x = \frac{V_{12}}{L_x}$$

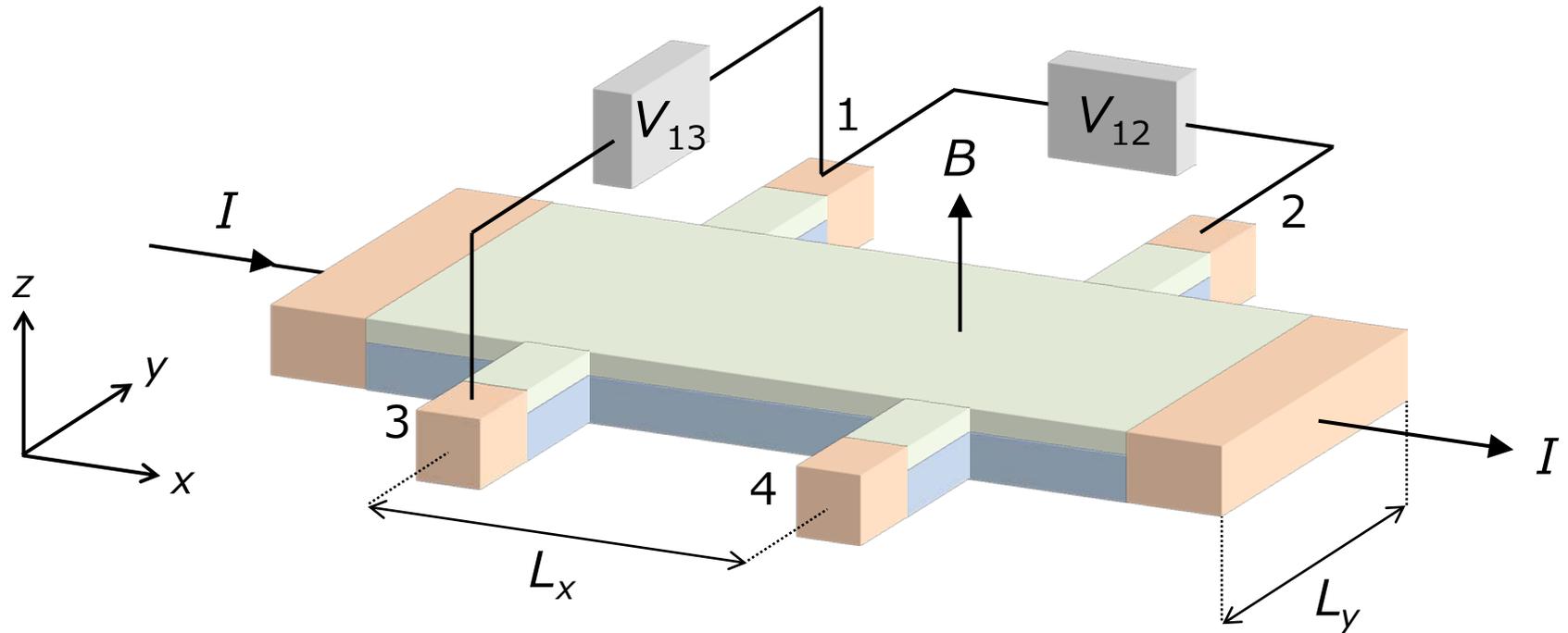
$$E_y = \frac{V_{13}}{L_y}$$

**抵抗**

$$R_{xx} = \frac{V_{12}}{I}$$

$$R_H = \frac{V_{13}}{I}$$

# ホールレバー



## 伝導率テンソル

$$\begin{pmatrix} i_x \\ i_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\sigma_{xx} = \sigma_{yy} \quad \sigma_{xy} = -\sigma_{yx}$$

## 抵抗率テンソル

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \begin{pmatrix} i_x \\ i_y \end{pmatrix}$$

$$\rho_{xx} = \rho_{yy} \quad \rho_{xy} = -\rho_{yx}$$

# 2次元の伝導率・抵抗率

電流密度

$$i_x = \frac{I}{L_y}$$

$$i_y = 0$$

電場

$$E_x = \frac{V_{12}}{L_x}$$

$$E_y = \frac{V_{13}}{L_y}$$

抵抗

$$R_{xx} = \frac{V_{12}}{I}$$

$$R_H = \frac{V_{13}}{I}$$

抵抗率テンソル

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \begin{pmatrix} i_x \\ i_y \end{pmatrix}$$

$$\rho_{xx} = \rho_{yy} \quad \rho_{xy} = -\rho_{yx}$$



$$\rho_{xx} = \frac{E_x}{i_x} = \frac{V_{12}L_y}{IL_x} = R_{xx} \frac{L_y}{L_x}$$

$$\rho_{yx} = \frac{E_y}{i_x} = \frac{V_{13}}{I} = R_H$$

2次元ではホール抵抗率はホール抵抗そのものの  
試料形状に依存しない高精度測定が可能

# 2次元の伝導度・抵抗率

## 伝導率テンソルと抵抗率テンソルの関係

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}^{-1} = \frac{1}{\sigma_{xx}^2 + \sigma_{xy}^2} \begin{pmatrix} \sigma_{xx} & -\sigma_{xy} \\ \sigma_{xy} & \sigma_{xx} \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{xx} \end{pmatrix}$$

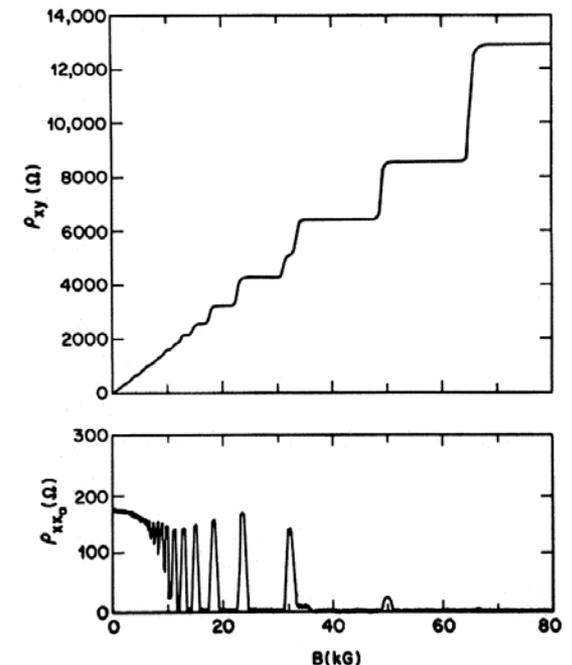
$$\Rightarrow \rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \quad \rho_{xy} = -\frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

## 量子ホール状態では

$$\rho_{xx} = 0 \Rightarrow \sigma_{xx} = 0$$

$$\rho_{xy} = \frac{h}{ie^2} = -\sigma_{xy}^{-1} \Rightarrow -\sigma_{xy} = \frac{e^2}{h} i$$

ランダウアー公式っぽい(?)



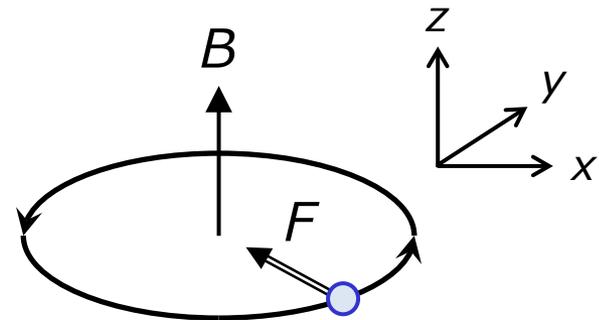
# 磁場・電場中の電子の運動

## 磁場: サイクロトロン円運動

$$m^* \frac{d^2 \mathbf{r}}{dt^2} = -e \mathbf{v} \times \mathbf{B}$$

$$\Rightarrow \frac{d}{dt} \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \frac{eB}{m^*} \begin{pmatrix} -v_y \\ v_x \end{pmatrix}$$

$$\Rightarrow \frac{d^2}{dt^2} \begin{pmatrix} v_x \\ v_y \end{pmatrix} = -\omega_c^2 \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

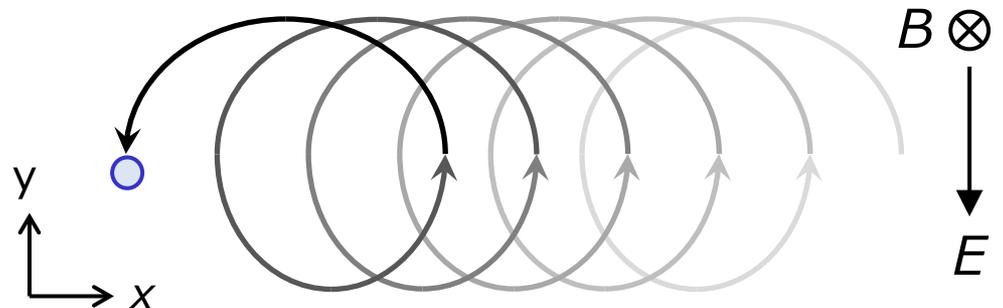


$$\text{サイクロトロン周波数 } \omega_c = \frac{eB}{m^*}$$
$$\text{サイクロトロン半径 } r_c = \frac{v}{\omega_c}$$

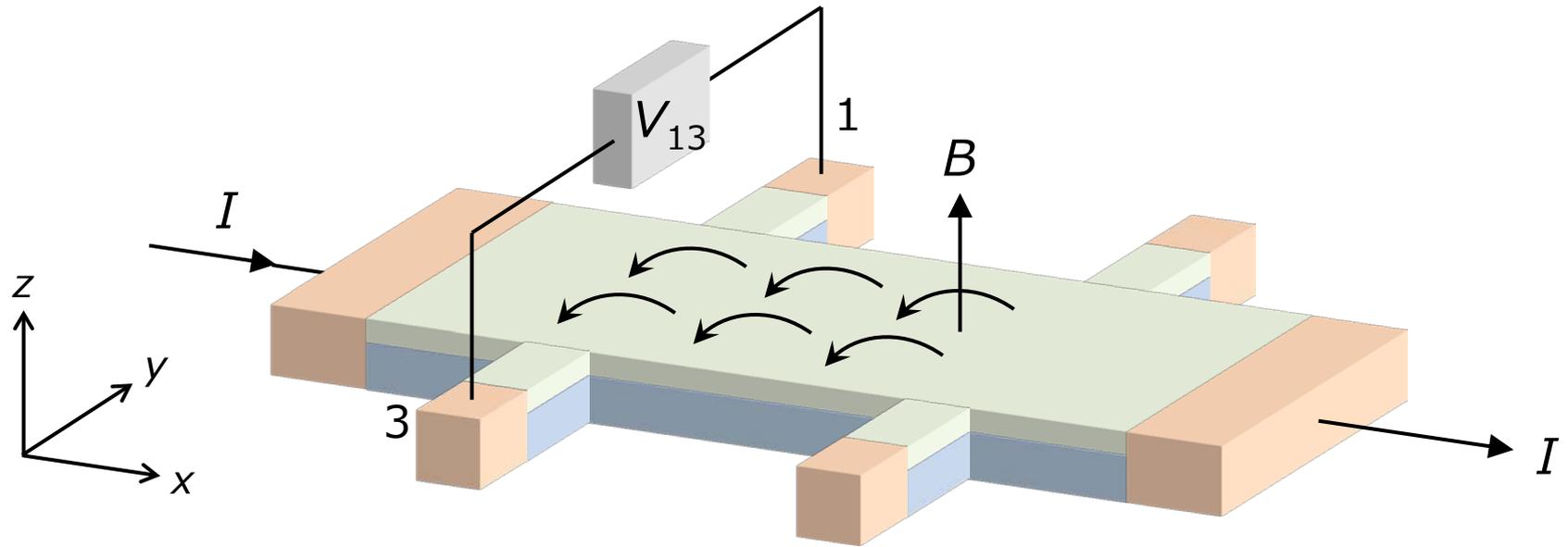
## 磁場+電場: ドリフト運動

$$m^* \frac{d^2 \mathbf{r}}{dt^2} = -e(\mathbf{v} \times \mathbf{B} + \mathbf{E})$$

$$\Rightarrow v_x = \frac{E_y}{B}$$

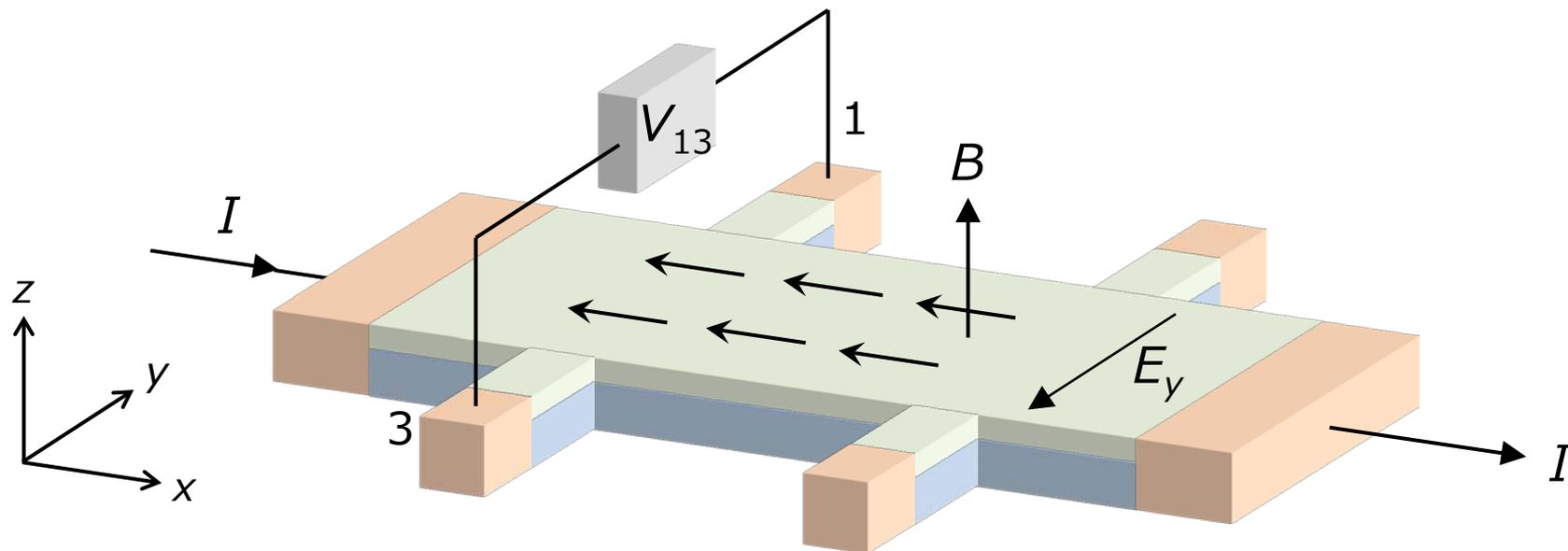


# ホール効果



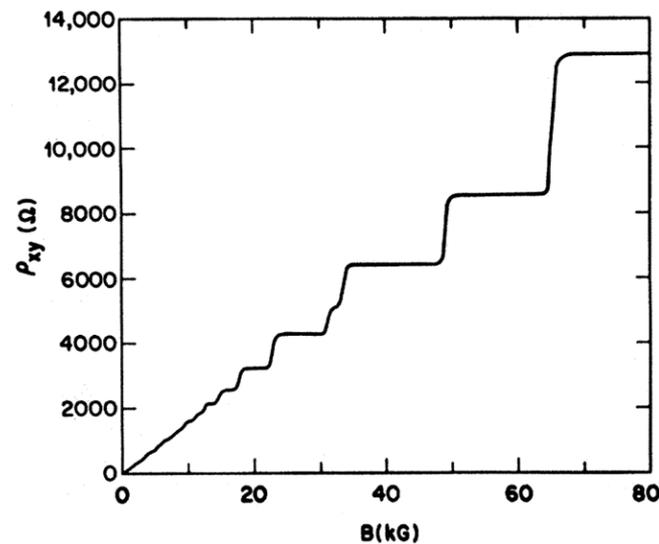
磁場で曲げられた電子がホールバーの片側に溜まりホール電場が発生  
ローレンツ力と釣り合って定常状態

# ホール効果



$$i_x = n_e e v_x = n_e e \frac{E_y}{B}$$

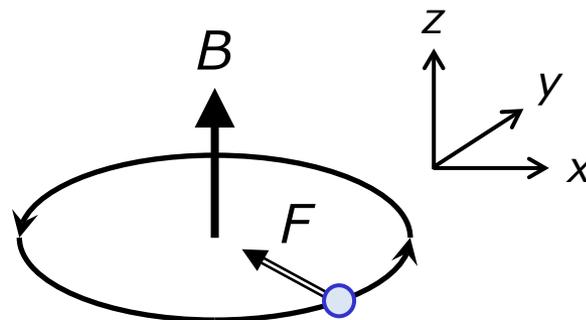
$$\Rightarrow \rho_{yx} = \frac{E_y}{i_x} = \frac{B}{n_e e}$$



# 磁場中の電子の量子論

古典論: サイクロトロン円運動

$$m^* \frac{d^2 \mathbf{r}}{dt^2} = -e \mathbf{v} \times \mathbf{B}$$



量子論: ランダウ量子化

$$H_c = \frac{1}{2m^*} (\mathbf{p} + e\mathbf{A})^2 \quad \Longrightarrow \quad E_n = \left( n - \frac{1}{2} \right) \hbar \omega_c \quad (n = 1, 2, \dots)$$

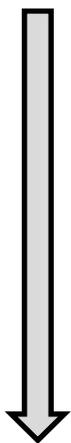
$$\text{運動量演算子 } \mathbf{p} = -i\hbar \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ 0 \end{pmatrix}$$

$$\text{ベクトルポテンシャル } \text{rot} \mathbf{A} = \mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ B \end{pmatrix} \text{ を満たせばOK(一意ではない)}$$

# ランダウ準位

## ハミルトニアン

$$H_c = \frac{1}{2m^*} (\mathbf{p} + e\mathbf{A})^2 = \frac{1}{2m^*} \left[ \left( i\hbar \frac{\partial}{\partial x} + eBy \right)^2 - \hbar^2 \frac{\partial^2}{\partial y^2} \right]$$



変数分離形

$$\Psi(x, y) = e^{ikx} u(y)$$

ランダウゲージ  $\mathbf{A} = (-By, 0, 0)$

ほかにも  $\mathbf{A}' = (0, Bx, 0)$

対称ゲージ  $\mathbf{A}'' = (-By/2, Bx/2, 0)$

固有関数はゲージの取り方に依存するが、  
固有値(エネルギー)は不変

## シュレディンガー方程式

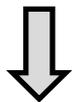
$$\left[ -\frac{\hbar^2}{2m^*} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \frac{i\hbar eBy}{m^*} \frac{\partial}{\partial x} + \frac{(eBy)^2}{2m^*} \right] e^{ikx} u(y) = E e^{ikx} u(y)$$

# ランダウ準位

$$\left[ -\frac{\hbar^2}{2m^*} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \frac{i\hbar eBy}{m^*} \frac{\partial}{\partial x} + \frac{(eBy)^2}{2m^*} \right] e^{ikx} u = E e^{ikx} u$$



$$-\frac{\hbar^2}{2m^*} e^{ikx} \frac{\partial^2 u}{\partial y^2} + \left( \frac{\hbar^2 k^2}{2m^*} - \frac{\hbar k eBy}{m^*} + \frac{(eBy)^2}{2m^*} \right) e^{ikx} u = E e^{ikx} u$$



$$\left[ -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial y^2} + \frac{(eB)^2}{2m^*} \left( y - \frac{\hbar k}{eB} \right)^2 \right] u = E u$$



$$\left[ -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial y^2} + \frac{1}{2} m^* \omega_c^2 (y - y_k)^2 \right] u = E u$$

$$y_k = \frac{\hbar k}{eB}$$

$$\omega_c = \frac{eB}{m^*}$$

# ランダウ準位

## 固有値と固有関数

$$E_n = \left(n - \frac{1}{2}\right) \hbar \omega_c \quad (n = 1, 2, \dots)$$

$$\text{磁気長 } l_B = \sqrt{\frac{\hbar}{eB}} \approx 26 \text{ nm@1T}$$

$$\Psi_{nk}(x, y) = e^{ikx} h_{n-1} \left(\frac{y - y_k}{l_B}\right) \exp\left(-\frac{(y - y_k)^2}{2l_B^2}\right)$$

## 状態数(縮重度)

$$0 < y_k < L_y \implies 0 < \frac{\hbar}{eB} \times \frac{2\pi}{L_x} j < L_y \implies 0 < j < \frac{eB}{h} L_x L_y$$

各準位に単位面積あたり最大

$$n_B = \frac{eB}{h} = \frac{1}{\pi(\sqrt{2}l_B)^2} \approx 2.4 \times 10^{10} \text{ cm}^{-2} @ 1\text{T} \quad \text{個の状態}$$

$$y_k = \frac{\hbar k}{eB}$$
$$k = \frac{2\pi}{L_x} j$$

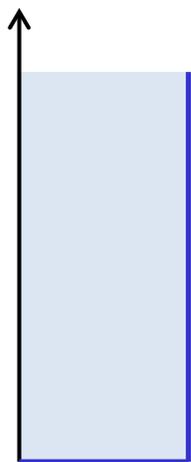
# ランダウ準位

## 固有値と固有関数

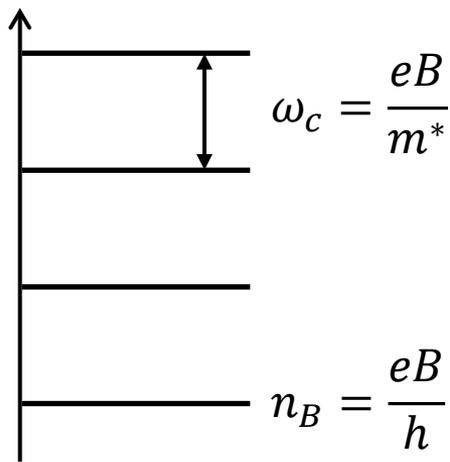
$$E_n = \left(n - \frac{1}{2}\right) \hbar \omega_c \quad (n = 1, 2, \dots)$$

$$\text{磁気長 } l_B = \sqrt{\frac{\hbar}{eB}} \approx 26 \text{ nm@1T}$$

$$\Psi_{nk}(x, y) = e^{ikx} h_{n-1} \left(\frac{y - y_k}{l_B}\right) \exp\left(-\frac{(y - y_k)^2}{2l_B^2}\right)$$



$B = 0$



$B \neq 0$

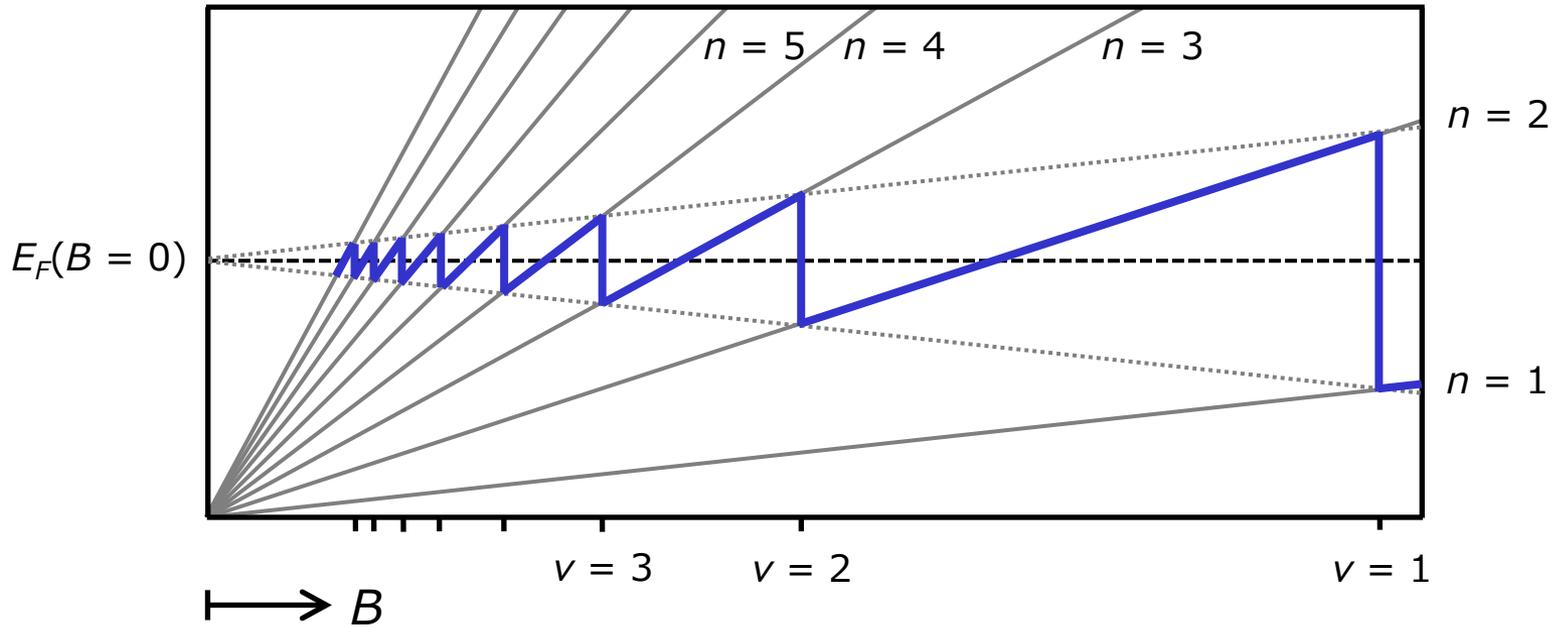
充填率

$$\nu = \frac{n_e}{n_B}$$

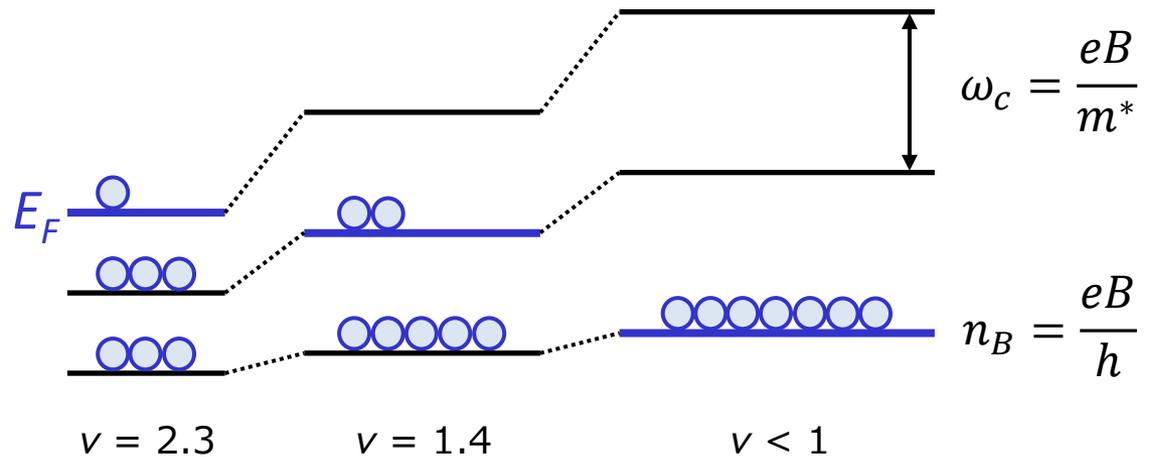
フェルミ面のある準位

$$n_F = [\nu]$$

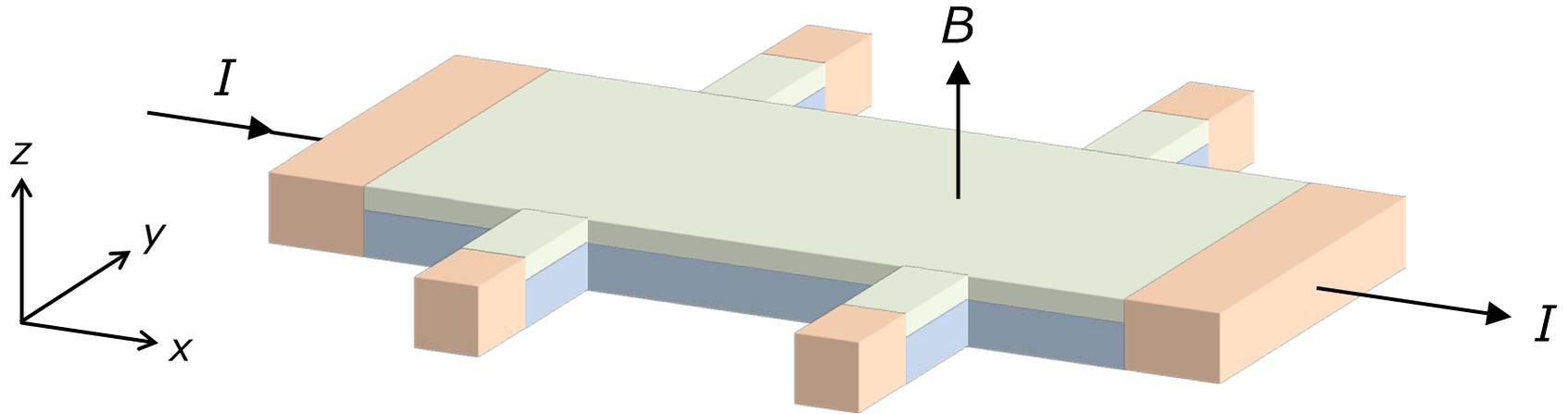
# シュブニコフ・ドハース振動



$$E_n = \left( n - \frac{1}{2} \right) \hbar \omega_c$$



# 量子化値



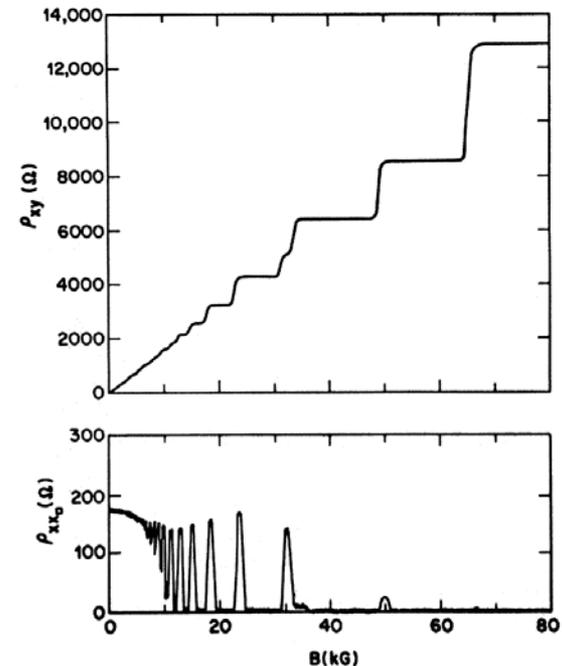
$$\rho_{xy} = \frac{B}{n_e e} = \frac{B}{n_B v e} = \frac{h}{v e^2}$$

ランダウ準位がちょうど埋まるところで量子化(?)

$$v = \frac{n_e}{n_B} \quad n_B = \frac{eB}{h}$$

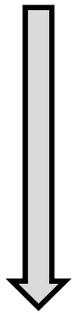
## 疑問点

プラトーの生じる理由  
SdH振動がゼロになる理由



# 確認: $\nu = i$ となる磁場

$$\rho_{xy} = \frac{B}{n_e e} = \frac{h}{\nu e^2}$$



$$e = 1.6 \times 10^{-19} \text{ C}$$

$$n_e = 4.0 \times 10^{11} \text{ cm}^{-2}$$

$$\frac{e}{h} = 2.4 \times 10^{10} \text{ cm}^{-2} \text{ T}^{-1}$$

$$\rho_{xy}(B) = 1560 \times B \text{ } \Omega$$

$$B(\nu) = \frac{16.7}{\nu} \text{ T}$$

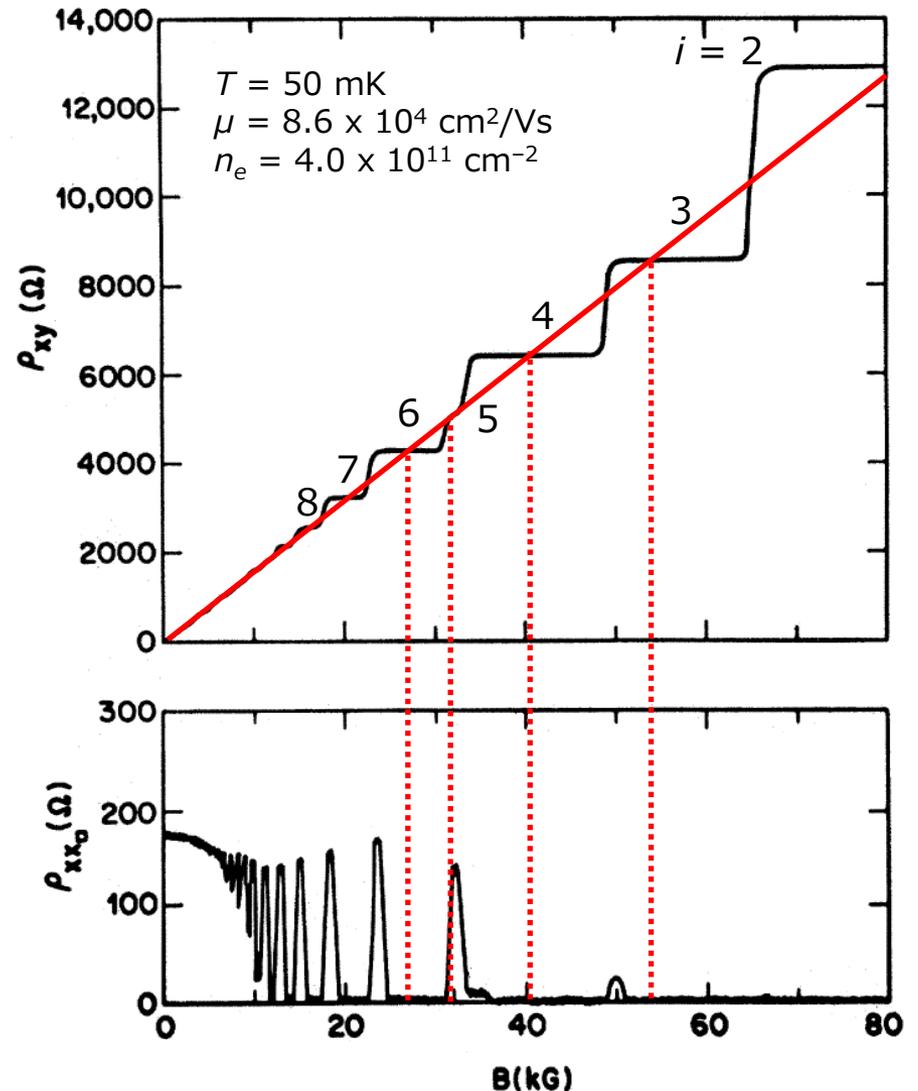
$$B(2) = 8.4 \text{ T}$$

$$B(3) = 5.6 \text{ T}$$

$$B(4) = 4.2 \text{ T}$$

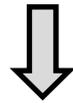
$$B(5) = 3.3 \text{ T}$$

$$B(6) = 2.8 \text{ T}$$

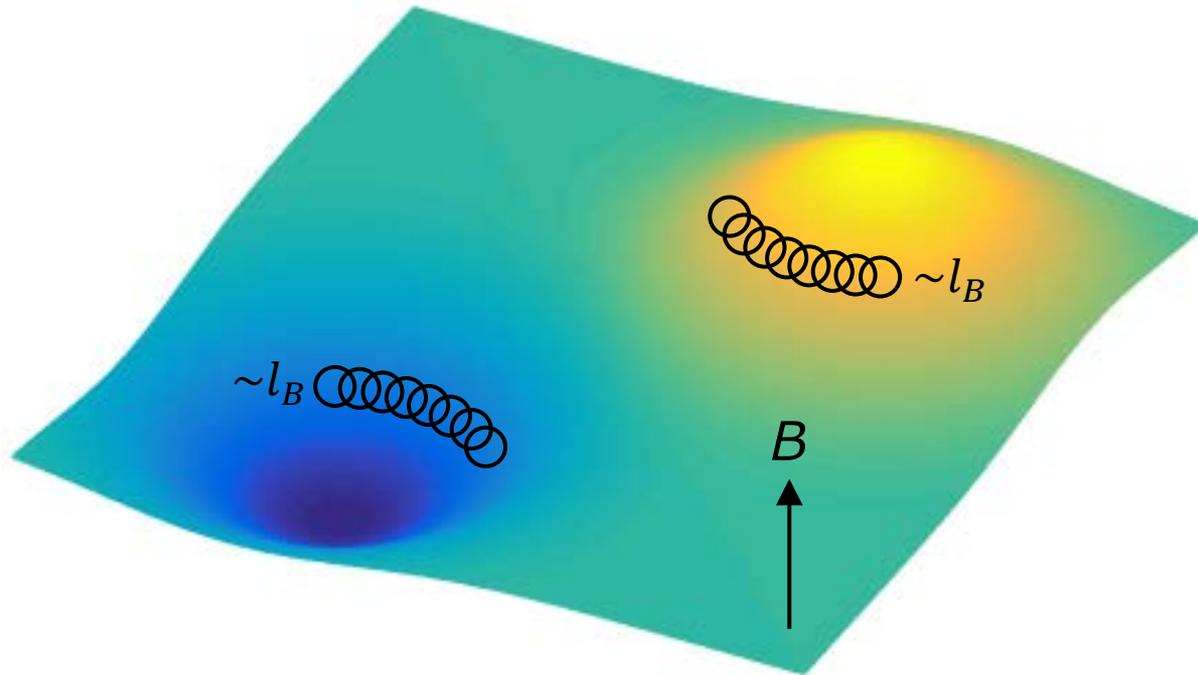


# 局在

乱れ(disorder)によって生じるポテンシャルの山・谷に電子軌道が巻き付く



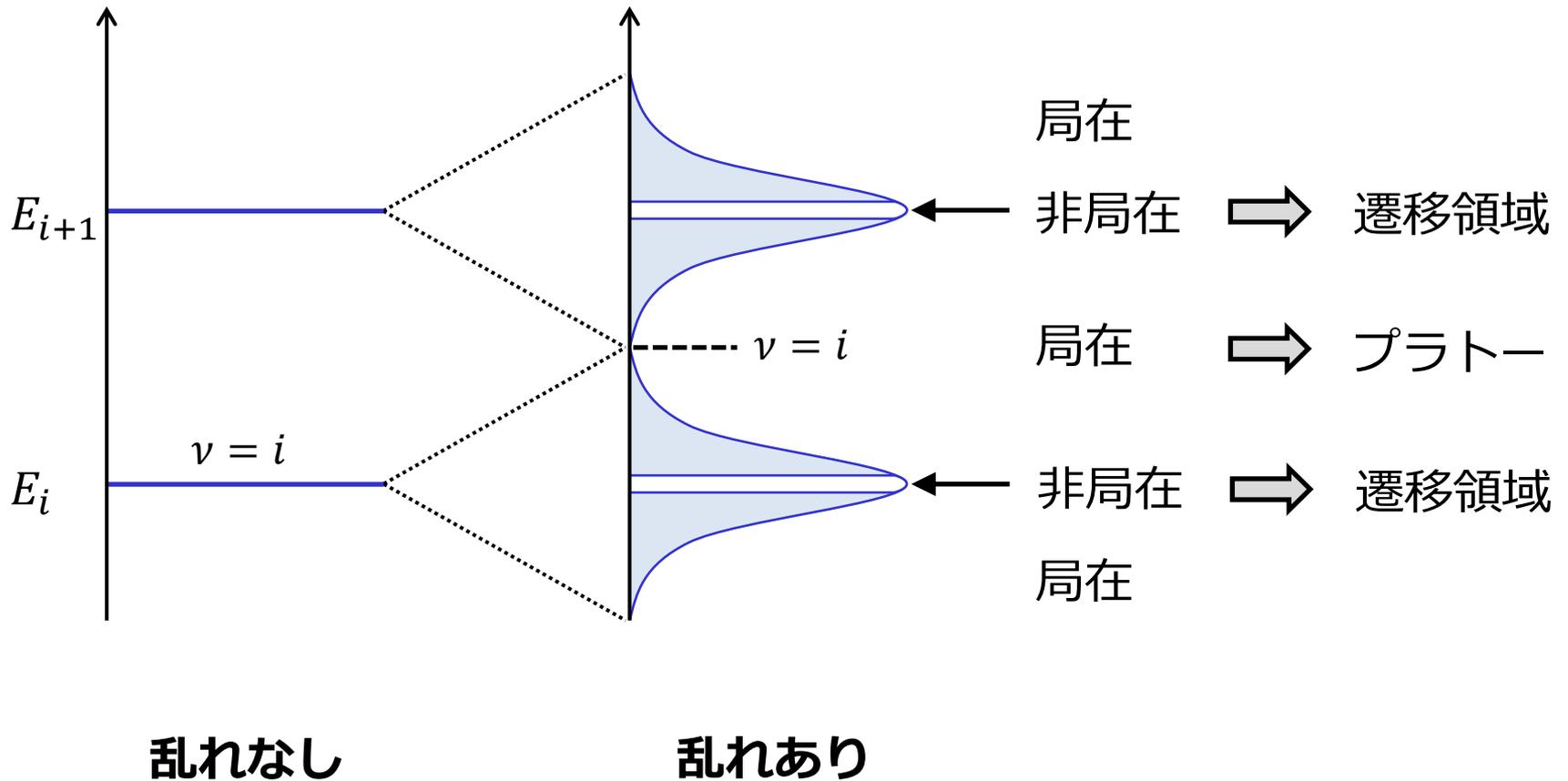
らせん軌道の波動関数は磁気長程度の幅で閉曲線を描いて局在



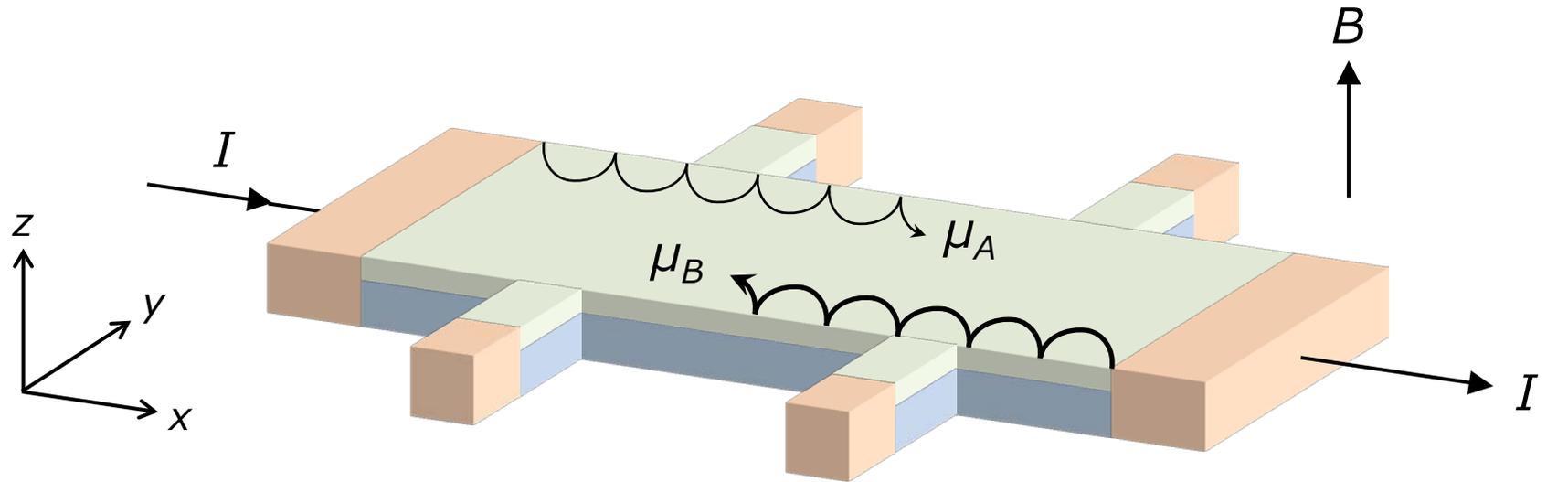
局在長(数値シミュレーション)  $\xi(E) \propto |E - E_i|^{-2.3 \pm 0.1}$

# 局在のランダウ準位への影響

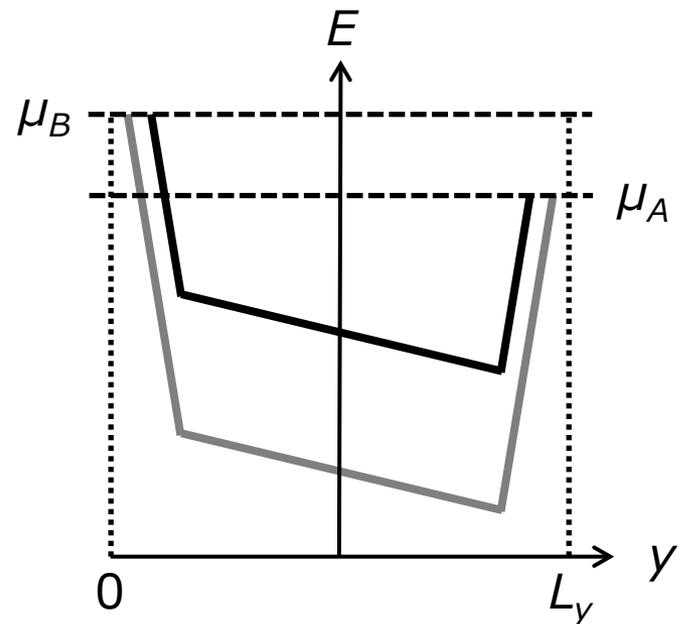
各準位が有限の幅を持ち、局在領域と非局在領域に分かれる



# 端(エッジ)状態



- 古典論のスキッピング軌道に対応
- 試料端の閉じ込めポテンシャルの存在
- 端状態は乱れの影響を受けにくい
- チャンネルの分離(後方散乱の抑制)



# 端電流

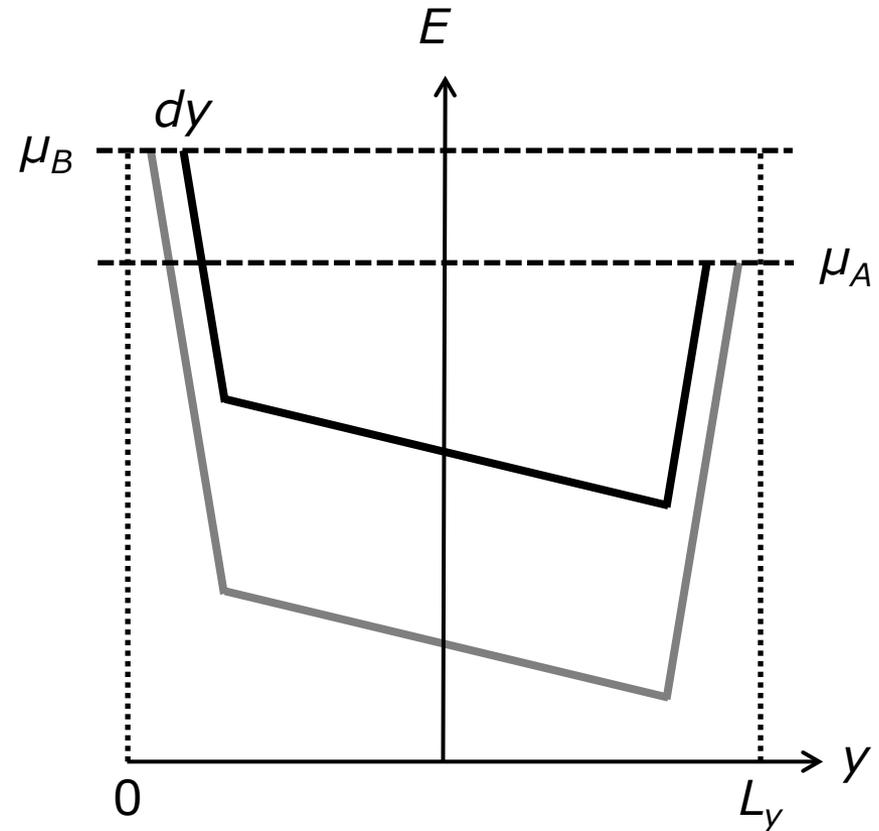
x軸に沿って移動する電子の速度

$$v_x = \frac{dE}{\hbar dk} = \frac{1}{eB} \frac{dE}{dy}$$

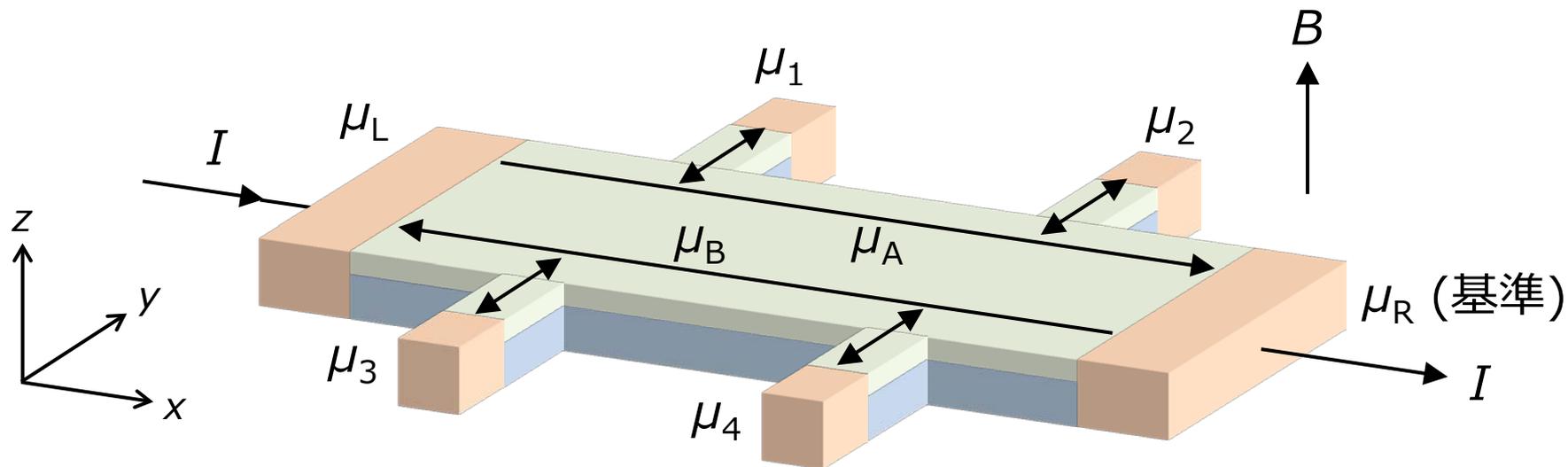
$$y_k = \frac{\hbar k}{eB} \quad n_B = \frac{eB}{h}$$

端状態によって運ばれる電流

$$\begin{aligned} I_L &= \int_{y_0}^{y_A} -e v_x n_B dy \\ &= \int_{y_0}^{y_A} -e \frac{1}{eB} \frac{dE}{dy} \frac{eB}{h} dy \\ &= -\frac{e}{h} \int_{\mu_0=0}^{\mu_B} dE = -\frac{e}{h} \mu_B \end{aligned}$$



# 端状態による伝導



$$\mu_L > \mu_R = 0$$

$$\mu_A = \mu_1 = \mu_2$$

$$\mu_B = \mu_3 = \mu_4$$

$$I = I_L - I_R$$

$$I_L = -\frac{e}{h}(\mu_A - \mu_R)$$

$$I_R = -\frac{e}{h}(\mu_B - \mu_R)$$

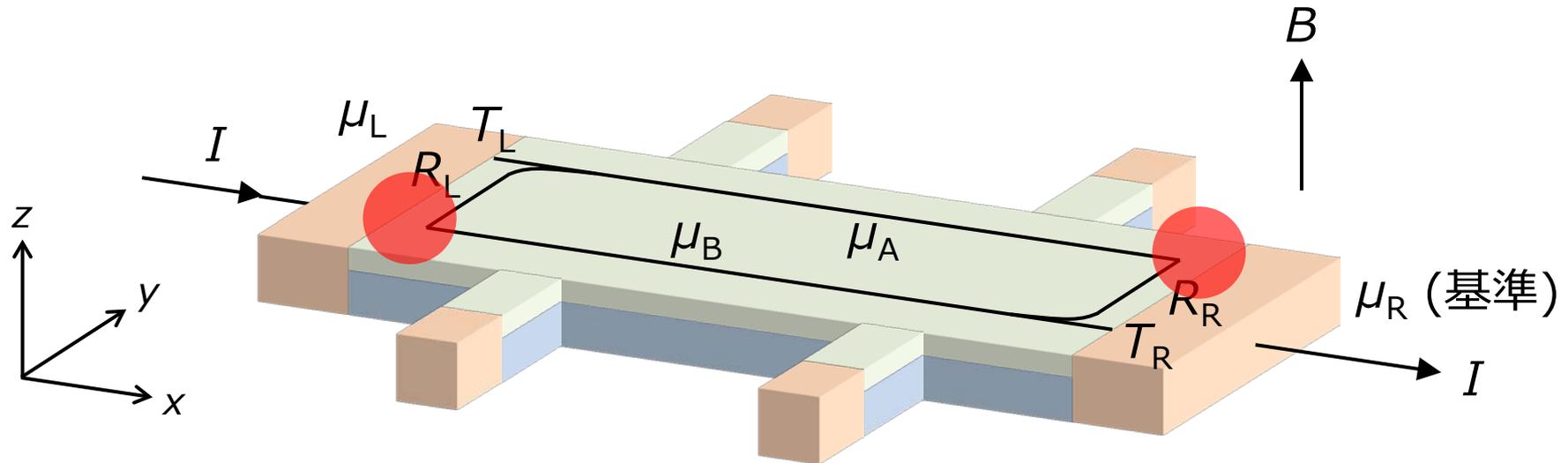
$$V_{13} = -\frac{\mu_A - \mu_B}{e}$$

## 量子化

$$\sigma_{xy} = \frac{I}{V_{13}} = \frac{e(I_L - I_R)}{\mu_A - \mu_B} = \frac{e^2}{h}$$

端チャンネル1本あたり  
(スピン分離している)

# 端状態による伝導



$$-I_L = \frac{e}{h} \mu_A = \frac{e}{h} T_L \mu_L + \frac{e}{h} R_L \mu_B$$

$$-I_R = \frac{e}{h} \mu_B = \frac{e}{h} T_R \cdot 0 + \frac{e}{h} R_R \mu_A$$

## 接触抵抗

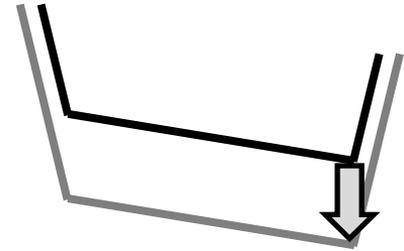
$$\mu_A - \mu_B = \frac{T_L T_R}{1 - R_L R_R} \mu_L < \mu_L - \mu_R \quad \text{通常}$$

$\mu_{A,B}$ が電流端子( $\mu_{L,R}$ )に流れ込む位置で発熱(ホットスポット)

# ホットスポットの観測

ランダウ準位間のサイクロトロン発光を可視化

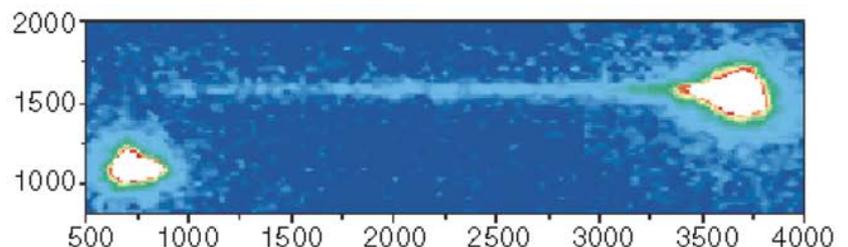
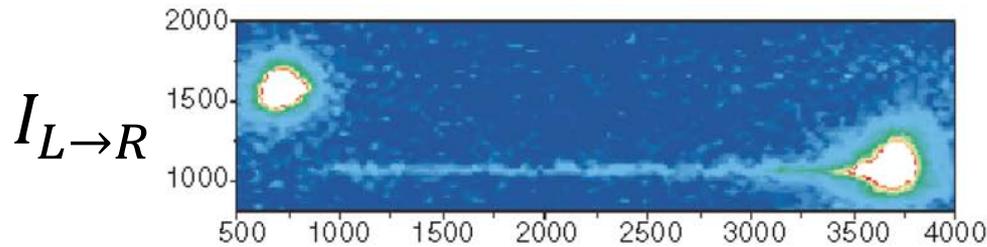
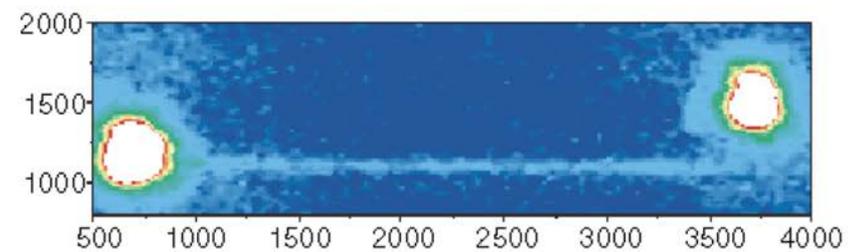
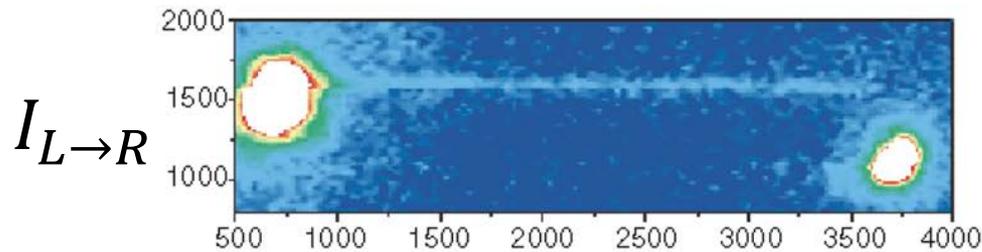
- 端状態による輸送
- カイラリティ
- 後方散乱の抑制



$y$  ( $\mu\text{m}$ )  
 $x$  ( $\mu\text{m}$ )

$B \otimes$

$B \odot$



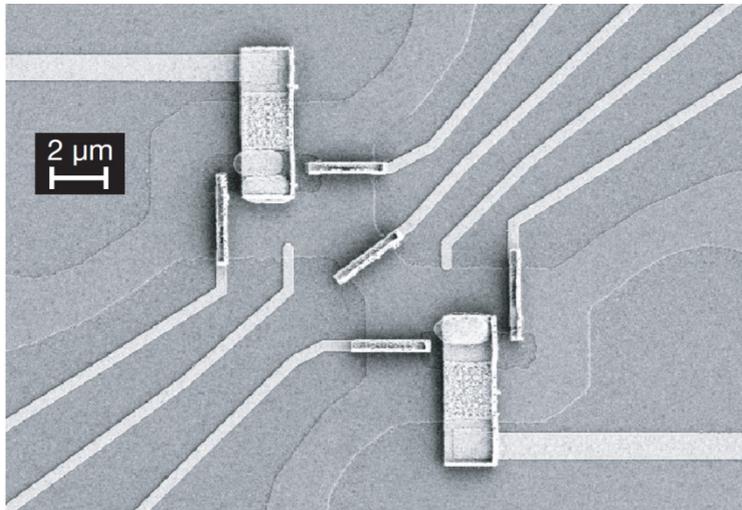
# 電子量子光学

現在では「端状態回路」を用いて電子同士を衝突・干渉させて粒子の統計性や波動性を調べる実験が活発に行われている

LETTERS

## Interference between two indistinguishable electrons from independent sources

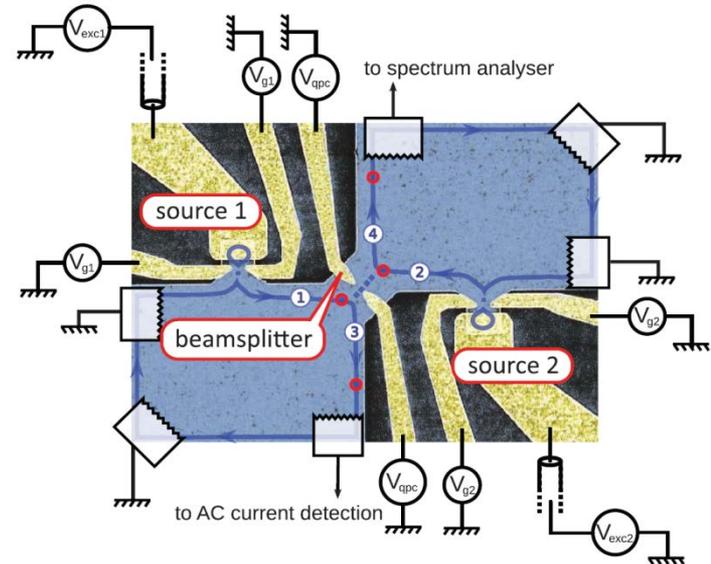
I. Neder<sup>1</sup>, N. Ofek<sup>1</sup>, Y. Chung<sup>2</sup>, M. Heiblum<sup>1</sup>, D. Mahalu<sup>1</sup> & V. Umansky<sup>1</sup>



Nature **448**, 333 (2007) Neder *et al.*

## Coherence and Indistinguishability of Single Electrons Emitted by Independent Sources

E. Bocquillon<sup>1</sup>, V. Freulon<sup>1</sup>, J.-M. Berroir<sup>1</sup>, P. Degiovanni<sup>2</sup>, B. Plaçais<sup>1</sup>, A. Cavanna<sup>3</sup>, Y. Jin<sup>3</sup>, G. Fève<sup>1\*</sup>



Science **339**, 1054 (2013) Bocquillon *et al.*

# 講義内容

- 量子輸送の基礎
  - 2次元電子系
  - ランダウアー公式
  - 量子ポイントコンタクト
- 整数量子ホール効果
- **量子ホール効果とノーベル賞**

# 量子ホール効果とノーベル賞

- **整数量子ホール効果**
  - von Klitzing (1980発見 → 1985受章)
- **分数量子ホール効果**
  - Laughlin, Störmer & Tsui (1982 → 1998)
- **2次元物質グラフェン**
  - Geim & Novoselov (2004 → 2010)

# 整数量子ホール効果

*“for the discovery of the quantized Hall effect” (Physics, 1985)*

VOLUME 45, NUMBER 6

PHYSICAL REVIEW LETTERS

11 AUGUST 1980

## New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

K. v. Klitzing

*Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and  
Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France*

and

G. Dorda

*Forschungslaboratorien der Siemens AG, D-8000 München, Federal Republic of Germany*

and

M. Pepper

*Cavendish Laboratory, Cambridge CB3 0HE, United Kingdom*

(Received 30 May 1980)

Measurements of the Hall voltage of a two-dimensional electron gas, realized with a silicon metal-oxide-semiconductor field-effect transistor, show that the Hall resistance at particular, experimentally well-defined surface carrier concentrations has fixed values which depend only on the fine-structure constant and speed of light, and is insensitive to the geometry of the device. Preliminary data are reported.

PACS numbers: 73.25.+i, 06.20.Jr, 72.20.My, 73.40.Qv

In this paper we report a new, potentially high-accuracy method for determining the fine-structure constant,  $\alpha$ .

$$\alpha = \frac{e^2}{\hbar c} = \frac{e^2}{h} \times \frac{2\pi}{c} \approx \frac{1}{137}$$



von Klitzing

© Nobel Foundation

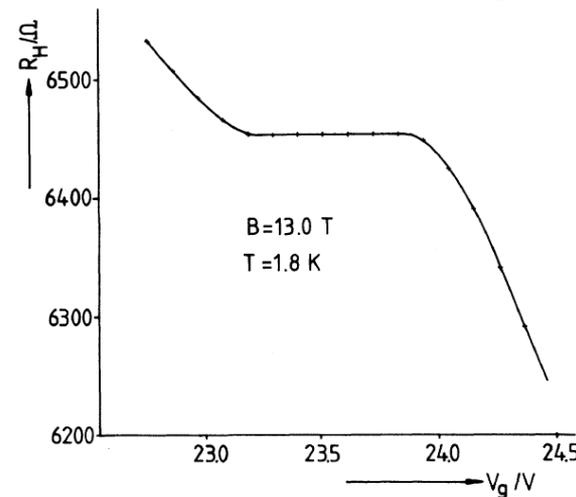
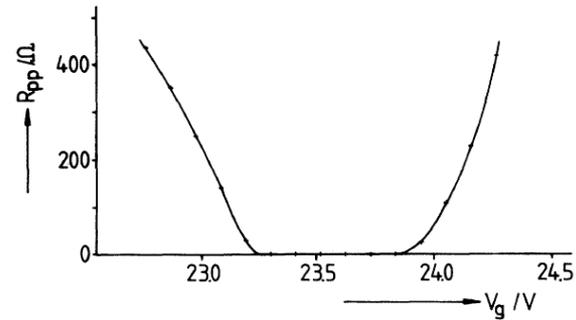
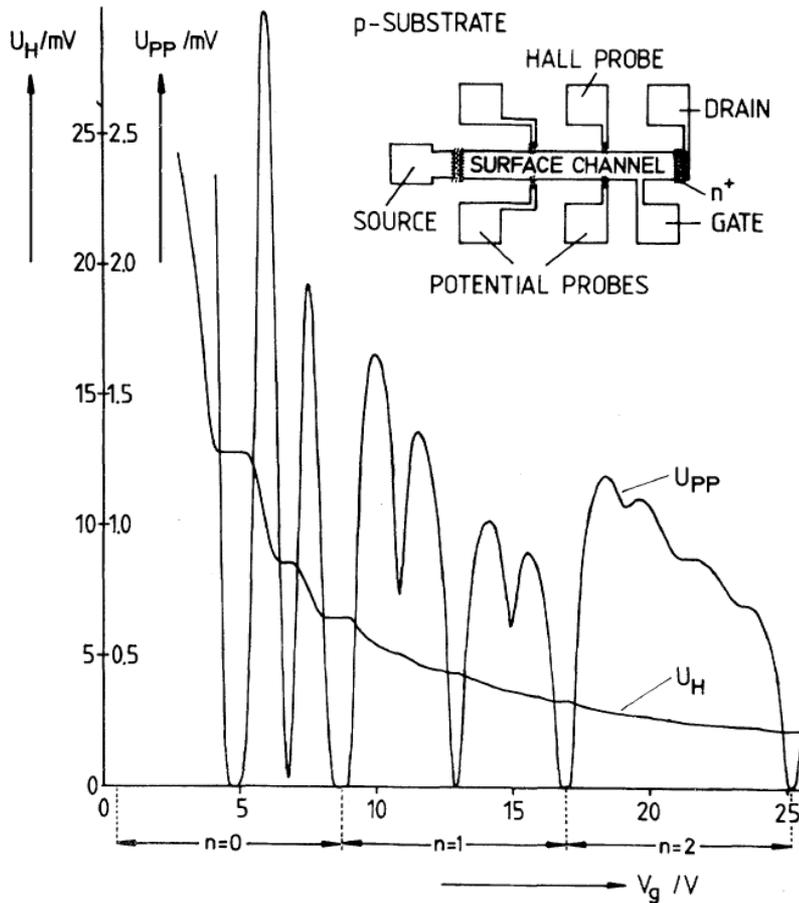
# 整数量子ホール効果

“for the discovery of the quantized Hall effect” (Physics, 1985)



von Klitzing

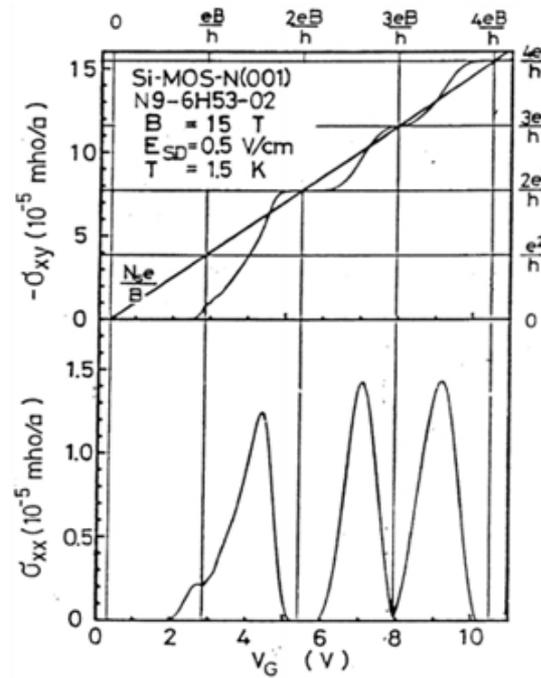
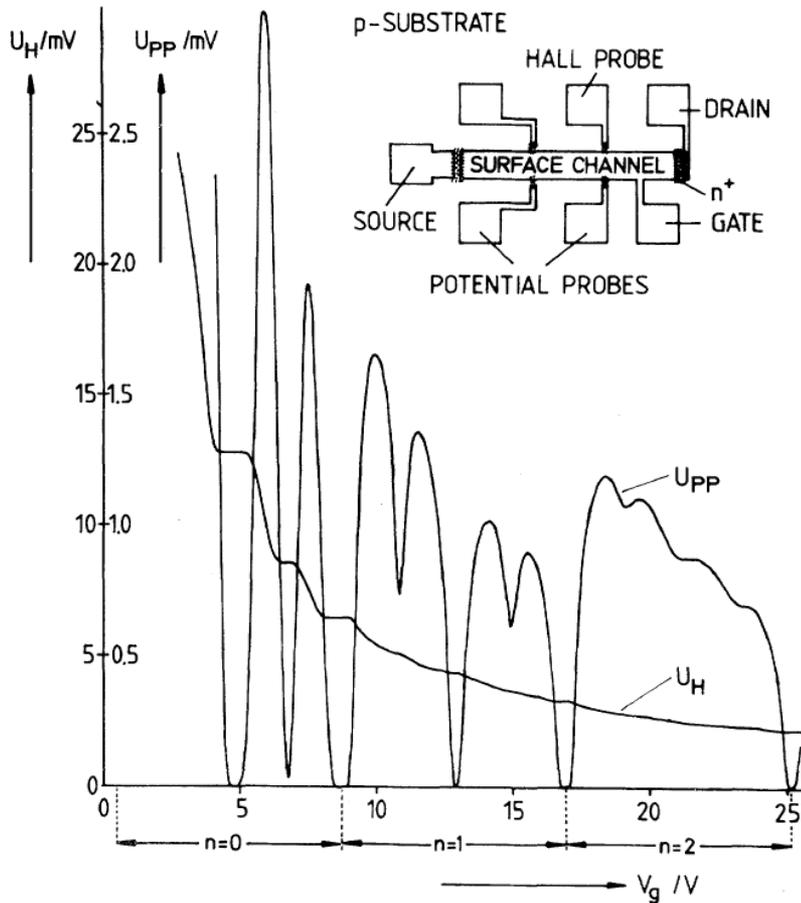
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Phys. Rev. Lett. **45**, 494 (1980) v. Klitzing, Dorda & Pepper

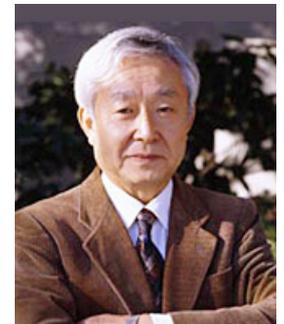
# 整数量子ホール効果

“for the discovery of the quantized Hall effect” (Physics, 1985)



von Klitzing

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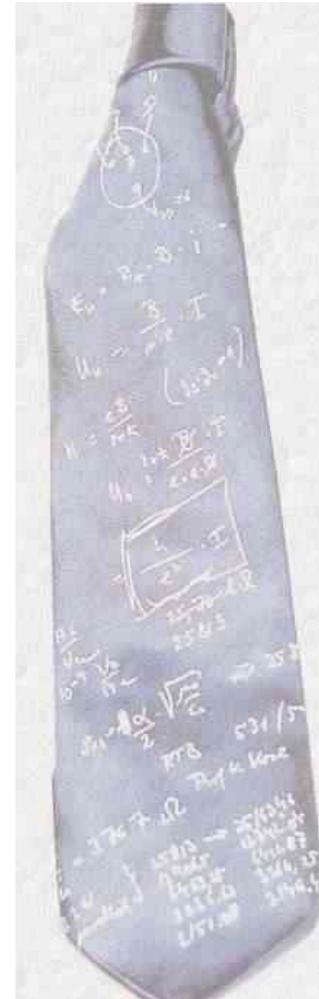
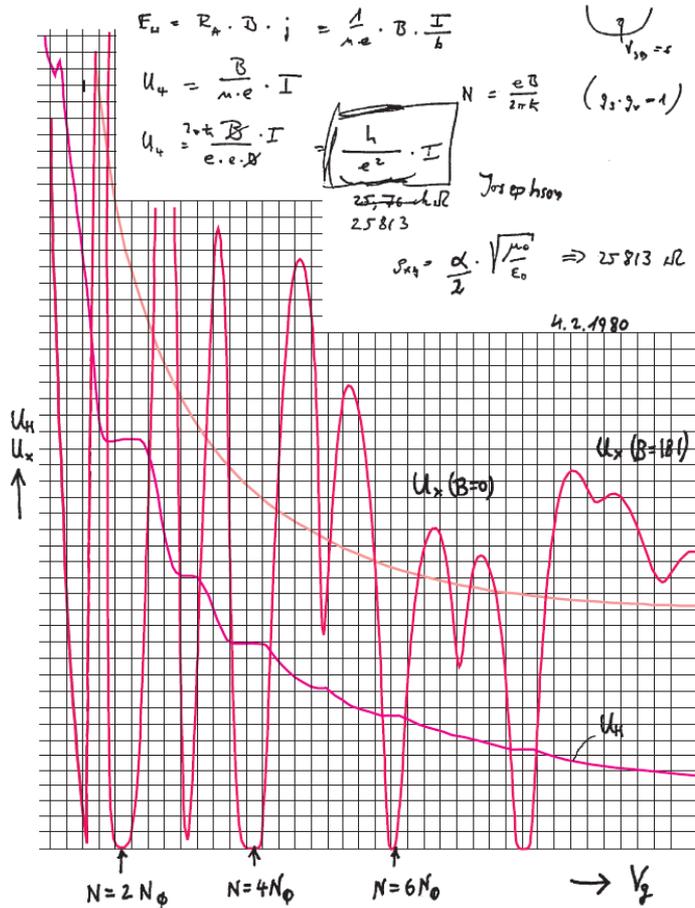
Kawaji

Wakabayashi, Kawaji 王子国際セミナー (1980)  
<http://www.gakushuin.ac.jp/univ/sci/top/interview/in01.html>

Phys. Rev. Lett. **45**, 494 (1980) v. Klitzing, Dorda & Pepper

# 整数量子ホール効果

“for the discovery of the quantized Hall effect” (Physics, 1985)



von Klitzing

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## 実際のデータと実験ノート

from *Fundamentals of Semiconductors*, Yu & Cardona

# 分数量子ホール効果

*“for their discovery of a new form of quantum fluid with fractionally charged excitations”*  
(Physics, 1998)



Laughlin

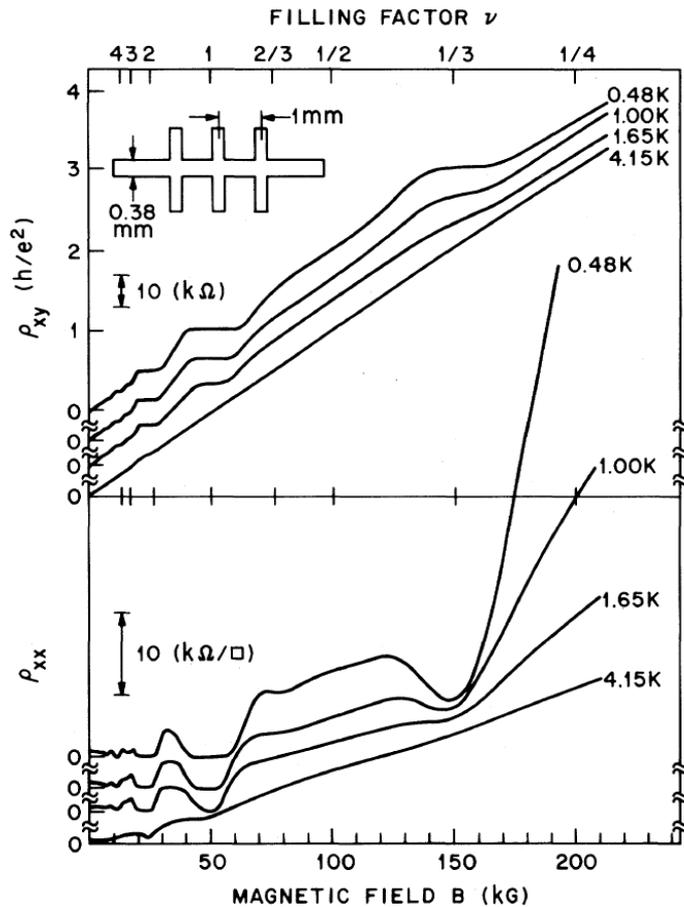


Störmer



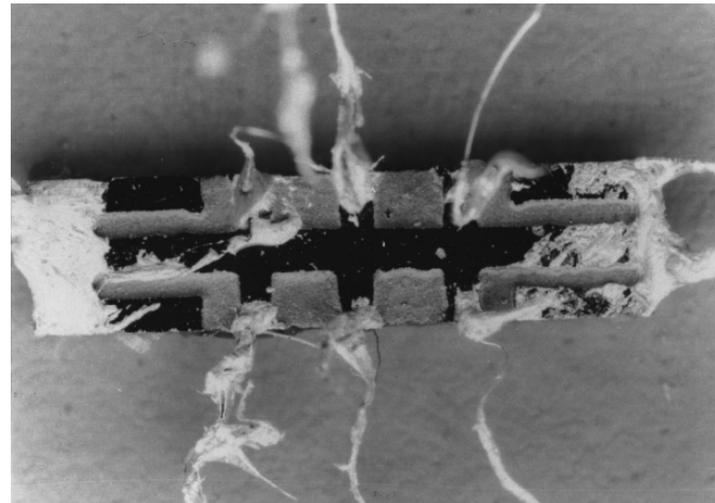
Tsui

© Nobel Foundation



## 実際のサンプル

from Nobel Lecture by Strörmer



Phys. Rev. Lett. **48**, 1559 (1982) Tsui, Stormer & Gossard

# 分数量子ホール効果

*“for their discovery of a new form of quantum fluid with fractionally charged excitations”*  
(Physics, 1998)



Laughlin



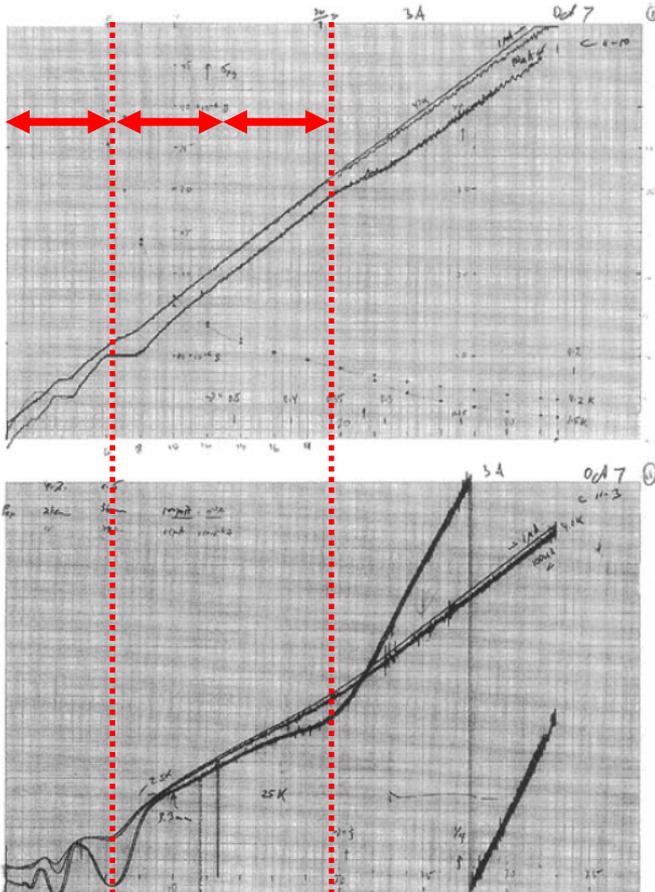
Störmer



Tsui

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“Quarks!”



実際のデータ

Lightheartedly, **Dan Tsui** enclosed the distance between  $B = 0$  and the position of the last IQHE between two fingers of one hand and measured the position of the new feature in this unit. He **determined it to be three** and exclaimed, **“quarks!”** Although obviously joking, **with finely honed intuition, he had hit on the very essence of the data.**

Nobel Lecture by Strömer

# 分数量子ホール効果

*“for their discovery of a new form of quantum fluid with fractionally charged excitations”*  
(Physics, 1998)



Laughlin



Störmer



Tsui

© Nobel Foundation

## Two-Dimensional Magnetotransport in the Extreme Quantum Limit

D. C. Tsui,<sup>(a), (b)</sup> H. L. Stormer,<sup>(a)</sup> and A. C. Gossard

*Bell Laboratories, Murray Hill, New Jersey 07974*

(Received 5 March 1982)

A quantized Hall plateau of  $\rho_{xy} = 3h/e^2$ , accompanied by a minimum in  $\rho_{xx}$ , was observed at  $T < 5$  K in magnetotransport of high-mobility, two-dimensional electrons, when the lowest-energy, spin-polarized Landau level is  $\frac{1}{3}$  filled. The formation of a Wigner solid or charge-density-wave state with triangular symmetry is suggested as a possible explanation.

# 分数量子ホール効果

*“for their discovery of a new form of quantum fluid with fractionally charged excitations”*  
(Physics, 1998)



Laughlin



Störmer



Tsui

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## **Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid with Fractionally Charged Excitations**

R. B. Laughlin

*Lawrence Livermore National Laboratory, University of California, Livermore, California 94550*

(Received 22 February 1983)

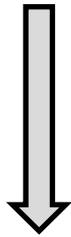
This Letter presents variational ground-state and excited-state wave functions which describe the condensation of a two-dimensional electron gas into a new state of matter.

Phys. Rev. Lett. **50**, 1395 (1983) Laughlin

# ラフリン波動関数

電子間相互作用を取り入れたハミルトニアン

$$H_{ee} = \sum_i \frac{1}{2m^*} (\mathbf{p}_i + e\mathbf{A})^2 + \sum_{i>j} \frac{e^2}{|z_i - z_j|}$$



近似解となる多体波動関数を書き下した

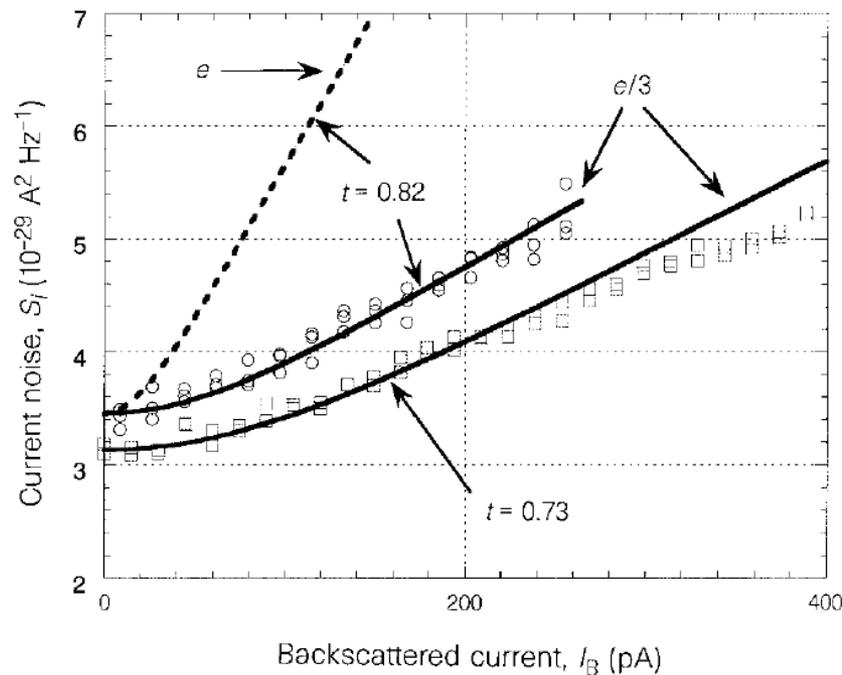
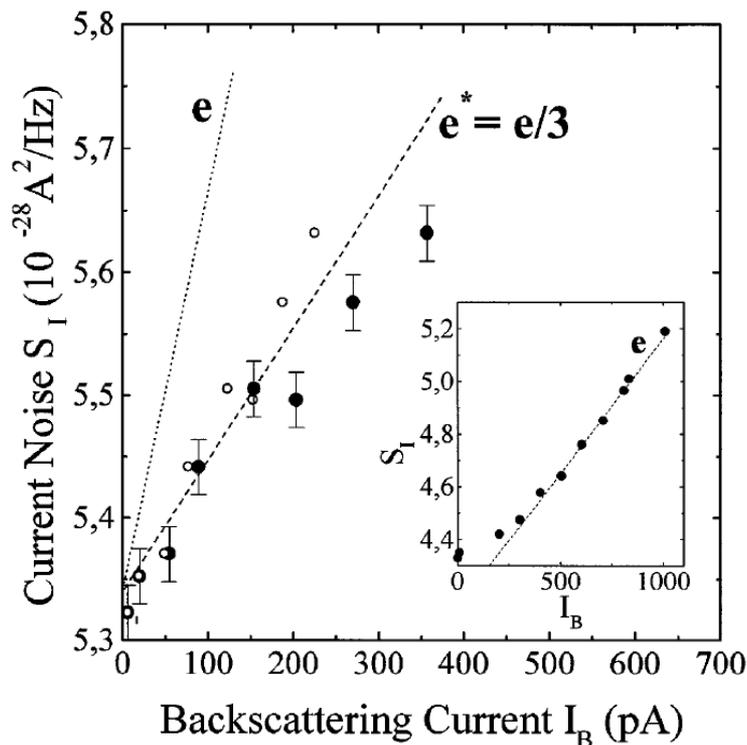
$$\Psi_q(z_1, z_2, \dots, z_N) = \prod_{i>j} (z_i - z_j)^q \exp\left(-\sum_i \frac{|z_i|^2}{4}\right)$$

- 電子間相互作用を下げる関数形
- 電子の反対称性より $q$ =奇数(占有率 $1/q$ )
- 厳密対角化の数値計算とよい一致
- 分数電荷 $e/3$ の素励起を予言 etc

$$z_i = \frac{x_i - iy_i}{l_B}$$

# 分数電荷の検証

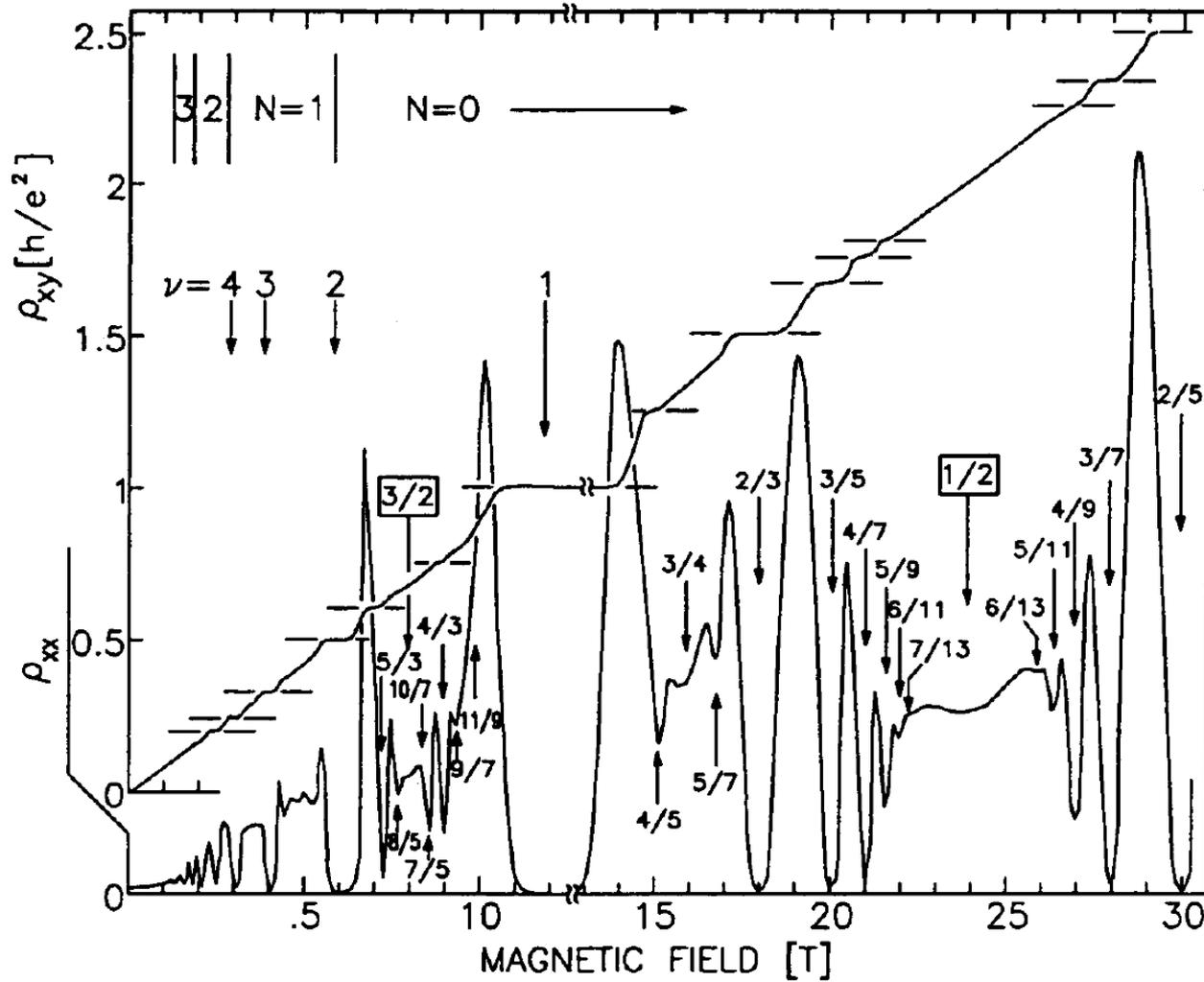
ショットノイズが準粒子の電荷(ここでは $e/3$ )に比例することを利用して翌年のノーベル賞の決め手になった(とされている)



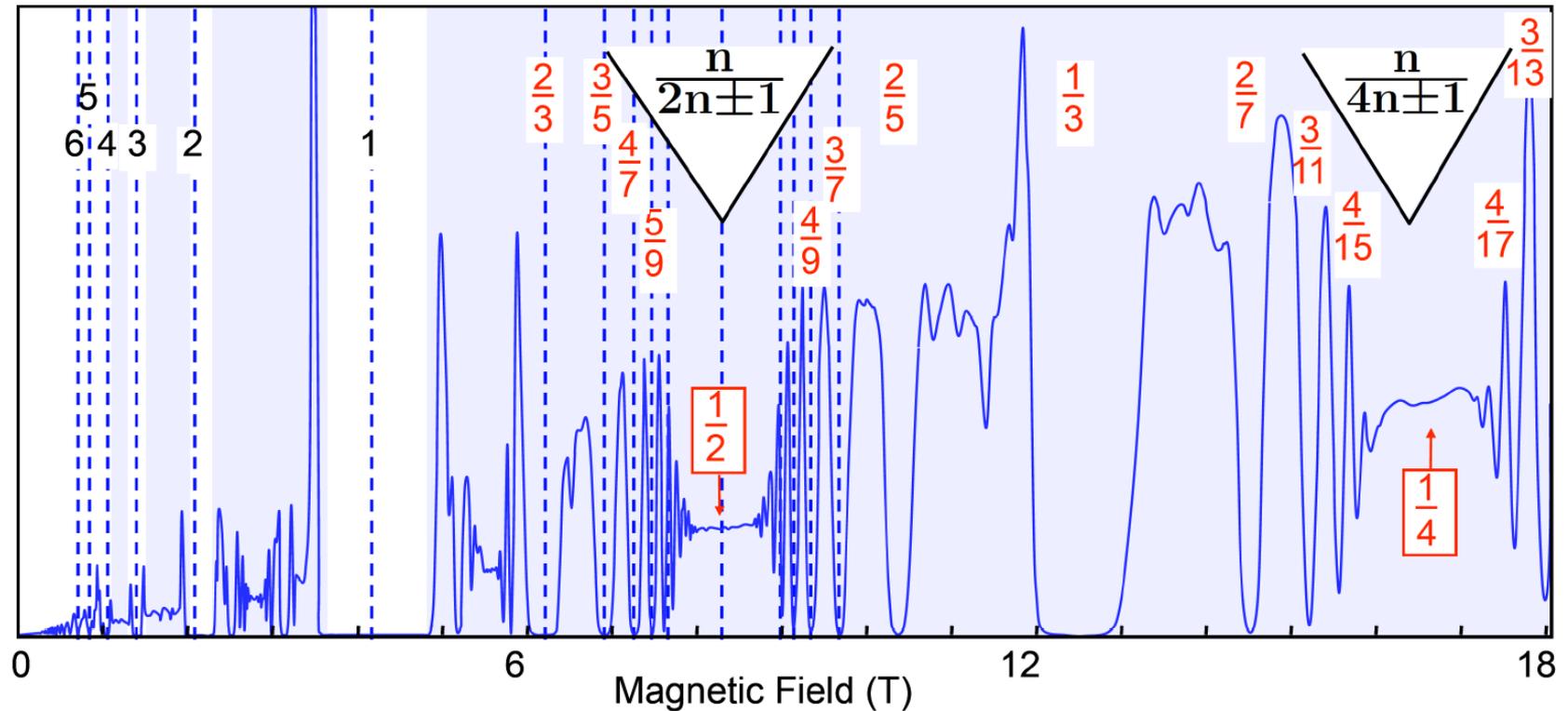
Nature **389**, 162 (1997) de-Picciotto *et al.*  
Phys. Rev. Lett. **79**, 2526 (1997) Saminadayar *et al.*

cf. 共鳴トンネルによる検証  
Science **267**, 1010 (1995) Goldman & Su

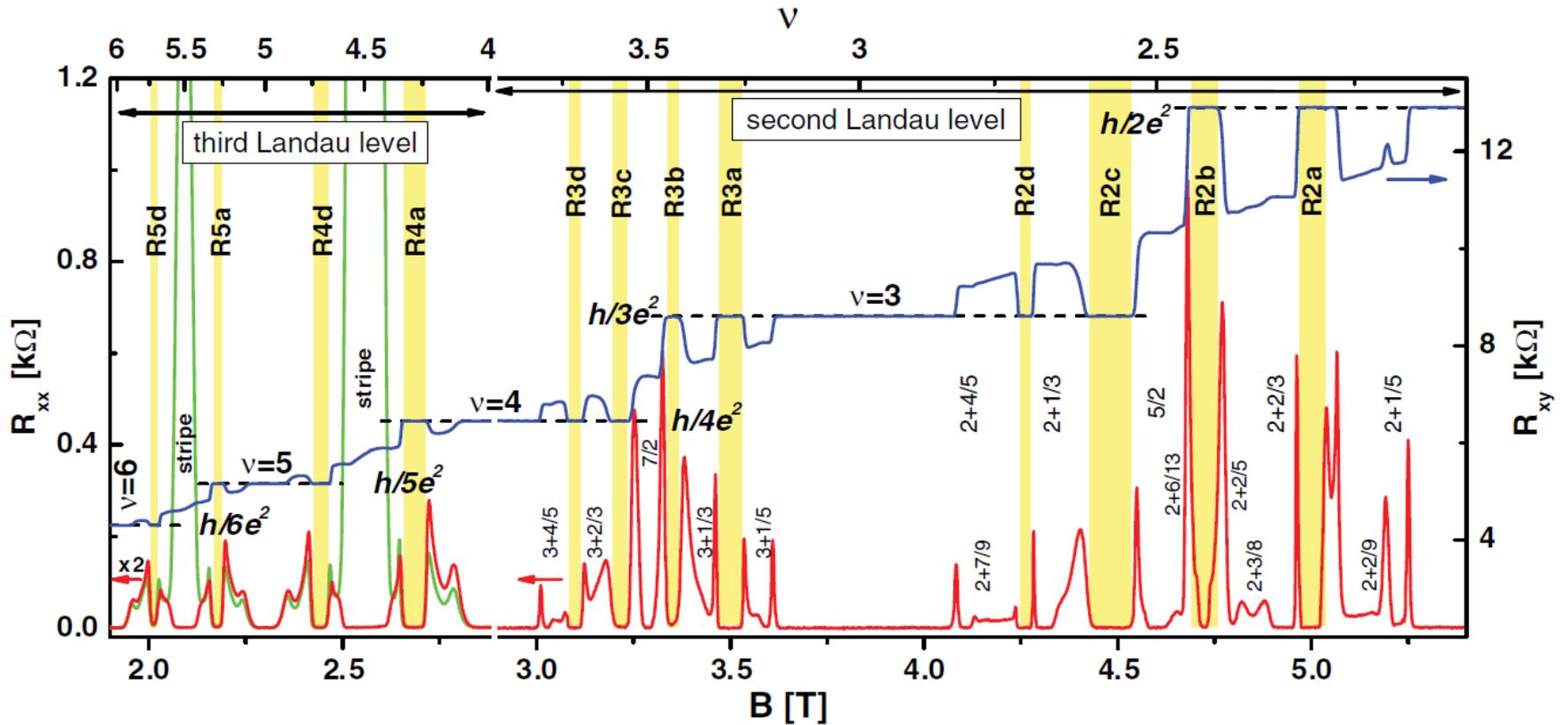
# その後のデータ



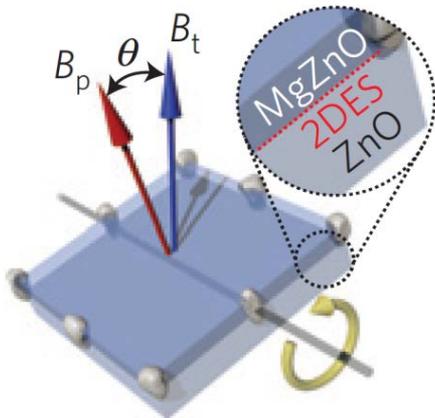
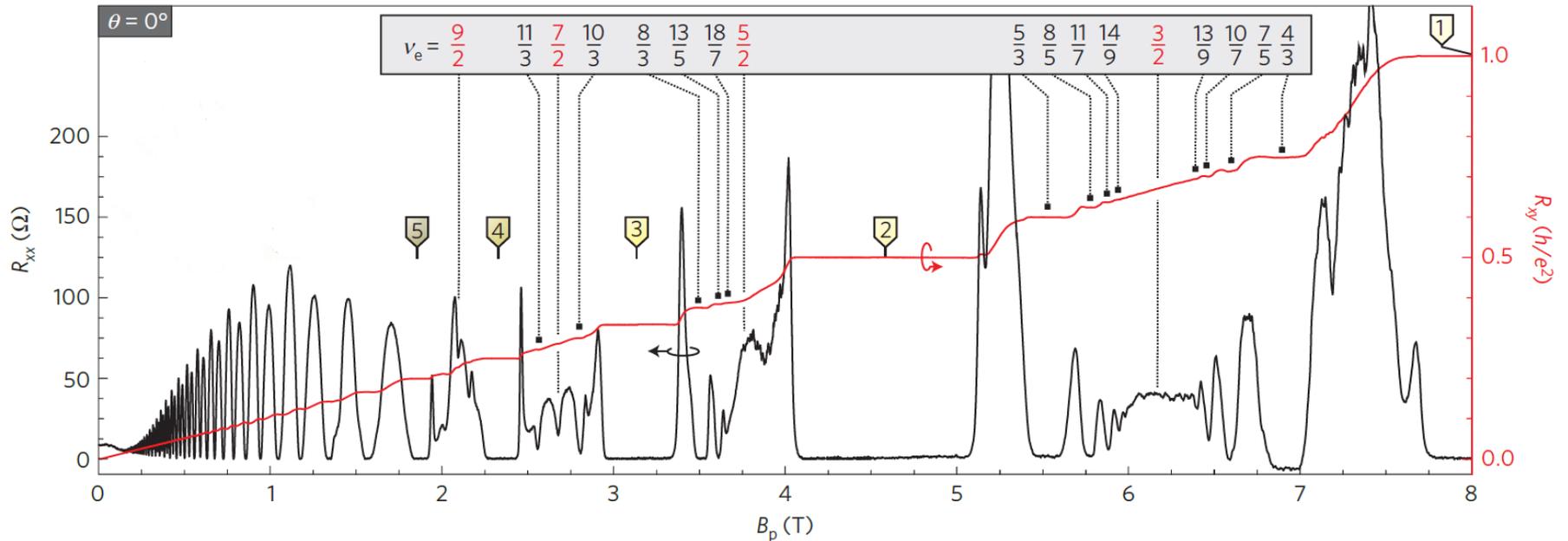
# 最新データ: GaAs系



# 最新データ: GaAs系



# 最新データ: ZnO系



# 2次元物質グラフェン

*“for groundbreaking experiments regarding the two-dimensional material graphene”* (Physics, 2010)

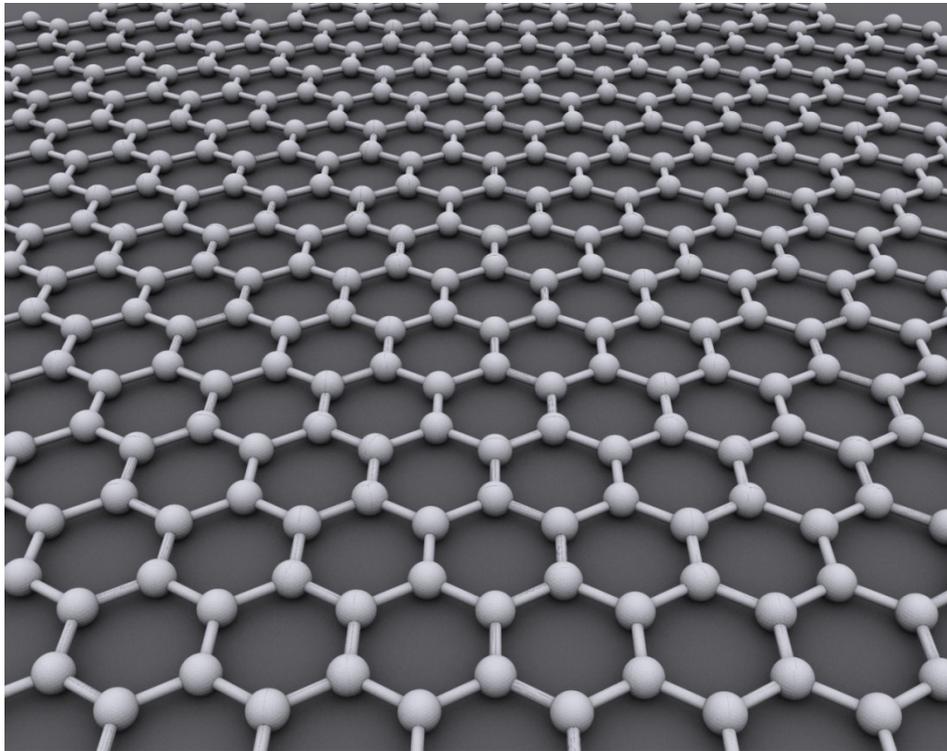


Geim



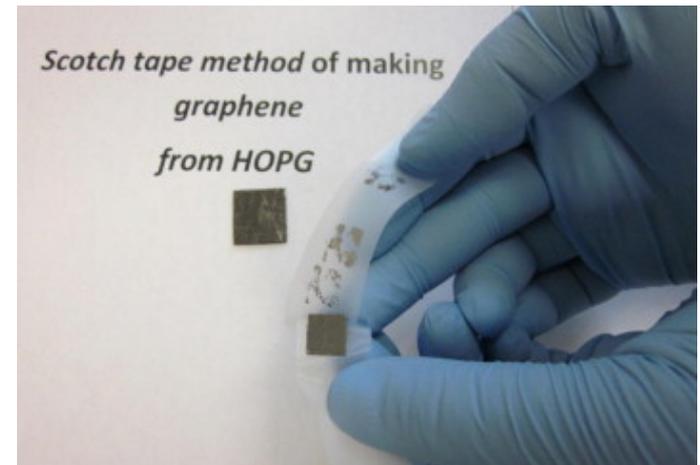
Novoselov

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from Wikipedia

*“Friday evening” experiment*



Prog. Mat. Sci. **56**, 1178 (2011) Singh *et al.*

# 2次元物質グラフェン

*“for groundbreaking experiments regarding the two-dimensional material graphene” (Physics, 2010)*



Geim

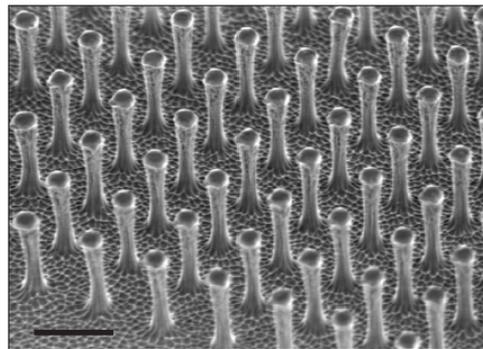


Novoselov

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授賞発表直後の電話インタビュー

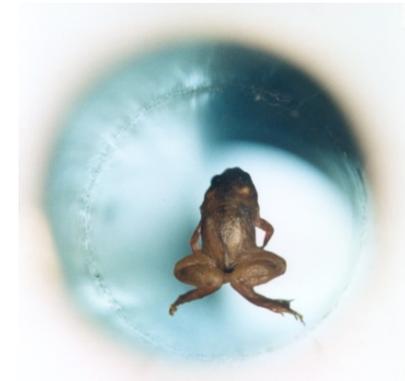
“I'm fine, I slept well. I didn't expect the Nobel Prize this year.”  
“Oh shit! I will not win many more prizes.”



**Gecko tape**

Nature Mat. **2**, 461 (2003) Geim *et al.*

**Levitating frog**  
Ig Nobel (with Berry, 2000)



© Radboud University

# 2次元物質グラフェン

“for groundbreaking experiments regarding the two-dimensional material graphene” (Physics, 2010)

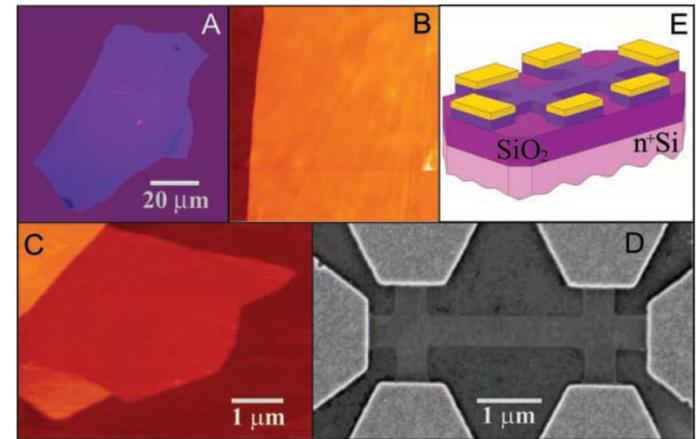
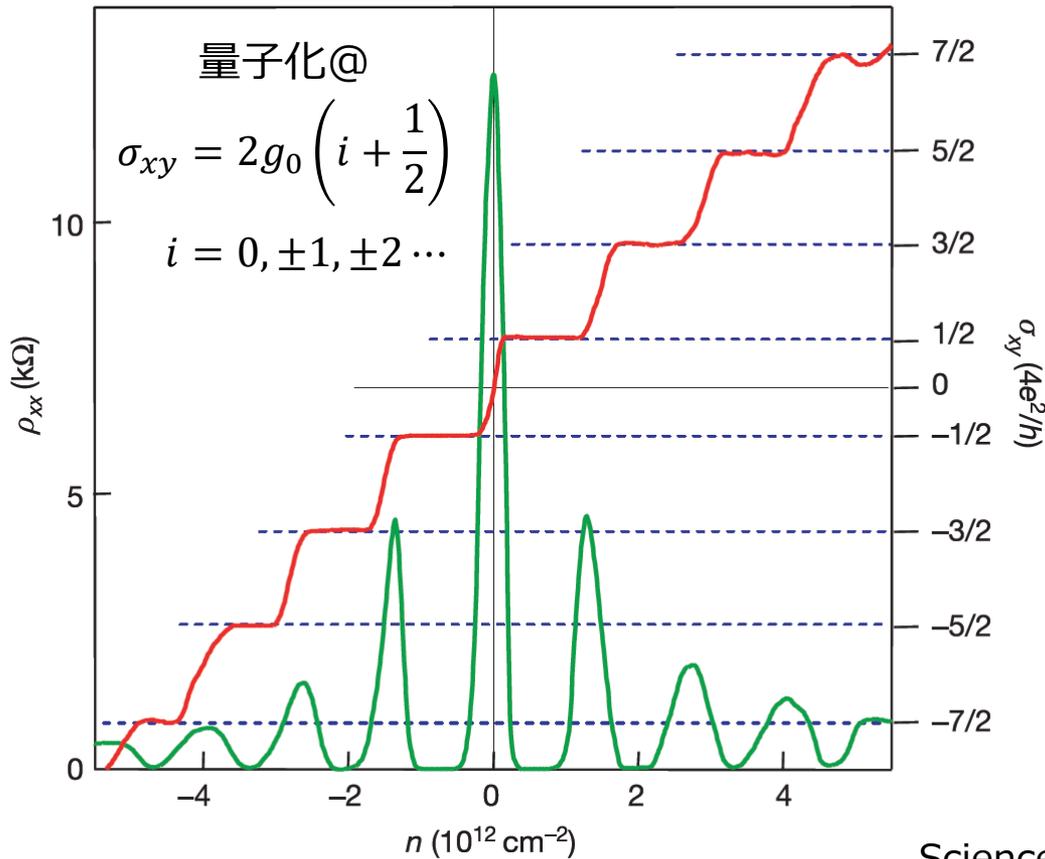


Geim



Novoselov

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Science **306**, 666 (2004) Novoselov, Geim *et al.*

Nature **438**, 197 (2005) Novoselov, Geim *et al.*

# 参考書

- J. H. Davies (1997)
  - **“The Physics of Low-Dimensional Semiconductors”** (邦訳あり)
- S. Datta (1997)
  - **“Electronic Transport in Mesoscopic Systems”** (邦訳あり)
- 吉岡大二郎 (1998)
  - **“量子ホール効果”** (英訳あり)